Solutions Manual

Second Edition

Discrete-Time Control Systems

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Preface

This solutions manual for Discrete-Time Control Systems, second edition, contains solutions to all B problems (unsolved problems in the text).

All the materials in the text may be covered in two quarters. In a semester course, the instructor will have some flexibility in choosing the subjects to be covered. If the student has an adequate background in the vector-matrix analysis, then by leaving approximately 10 percent of the text material to the student's self study, most of the important subjects of the text may be covered in one semester. In a quarter course, a good part of the first six chapters may be covered.

Katsuhiko Ogata

CHAPTER 2

B-2-1.

$$X(z) = \mathcal{Z}\left[\frac{1}{a}(1 - e^{-at})\right] = \frac{1}{a}\left(\frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT}z^{-1}}\right)$$

$$= \frac{1}{a}\,\frac{z^{-1}(1 - e^{-aT})}{(1 - z^{-1})(1 - e^{-aT}z^{-1})}$$

B-2-2.

Method 1: Noting that

$$\mathcal{Z}[k] = \frac{z^{-1}}{(1 - z^{-1})^2}, \qquad \mathcal{Z}[k^2] = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$$

it can be expected that $\mathcal{Z}[k^3]$ will involve a term $(1 - z^{-1})^4$ in the denominator. Since

$$\mathcal{Z}[k^3] = \sum_{k=0}^{\infty} k^3 z^{-k} = z^{-1} + 2^3 z^{-2} + 3^3 z^{-3} + 4^3 z^{-4} + \cdots$$

$$= z^{-1} + 8z^{-2} + 27z^{-3} + 64z^{-4} + \cdots$$

and

$$(z^{-1} + 8z^{-2} + 27z^{-3} + 64z^{-4} + \cdots)(1 - z^{-1})^4$$

$$= (z^{-1} + 7z^{-2} + 19z^{-3} + 37z^{-4} + \cdots)(1 - z^{-1})^3$$

$$= (z^{-1} + 6z^{-2} + 12z^{-3} + 18z^{-4} + \cdots)(1 - z^{-1})^2$$

$$= (z^{-1} + 5z^{-2} + 6z^{-3} + 6z^{-4} + \cdots)(1 - z^{-1})$$

$$= z^{-1} + 4z^{-2} + z^{-3}$$

we find

$$\mathcal{Z}[k^3] = \frac{z^{-1} + 4z^{-2} + z^{-3}}{(1 - z^{-1})^4} = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$$

Method 2:

$$\mathcal{Z}[k^3] = \mathcal{Z}[k \cdot k^2] = -z\frac{dX(z)}{dz}$$

where

$$X(z) = \mathcal{Z}[k^2] = \frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$$

Since

$$\frac{dX(z)}{dz} = \frac{d}{dz}\left[\frac{z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}\right] = -\frac{z^{-2} + 4z^{-3} + z^{-4}}{(1 - z^{-1})^4}$$

we have

$$\mathcal{Z}\left[k^3\right] = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$$

B-2-3.

Method 1: Noting that

$$\mathcal{Z}\left[te^{-at}\right] = \frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$$

we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \mathcal{Z}\left[-\frac{\partial}{\partial a}\,te^{-at}\right] = \frac{\partial}{\partial a}\left[\frac{-\,Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}\right]$$

$$= \frac{T^2e^{-aT}(1 + e^{-aT}z^{-1})z^{-1}}{(1 - e^{-aT}z^{-1})^3}$$

Method 2:

$$\mathcal{Z}\left[t^2e^{-at}\right] = \sum_{k=0}^{\infty} (kT)^2e^{-akT}z^{-k}$$

$$= T^2\left[(e^{aT}z)^{-1} + 4(e^{aT}z)^{-2} + 9(e^{aT}z)^{-3} + 16(e^{aT}z)^{-4} + \cdots\right]$$

Referring to Problem A-2-2, we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \frac{T^2e^{-aT}z^{-1}(1 + e^{-aT}z^{-1})}{(1 - e^{-aT}z^{-1})^3}$$

B-2-4.

$$\mathcal{Z}\left[x(k)\right] = \mathcal{Z}\left[9k(2^{k-1})\right] - \mathcal{Z}\left[2^k\right] + \mathcal{Z}\left[3\right]$$

$$= \frac{9z^{-1}}{(1 - 2z^{-1})^2} - \frac{1}{1 - 2z^{-1}} + \frac{3}{1 - z^{-1}}$$

$$= \frac{2 + z^{-2}}{(1 - 2z^{-1})^2(1 - z^{-1})}$$

B-2-5. Referring to Problem A-2-4, we have

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}}\,X(z)$$

where

$$X(z) = \mathcal{Z}\left[a^h\right] = \frac{1}{1 - az^{-1}}$$

Hence

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}} \cdot \frac{1}{1 - az^{-1}}$$

B-2-6.

$$\mathcal{Z}\left[ka^{k-1}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a} a^k\right] = \frac{\partial}{\partial a}\mathcal{Z}\left[a^k\right] = \frac{\partial}{\partial a}\left(\frac{1}{1 - az^{-1}}\right)$$

$$= \frac{z^{-1}}{(1 - az^{-1})^2} = \frac{z}{(z - a)^2}$$

$$\mathcal{Z}\left[k(k - 1)a^{k-2}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a}(ka^{k-1})\right] = \frac{\partial}{\partial a}\left[\frac{z}{(z - a)^2}\right]$$

$$= \frac{2z}{(z - a)^3} = \frac{(2!)z}{(z - a)^3}$$

$$\mathcal{Z}\left[k(k - 1) \cdots (k - h + 1)a^{k-h}\right]$$

$$= \mathcal{Z}\left[\frac{\partial}{\partial a} k(k - 1) \cdots (k - h + 2)a^{k-h+1}\right]$$

$$= \frac{\partial}{\partial a} X(z, a)$$

where

$$X(z, a) = \frac{(h - 1)!\, z}{(z - a)^h}$$

Hence

$$\mathcal{Z}\left[k(k - 1) \cdots (k - h + 1)a^{k-h}\right] = \frac{\partial}{\partial a} X(z, a)$$

$$= (h - 1)!\, zh(z - a)^{-h-1} = \frac{h!\, z}{(z - a)^{h+1}}$$

B-2-7. From Figure 2-8 we have

$$x(0) = 0, \qquad x(1) = 0, \qquad x(2) = 0, \qquad x(3) = \frac{1}{3}$$

$$x(4) = \frac{2}{3}, \qquad x(k) = 1 \quad \text{for } k = 5, 6, 7, \ldots$$

Then

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-k}$$

$$= \frac{1}{3} z^{-3} + \frac{2}{3} z^{-4} + z^{-5} + z^{-6} + z^{-7} + \cdots$$

$$= \frac{1}{3} (z^{-3} + 2z^{-4}) + \frac{z^{-5}}{1 - z^{-1}}$$

$$= \frac{1}{3} \frac{z^{-3} + z^{-4} + z^{-5}}{1 - z^{-1}}$$

B-2-8. By dividing both numerator and denominator by z^4, we have

$$X(z) = 5 + 4z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

This last equation is already in the form of a power series in z^{-1}. By inspection, we have

$$x(0) = 5$$
$$x(1) = 4$$
$$x(2) = 3$$
$$x(3) = 2$$
$$x(4) = 1$$
$$x(k) = 0 \qquad k \geqslant 5$$

Note that the given X(z) is the z transform of a signal of finite length.

B-2-9.

1. Partial-fraction-expansion method:

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2} = \frac{z(0.5z - 1)}{(z - 0.5)(z - 0.8)^2}$$

Hence,

$$\frac{X(z)}{z} = - \frac{8.3333}{z - 0.5} + \frac{8.3333}{z - 0.8} - \frac{2}{(z - 0.8)^2}$$

or

$$X(z) = - \frac{8.3333}{1 - 0.5z^{-1}} + \frac{8.3333}{1 - 0.8z^{-1}} - \frac{2z^{-1}}{(1 - 0.8z^{-1})^2}$$

Thus,

$$x(k) = -8.3333(0.5)^k + 8.3333(0.8)^k - 2k(0.8)^{k-1}, \qquad k = 0, 1, 2, \ldots$$

2. Computational solution with MATLAB:

```
»% MATLAB Program for Problem B-2-9
»
»% ----- Finding inverse z transform -----
»
»num = [0   0.5   -1   0];
»den = [1   -2.1   1.44   -0.32];
»u = [1   zeros(1,40)];
»x = filter(num,den,u)

x =

 Columns 1 through 7

       0    0.5000    0.0500   -0.6150   -1.2035   -1.6257   -1.8778

 Columns 8 through 14

   -1.9875   -1.9899   -1.9177   -1.7977   -1.6505   -1.4910   -1.3296

 Columns 15 through 21

   -1.1733   -1.0265   -0.8915   -0.7694   -0.6606   -0.5645   -0.4804

 Columns 22 through 28

   -0.4074   -0.3443   -0.2902   -0.2440   -0.2046   -0.1713   -0.1431

 Columns 29 through 35

   -0.1193   -0.0993   -0.0825   -0.0685   -0.0568   -0.0470   -0.0389

 Columns 36 through 41

   -0.0321   -0.0265   -0.0219   -0.0180   -0.0148   -0.0122
```

B-2-10.

$$x(0) = \lim_{z\to\infty} X(z) = \lim_{z\to\infty} \frac{z^{-1}}{(1 - z^{-1})(1 + 1.3z^{-1} + 0.4z^{-2})} = 0$$

$$x(\infty) = \lim_{z\to 1} \left[(1 - z^{-1})X(z)\right] = \lim_{z\to 1} \frac{z^{-1}}{1 + 1.3z^{-1} + 0.4z^{-2}} = \frac{1}{2.7}$$

Notice that

$$X(z) = \frac{z^{-1}}{(1 - z^{-1})(1 + 1.3z^{-1} + 0.4z^{-2})}$$

$$= \frac{z^2}{(z - 1)(z + 0.8)(z + 0.5)}$$

$$= \frac{1}{2.7}\left(\frac{z}{z - 1} - \frac{4z}{z + 0.8} + \frac{3z}{z + 0.5}\right)$$

Hence

$$x(k) = \frac{1}{2.7}\left[1 - 4(-0.8)^k + 3(-0.5)^k\right]$$

B-2-11.

1. Inversion integral method:

$$X(z) = \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}} = \frac{z^2 + z - 1}{(z - 1)z}$$

Hence

$$X(z)z^{k-1} = \frac{(z^2 + z - 1)z^{k-1}}{(z - 1)z}$$

For k = 0:

$$X(z)z^{k-1} = \frac{z^2 + z - 1}{(z - 1)z^2}$$

Thus,

$$x(0) = \left[\text{residue of } \frac{z^2 + z - 1}{(z - 1)z^2} \text{ at pole } z = 1\right]$$

$$+ \left[\text{residue of } \frac{z^2 + z - 1}{(z - 1)z^2} \text{ at double pole } z = 0\right]$$

$$= \lim_{z\to 1}\left[(z - 1)\frac{z^2 + z - 1}{(z - 1)z^2}\right]$$

$$+ \frac{1}{(2 - 1)!}\lim_{z\to 0}\frac{d}{dz}\left[z^2\frac{z^2 + z - 1}{(z - 1)z^2}\right] = 1 + 0 = 1$$

For k = 1:

$$X(z)z^{k-1} = \frac{z^2 + z - 1}{(z - 1)z}$$

$$x(1) = \left[\text{residue of } \frac{z^2 + z - 1}{(z - 1)z} \text{ at pole } z = 1\right]$$

$$+ \left[\text{residue of } \frac{z^2 + z - 1}{(z - 1)z} \text{ at pole } z = 0\right]$$

$$= \lim_{z \to 1} \left[(z - 1)\frac{z^2 + z - 1}{(z - 1)z}\right] + \lim_{z \to 0} \left[z \frac{z^2 + z - 1}{(z - 1)z}\right]$$

$$= 1 + 1 = 2$$

For k = 2, 3, 4, ...:

$$X(z)z^{k-1} = \frac{(z^2 + z - 1)z^{k-2}}{z - 1}$$

Hence

$$x(k) = \left[\text{residue of } \frac{(z^2 + z - 1)z^{k-2}}{z - 1} \text{ at pole } z = 1\right]$$

$$= \lim_{z \to 1} \left[(z - 1)\frac{(z^2 + z - 1)z^{k-2}}{z - 1}\right] = 1$$

Therefore,

$$x(0) = 1$$

$$x(1) = 2$$

$$x(k) = 1 \qquad \text{for } k = 2, 3, 4, \ldots$$

2. Computational solution with MATLAB:

```
»% MATLAB Program for Problem B-2-11
»
»% ----- Finding inverse z transform -----
»
»num = [1   1   -1];
»den = [1   -1   0];
»u = [1   zeros(1,40)];
»x = filter(num,den,u)
```

```
x =

  Columns 1 through 12

    1    2    1    1    1    1    1    1    1    1    1    1

  Columns 13 through 24

    1    1    1    1    1    1    1    1    1    1    1    1

  Columns 25 through 36

    1    1    1    1    1    1    1    1    1    1    1    1

  Columns 37 through 41

    1    1    1    1    1
```

B-2-12.

$$X(z) = \frac{z^{-3}}{(1 - z^{-1})(1 - 0.2z^{-1})} = \frac{1}{z(z - 1)(z - 0.2)}$$

$$= \frac{5}{z} + \frac{1.25}{z - 1} - \frac{6.25}{z - 0.2}$$

Hence

$$x(k) = 5\,\delta_0(k - 1) + 1.25 - 6.25(0.2)^{k-1} \qquad \text{for } k = 1, 2, 3, \ldots$$

That is,

$$x(k) = 0 \qquad \text{for } k = 0, 1, 2$$

$$= 1.25(1 - 0.2^{k-2}) \qquad \text{for } k = 3, 4, 5, \ldots$$

B-2-13.

$$X(z) = \frac{1 + 6z^{-2} + z^{-3}}{(1 - z^{-1})(1 - 0.2z^{-1})} = \frac{z^3 + 6z + 1}{z(z - 1)(z - 0.2)}$$

$$X(z)z^{k-1} = \frac{(z^3 + 6z + 1)z^{k-1}}{z(z - 1)(z - 0.2)}$$

For k = 0:

$$X(z)z^{k-1} = \frac{z^3 + 6z + 1}{z^2(z - 1)(z - 0.2)}$$

Hence

$$x(0) = \left[\text{residue of } \frac{z^3 + 6z + 1}{z^2(z - 1)(z - 0.2)} \text{ at double pole } z = 0\right]$$

$$+ \left[\text{residue of } \frac{z^3 + 6z + 1}{z^2(z - 1)(z - 0.2)} \text{ at pole } z = 1\right]$$

$$+ \left[\text{residue of } \frac{z^3 + 6z + 1}{z^2(z - 1)(z - 0.2)} \text{ at pole } z = 0.2\right]$$

$$= \frac{1}{(2 - 1)!} \lim_{z \to 0} \frac{d}{dz}\left[\frac{z^3 + 6z + 1}{(z - 1)(z - 0.2)}\right]$$

$$+ \lim_{z \to 1}\left[\frac{z^3 + 6z + 1}{z^2(z - 0.2)}\right] + \lim_{z \to 0.2}\left[\frac{z^3 + 6z + 1}{z^2(z - 1)}\right]$$

$$= 60 + 10 - 69 = 1$$

For k = 1:

$$X(z)z^{k-1} = \frac{z^3 + 6z + 1}{z(z - 1)(z - 0.2)}$$

Hence

$$x(1) = \left[\text{residue of } \frac{z^3 + 6z + 1}{z(z - 1)(z - 0.2)} \text{ at pole } z = 0\right]$$

$$+ \left[\text{residue of } \frac{z^3 + 6z + 1}{z(z - 1)(z - 0.2)} \text{ at pole } z = 1\right]$$

$$+ \left[\text{residue of } \frac{z^3 + 6z + 1}{z(z - 1)(z - 0.2)} \text{ at pole } z = 0.2\right]$$

$$= \lim_{z \to 0}\left[\frac{z^3 + 6z + 1}{(z - 1)(z - 0.2)}\right] + \lim_{z \to 1}\left[\frac{z^3 + 6z + 1}{z(z - 0.2)}\right]$$

$$+ \lim_{z \to 0.2}\left[\frac{z^3 + 6z + 1}{z(z - 1)}\right] = 5 + 10 - 13.8 = 1.2$$

For k = 2, 3, 4, ...:

$$X(z)z^{k-1} = \frac{(z^3 + 6z + 1)z^{k-2}}{(z - 1)(z - 0.2)}$$

$$x(k) = \left[\text{residue of } \frac{(z^3 + 6z + 1)z^{k-2}}{(z - 1)(z - 0.2)} \text{ at pole } z = 1\right]$$

$$+ \left[\text{residue of } \frac{(z^3 + 6z + 1)z^{k-2}}{(z - 1)(z - 0.2)} \text{ at pole } z = 0.2\right]$$

$$= \lim_{z \to 1} \left[\frac{(z^3 + 6z + 1)z^{k-2}}{z - 0.2} \right] + \lim_{z \to 0.2} \left[\frac{(z^3 + 6z + 1)z^{k-2}}{z - 1} \right]$$

$$= 10 - 2.76(0.2)^{k-2}$$

In summarizing, we have

$$x(0) = 1$$

$$x(1) = 1.2$$

$$x(k) = 10 - 2.76(0.2)^{k-2} \qquad \text{for } k = 2, 3, 4, \ldots$$

<u>B-2-14</u>.

1. <u>Direct division method</u>:

$$X(z) = \frac{z^{-1} - z^{-3}}{1 + 2z^{-2} + z^{-4}} = z^{-1} - 3z^{-3} + 5z^{-5} - 7z^{-7} + \ldots$$

Hence

$$x(0) = 0, \quad x(1) = 1, \quad x(2) = 0, \quad x(3) = -3$$

$$x(4) = 0, \quad x(5) = 5, \quad x(6) = 0, \quad x(7) = -7, \ldots$$

2. <u>Computational solution with MATLAB</u>:

```
»% MATLAB Program for Problem B-2-14
»
»% ----- Finding inverse z transform -----
»
»num = [0   1   0   -1   0];
»den = [1   0   2   0   1];
»u = [1   zeros(1,40)];
»x = filter(num,den,u)
```

```
x =

 Columns 1 through 12

   0   1   0  -3   0   5   0  -7   0   9   0  -11

 Columns 13 through 24

   0   13   0  -15   0   17   0  -19   0   21   0  -23
```

Columns 25 through 36

0 25 0 -27 0 29 0 -31 0 33 0 -35

Columns 37 through 41

0 37 0 -39 0

B-2-15.

$$X(z) = \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z^2}$$

$$X(z)z^{k-1} = \frac{(0.368z^2 + 0.478z + 0.154)z^{k-1}}{(z-1)z^2}$$

For k = 0:

$$X(z)z^{k-1} = \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z^3}$$

$$x(0) = \left[\text{residue of } \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z^3} \text{ at triple pole } z = 0\right]$$

$$+ \left[\text{residue of } \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z^3} \text{ at pole } z = 1\right]$$

$$= \frac{1}{(3-1)!} \lim_{z \to 0} \frac{d^2}{dz^2}\left[\frac{0.368z^2 + 0.478z + 0.154}{z-1}\right]$$

$$+ \lim_{z \to 1}\left[\frac{0.368z^2 + 0.478z + 0.154}{z^3}\right] = -1 + 1 = 0$$

For k = 1:

$$X(z)z^{k-1} = \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z^2}$$

$$x(1) = \frac{1}{(2-1)!} \lim_{z \to 0} \frac{d}{dz}\left[\frac{0.368z^2 + 0.478z + 0.154}{z-1}\right]$$

$$+ \lim_{z \to 1}\left[\frac{0.368z^2 + 0.478z + 0.154}{z^2}\right] = -0.632 + 1 = 0.368$$

For k = 2:

$$X(z)z^{k-1} = \frac{0.368z^2 + 0.478z + 0.154}{(z-1)z}$$

$$x(2) = \lim_{z \to 0} \left[\frac{0.368z^2 + 0.478z + 0.154}{z - 1} \right] + \lim_{z \to 1} \left[\frac{0.368z^2 + 0.478z + 0.154}{z} \right] = -0.154 + 1 = 0.846$$

For $k = 3, 4, 5, \ldots$:

$$X(z)z^{k-1} = \frac{(0.368z^2 + 0.478z + 0.154)z^{k-3}}{z - 1}$$

$$x(k) = \lim_{z \to 1} \left[(0.368z^2 + 0.478z + 0.154)z^{k-3} \right] = 1$$

In summary, we have

$$x(0) = 0$$

$$x(1) = 0.368$$

$$x(2) = 0.846$$

$$x(k) = 1 \qquad \text{for } k = 3, 4, 5, \ldots$$

B-2-16.

Case 1:

$$u(k) = 1 \qquad \text{for } k = 0, 1, 2, \ldots$$
$$= 0 \qquad \text{for } k < 0$$

The z transform of the given difference equation is

$$z^2X(z) - 1.3zX(z) + 0.4X(z) = U(z) = \frac{z}{z - 1}$$

or

$$X(z) = \frac{z}{(z - 0.8)(z - 0.5)(z - 1)} = -\frac{16.6666z}{z - 0.8} + \frac{6.6666z}{z - 0.5} + \frac{10z}{z - 1}$$

Hence

$$x(k) = -16.6666(0.8)^k + 6.6666(0.5)^k + 10 \qquad \text{for } k = 0, 1, 2, \ldots$$

Case 2:

$$u(0) = 1$$

$$u(k) = 0 \qquad \text{for } k \neq 0$$

The z transform of the given difference equation is

$$(z^2 - 1.3z + 0.4)X(z) = U(z) = 1$$

or

$$X(z) = \frac{1}{(z - 0.8)(z - 0.5)} = \frac{3.3333}{z - 0.8} - \frac{3.3333}{z - 0.5}$$

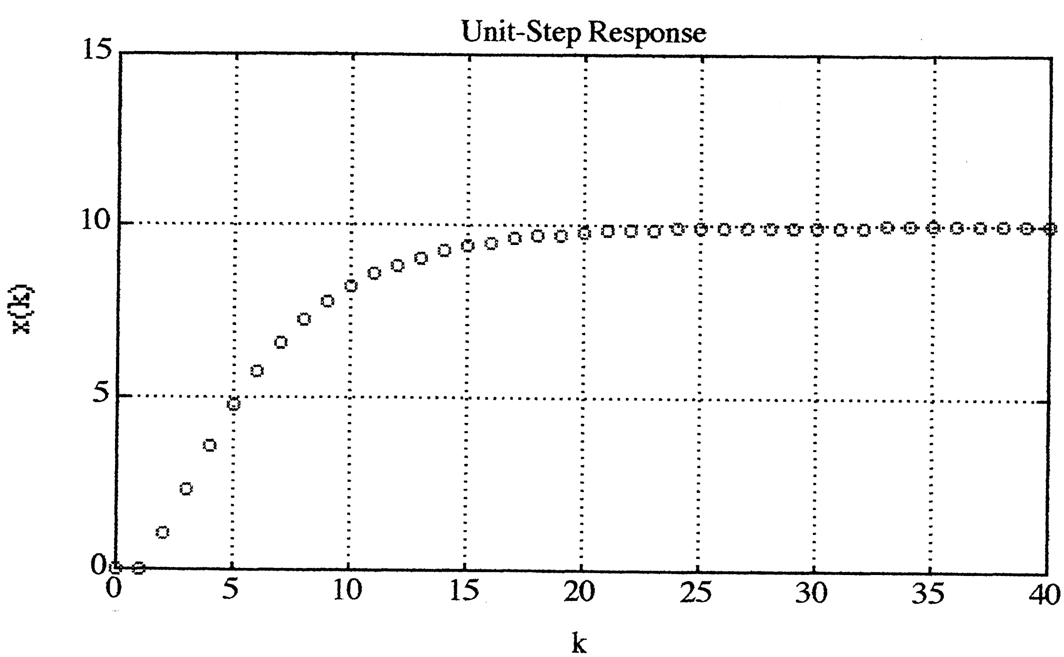
Hence,

$$x(0) = 0$$

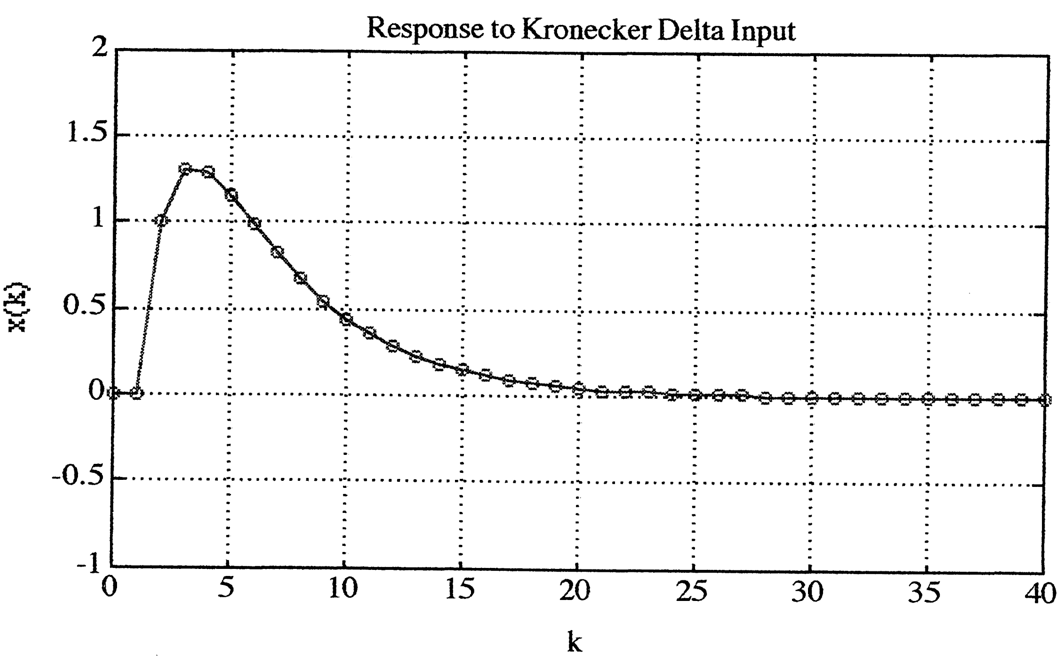
$$x(k) = 3.3333(0.8)^{k-1} - 3.3333(0.5)^{k-1}, \quad \text{for } k = 1, 2, 3, \ldots$$

<u>Computational solution with MATLAB</u>:

```
»% MATLAB Program for Problem B-2-16 (Part 1)
»
»% ----- Unit-step response -----
»
»num = [0   0   1];
»den = [1   -1.3   0.4];
»u = ones(1,41);
»v = [0   40   0   15];
»axis(v);
»k = 0:40;
»x = filter(num,den,u);
»plot(k,x,'o')
»grid
»title('Unit-Step Response')
»xlabel('k')
»ylabel('x(k)')
```



```
»% MATLAB Program for Problem B-2-16 (Part 2)
»
»% ----- Response to Kronecker delta input -----
»
»num =[0  0  1];
»den = [1  -1.3  0.4];
»u = [1  zeros(1,40)];
»v = [0  40  -1  2];
»axis(v);
»k = 0:40;
»x = filter(num,den,u);
»plot(k,x,'o',k,x,'-')
»grid
»title('Response to Kronecker Delta Input')
»xlabel('k')
»ylabel('x(k)')
»
```



B-2-17.

$$x(k+2) - x(k+1) + 0.25x(k) = u(k+2)$$

The z transform of this difference equation is

$$\left[z^2X(z) - z^2x(0) - zx(1)\right] - \left[zX(z) - zx(0)\right] + 0.25X(z)$$
$$= z^2U(z) - z^2u(0) - zu(1)$$

Substituting the initial data into this last equation, we get

$$(z^2 - z + 0.25)X(z) = \frac{z^3}{z-1}$$

or

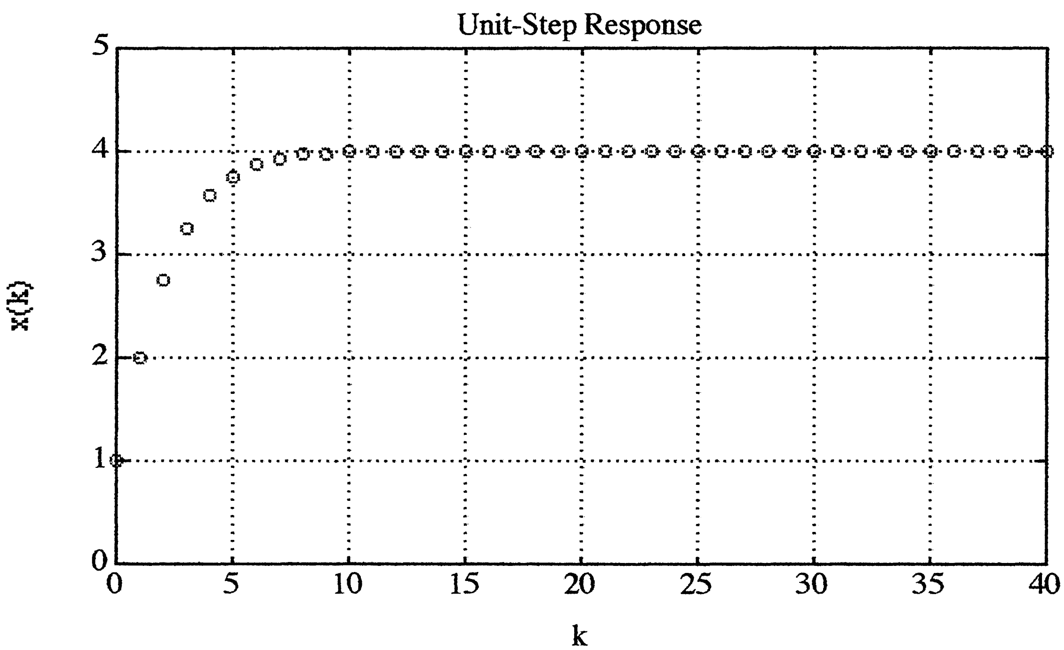
$$X(z) = \frac{z^3}{(z-1)(z^2 - z + 0.25)}$$
$$= \frac{4z}{z-1} - \frac{3z}{z-0.5} - \frac{0.5z}{(z-0.5)^2}$$

Hence

$$x(k) = 4 - (3+k)(0.5)^k \qquad \text{for } k = 0, 1, 2, \ldots$$

Computational solution with MATLAB:

```
»% MATLAB Program for Problem B-2-17
»
»% ----- Unit-step repone -----
»
»num = [1  0  0];
»den = [1  -1  0.25];
»u = ones(1,41);
»v = [0  40  0  5];
»axis(v);
»k = 0:40;
»x = filter(num,den,u);
»plot(k,x,'o')
»grid
»title('Unit-Step Response')
»xlabel('k')
»ylabel('x(k)')
```



B-2-18.

$$x(k + 2) - 1.3679x(k + 1) + 0.3679x(k) = 0.3679u(k + 1) + 0.2642u(k)$$

The z transform of this equation is

$$z^2X(z) - z^2x(0) - zx(1) - 1.3679\left[zX(z) - zx(0)\right] + 0.3679X(z)$$
$$= 0.3679\left[zU(z) - zu(0)\right] + 0.2642\,U(z)$$

Noting that $x(0) = 0$ and $x(1) = 0.5820$, we have

$$(z^2 - 1.3679z + 0.3679)X(z) = (0.3679z + 0.2642)U(z)$$

or

$$\frac{X(z)}{U(z)} = \frac{0.3679z^{-1} + 0.2642z^{-2}}{1 - 1.3679z^{-1} + 0.3679z^{-2}}$$

Since

$$U(z) = 1.5820 - 0.5820z^{-1}$$

we have

$$X(z) = \frac{0.5820z^{-1} + 0.2038z^{-2} - 0.1538z^{-3}}{1 - 1.3679z^{-1} + 0.3679z^{-2}}$$

$$= 0.5820z^{-1} + z^{-2} + z^{-3} + \cdots$$

Hence

$$x(0) = 0$$

$$x(1) = 0.5820$$

$$x(k) = 1 \qquad \text{for } k = 2, 3, 4, \ldots$$

Computational solution with MATLAB:

```
»% MATLAB Program for Problem B-2-18
»
»% ----- Response to arbitrary inpt -----
»
»num = [0   0.3679   0.2642];
»den = [1   -1.3679   0.3679];
»u = [1.582   -0.5820   zeros(1,24)];
»v = [0   25   0   2];
»axis(v);
»k = 0;25;
»x = filter(num,den,u);
»plot(k,x,'o')
»grid
»title('Reponse of System to Arbitrarily Specified Input')
»xlabel('k')
»ylabel('x(k)')
```