

SOLUTIONS MANUAL

POWER SYSTEMS ANALYSIS

SECOND EDITION

**ARTHUR R. BERGEN
VIJAY VITTAL**

**PART I. SOLUTIONS TO PROBLEM SETS
PART II. DISCUSSION OF SOLUTIONS TO
DESIGN EXERCISES**

PRENTICE HALL, Upper Saddle River, New Jersey 07458

Associate Editor: Alice Dworkin
Special Projects Manager: Barbara A. Murray
Production Editor: Meaghan Forbes
Supplement Cover Manager: Paul Gourhan
Supplement Cover Designer: Liz Nemeth
Manufacturing Buyer: Pat Brown

© 2000 by Prentice Hall
Upper Saddle River, NJ 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-022561-4

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada, Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall (Singapore) Pte. Ltd.
Prentice-Hall of Japan, Inc., Tokyo
Editora Prentice-Hall do Brazil, Ltda., Rio de Janeiro

CONTENTS

PART I.	SOLUTIONS TO PROBLEM SETS	1
PART II.	DISCUSSION OF SOLUTIONS TO DESIGN EXERCISES	87

PART I. SOLUTIONS TO PROBLEM SETS

$$\underline{2.1} \quad v = \sqrt{2} \times 120 \cos(\omega t + 30^\circ) \Rightarrow V = 120 \angle 30^\circ \text{ V}$$

$$i = \sqrt{2} \times 10 \cos(\omega t - 30^\circ) \Rightarrow I = 10 \angle -30^\circ \text{ A}$$

$$(a) \quad p(t) = |V||I| [\cos \phi + \cos(2\omega t + \angle V + \angle I)]$$

$$= 600 + 1200 \cos 2\omega t \text{ W}$$

$$S = VI^* = 1200 \angle 60^\circ = P + jQ \Rightarrow P = 600 \text{ W}, Q = 1039 \text{ VAR}$$

$$(b) \quad Z = V/I = 12 \angle 60^\circ = 6 + j10.39 = R + jX \Rightarrow R = 6, X = 10.39$$

$$\underline{2.2} \quad (a) \text{ Using (2.3) we find } P_{\max} = 1707 = |V||I|(\cos \phi + 1)$$

and $P_{\min} = -293 = |V||I|(\cos \phi - 1)$. Then, since $|V| = 100$, we get $|I| = 10$ and $\cos \phi = \pm 45^\circ$. Pick $\phi = 45^\circ \Rightarrow Z = 10 \angle 45^\circ = 7.07 + j7.07 = R + jX \Rightarrow R = 7.07, X = 7.07$

$$(b) \quad S = VI^* = Z|I|^2 = (7.07 + j7.07) 10^2 \Rightarrow P = 707, Q = 707$$

(c) For simplicity assume $i(t) = \sqrt{2}|I| \cos \omega t$. Then

$$p_L(t) = v_L(t) i(t) = L \frac{di}{dt} i = -2\omega L |I|^2 \cos \omega t \sin \omega t = -\omega L |I|^2 \sin 2\omega t$$

$$P_{L\max} = \omega L |I|^2 = 707 = Q. \text{ Thus } P_{L\max} = Q. \text{ The same!}$$

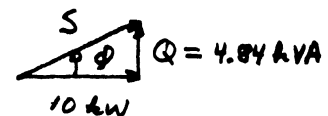
$$\underline{2.3} \quad 0.7 \text{ PF lagging} \Rightarrow \phi = 45.57^\circ, Q = 5.10 \text{ MVAR}$$

$$0.9 \text{ PF lagging} \Rightarrow \phi = 25.84^\circ, Q = 2.42 \text{ MVAR. Capacitor must supply } 5.10 - 2.42 = 2.68 \text{ MVAR.}$$

$$\underline{2.4} \quad 0.707 \text{ PF lagging} \Rightarrow S_{3\phi} = 200 + j200 \text{ kVA. Cap supplies } 50 \text{ kVAR. Resultant } S_{3\phi} = 200 + j150 \text{ kVA} \Rightarrow \text{PF} = 0.80.$$

$$|S| = \frac{|S_{3\phi}|}{3} = \frac{250 \times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}} |I| \Rightarrow |I| = 328 \text{ A}$$

$$\underline{2.5} \quad 0.9 \text{ PF lagging} \Rightarrow \phi = 25.84^\circ$$



$$(a) \quad S = 10 + j4.84 \text{ kVA}$$

$$(b) \quad 10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71 \text{ A}$$

(c) Using (2.3), (or first principles) we get

$$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84^\circ)$$

Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208 \angle 120^\circ$, $V_{bc} = 208 \angle 0^\circ$.
 Using (2.17) or Fig 2.11, $V_{an} = 120 \angle 90^\circ \Rightarrow V_{bn} = 120 \angle -30^\circ$,
 $V_{cn} = 120 \angle -150^\circ$. Using per phase analysis, $I_a = 12 \angle 105^\circ \Rightarrow$
 $I_b = 12 \angle -15^\circ$, $I_c = 12 \angle -135^\circ$.

2.7 $S = VI^* = V(YV)^* = Y^* |V|^2 = Y_C^* + Y_L^* + Y_R^*$
 $= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,
 $I_a = 0.9123 \angle -90.351^\circ$, $I_b = 0.9123 \angle -209.65^\circ$, $I_c = 0.9929 \angle 30^\circ$.

(b) Using per phase analysis $I_a = 1 \angle 0^\circ / j1.1 = 0.9091 \angle -90^\circ$,
 then, $I_b = 0.9091 \angle -210^\circ$, $I_c = 0.9091 \angle 30^\circ$.

2.9 Proceeding by analogy with 3ϕ , we note
 $E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2} E_{an} e^{j\pi/4}$.
 Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab} e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form
 a pos. seq. set of 4ϕ voltages. Doing per phase (phase a)
 analysis we have

$\frac{1}{\sqrt{2}} \angle 45^\circ \text{ (source)} \text{ --- } j1 \text{ (impedance)} \text{ --- } -j0.5 \text{ (load)} \Rightarrow I_a = \frac{\frac{1}{\sqrt{2}} \angle -45^\circ}{j0.5} = \sqrt{2} \angle -135^\circ$

Then $I_b = \sqrt{2} \angle -225^\circ$, $I_c = \sqrt{2} \angle -315^\circ$, $I_d = \sqrt{2} \angle -405^\circ$

2.10 Using per phase circuit we find $I_a = 103.8 \angle 41.5^\circ$.

$\frac{240}{\sqrt{3}} \text{ (source)} \text{ --- } Z_L \text{ (load)} \text{ --- } I_a$
 $Z_L = \frac{1}{3} \frac{1}{j\omega C} = -j0.884$

Then $|I_b| = 103.8 \text{ A}$

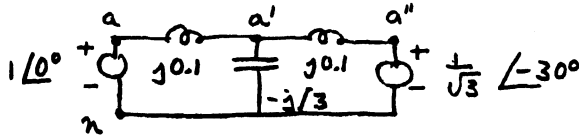
$V_{a'n} = Z_L I_a = 91.76 \angle -48.5^\circ \Rightarrow |V_{a'b'}| = \sqrt{3} \cdot 91.76 = 158.9 \text{ V}$

$S_{\text{load}} = V_{a'n} I_a^* = 9524 \angle -90^\circ$

$S_{\text{load}}^{3\phi} = 3 S_{\text{load}} = 28574 \angle -90^\circ \text{ W}$

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a'n} - V_{b'n} = \sqrt{3} V_{a'n} e^{j\pi/6} \Rightarrow V_{a'n} = \frac{1}{\sqrt{3}} V_{a''b''} e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle -30^\circ$

Per Phase Ckt

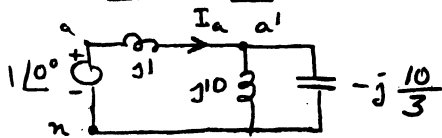


Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle -10.89^\circ \Rightarrow V_{b'n} = 0.899 \angle -130.89^\circ$ and
 $V_{c'n} = 0.899 \angle -250.89^\circ$. Then $V_{a'b'c'} = 1.557 \angle 19.11^\circ$

2.12 Assume pos. seq..

Per Phase Ckt



Combining parallel elements we have $Z_{||} = -j5$. $I_a = 0.25 \angle 90^\circ$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle 0^\circ$

$V_{a'b'c'} = 2.165 \angle 30^\circ \Rightarrow I_{cap} = 2.165 \angle 120^\circ$

$S_{load} = 3 V_{a'n} I_a^* = 0.3125 \angle -90^\circ$

2.13

(a) $V_{bc} = 208 \angle -120^\circ$, $V_{ca} = 208 \angle 120^\circ$

$V_{an} = \frac{208}{\sqrt{3}} \angle -30^\circ \Rightarrow I_a = 1.20 \angle -90^\circ$

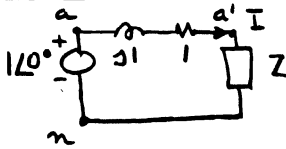
Then $I_b = 1.20 \angle -210^\circ$, $I_c = 1.20 \angle -330^\circ$

(b) $V_{bc} = 208 \angle 120^\circ$, $V_{ca} = 208 \angle -120^\circ$

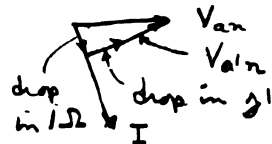
$V_{an} = \frac{208}{\sqrt{3}} \angle 30^\circ \Rightarrow I_a = 1.20 \angle -30^\circ$

$I_b = 1.20 \angle 90^\circ$, $I_c = 1.20 \angle -150^\circ$

2.14 Per Phase Ckt. Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.



I. $Z = j\omega L$



Clearly $|V_{a'n}| < |V_{an}|$

II $Z = R$



Clearly $|V_{a'n}| < |V_{an}|$

III $Z = -j \frac{1}{\omega C}$

For example $Z = -j2$

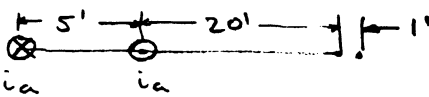
$I = \frac{1}{\sqrt{2}} \angle 45^\circ$

$V_{a'n} = \sqrt{2} \angle -45^\circ$

$|V_{a'n}| > |V_{an}|$

This is O.K.

2.15 Since $E_a + E_b + E_c = 0$, neutrals are at same potential.
 $Z = E_a / I_a = \sqrt{2} \angle 55^\circ$. For each Z , $S = VI^* = |V|^2 / Z^*$. Thus
 $S^{3\phi} = \frac{(\sqrt{2})^2 + 1^2 + 1^2}{\sqrt{2} \angle -55^\circ} = 2\sqrt{2} \angle 55^\circ$.

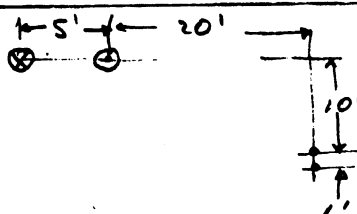
3.1  $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}, |I_a| = 100$

(Upward) flux linkages of telephone wires

$$\lambda = 2 \times 10^{-7} \left[i_a \ln \frac{26}{25} - i_a \ln \frac{21}{20} \right] = -0.00957 \times 2 \times 10^{-7} i_a$$

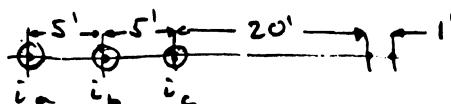
$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = \omega \times (-0.00957 \times 2 \times 10^{-7}) \times 100 \text{ V/meter}$$

Since 1 mile = 1.609 km, $|V_{tel}| = 0.116 \text{ V/mile}$

3.2 
$$\lambda = 2 \times 10^{-7} i_a \left[\ln \frac{\sqrt{25^2+11^2}}{\sqrt{25^2+10^2}} - \ln \frac{\sqrt{20^2+11^2}}{\sqrt{20^2+10^2}} \right]$$

$$= -0.066294 \times 2 \times 10^{-7} i_a$$

$$\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$$

3.3 
$$\lambda = 2 \times 10^{-7} \left[i_a \ln \frac{31}{30} + i_b \ln \frac{26}{25} + i_c \ln \frac{21}{20} \right]$$

$$= 2 \times 10^{-7} [0.0328 i_a + 0.0392 i_b + 0.0488 i_c]$$

$$V_{tel} = \omega \times 2 \times 10^{-7} [0.0328 I_a + 0.0392 I_b + 0.0488 I_c]$$

$$|V_{tel}| = 377 \times 2 \times 10^{-7} \times 100 \times |0.0328 \angle -120^\circ + 0.0392 \angle -120^\circ + 0.0488 \angle 120^\circ|$$

$$= 1048 \times 10^{-7} \text{ V/meter} = 0.1692 \text{ V/mile}$$

3.4 The telephone wires are transposed every 1000 ft.
 Cancellation occurs in all but 720' of line $\Rightarrow 0.0158 \text{ V}$.