

Chapter 1

BASIC ALGEBRAIC OPERATIONS

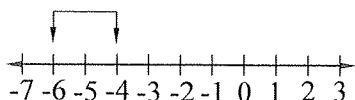
1.1 Numbers

1. The numbers -3 and 14 are integers. They are also rational numbers since they can be written

$$\text{as } \frac{-3}{1} \text{ and } \frac{14}{1}.$$

2. The absolute value of -6 and 6 and the absolute value of -7 is 7 . We write these as $|-6| = 6$ and $|-7| = 7$.

3. $-6 < -4$; -6 is to the left of -4



4. The reciprocal of $\frac{3}{2}$ is $\frac{1}{\frac{3}{2}} = 1\left(\frac{2}{3}\right) = \frac{2}{3}$.

5. 3 : integer, rational $\left(\frac{3}{1}\right)$, real

$\sqrt{-4}$: imaginary

$-\frac{\pi}{6}$: irrational, real

6. $-\sqrt{-6}$: imaginary

$-2.33 = -\frac{233}{100}$: rational, real

$\frac{\sqrt{7}}{3}$: irrational

7. $|3| = 3$, $|-4| = 4$, $\left|-\frac{\pi}{2}\right| = \frac{\pi}{2}$

8. $|-0.857| = 0.857$, $|\sqrt{2}| = \sqrt{2}$, $\left|-\frac{19}{4}\right| = \frac{19}{4}$

9. $6 < 8$

10. $7 > 5$

11. $\pi > -3.2$, $\pi = 3.14$ is greater than -3.2 .

12. $-4 < 0$ 13. $-|-3| = -3 \Rightarrow -4 < -3 = -|-3|$

14. $-\sqrt{2} > -1.42$ 15. $-\frac{1}{3} > -\frac{1}{2}$ 16. $-0.6 < 0.2$

17. The reciprocal of $3 = \frac{1}{3}$. The reciprocal of

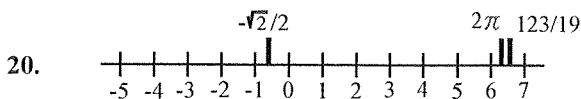
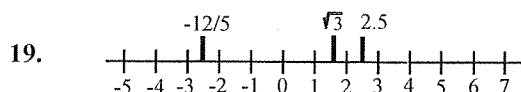
$$-\frac{4}{\sqrt{3}} \text{ is } \frac{1}{-\frac{4}{\sqrt{3}}} = -\frac{\sqrt{3}}{4}.$$

The reciprocal of $\frac{y}{b}$ is $\frac{1}{\frac{y}{b}} = \frac{b}{y}$.

18. The reciprocal of $-\frac{1}{3}$ is $\frac{1}{-\frac{1}{3}} = -3$.

The reciprocal of 0.25 is $\frac{1}{0.25} = 4$.

The reciprocal of x is $\frac{1}{x}$.



21. An absolute value is not always positive, $|0| = 0$ which is not positive.

22. Since $2.17 = \frac{217}{100}$ it is rational.

23. The reciprocal of the reciprocal of a number is the number itself.

$$\frac{1}{\frac{1}{n}} = 1 \times \frac{n}{1} = n.$$

24. $-\frac{10}{11} = -0.909090... = -0.\overline{90}$ is a rational number between -0.9 and -1.0 since $-0.9 < -0.\overline{90} < -1.0$.

25. Since $0.13 = \frac{13}{100} \cdot \frac{\frac{3}{13}}{\frac{3}{13}} = \frac{3}{\frac{300}{13}} = \frac{3}{23.0769} \dots <$

$$\frac{3}{23} = \frac{3}{23} \cdot \frac{\frac{14}{3}}{\frac{14}{3}} = \frac{14}{107.3} < \frac{14}{100} = 0.14$$

$\frac{3}{23}$ is a rational number between 0.13 and 0.14 with numerator 3 and an integer, 23 , in the denominator.

26. Yes, $|b-a| = |b|-|a|$ as shown below.

$$a > 0 \Rightarrow |a| = a$$

$$b > a \text{ and } a > 0 \Rightarrow b > 0 \Rightarrow |b| = b$$

$$b > a \Rightarrow b-a > 0 \Rightarrow |b-a| = b-a$$

$$|b-a| = b-a = |b|-|a|$$

27. $\frac{-3.1 \quad -3 \quad -1 \quad \sqrt{5} \quad \pi \quad |-8| \quad 9}{-3.1 \quad -|-3| \quad -1 \quad 2.236 \quad 3.14 \quad 8 \quad 9}$

28. $-|-6|, -4, -\sqrt{10}, \frac{1}{5}, 0.25, |\pi|$

29. (a) $b-a$; $b > a$, positive integer
 (b) $a-b$; $b > a$, negative integer
 (c) $\frac{b-a}{b+a}$, positive rational number less than 1

30. (a) $a+b$ positive integer

(b) $\frac{a}{b}$, positive rational integer

(c) $a \times b$, positive integer

31. (a) Let x be a positive integer, then $|x| = x$ which is positive integer and thus an integer. Yes. Let x be a negative integer, then $|x| = -x$ which is a positive integer and thus an integer. Yes.
 (b) Let x be a positive or negative integer, then the reciprocal is $\frac{1}{x}$ which is the ratio of two integers, 1 and x , and is thus a rational number. Yes.

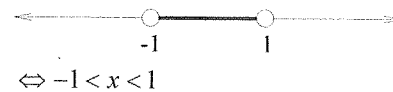
32. (a) Yes, $|\text{positive or negative rational}| = \text{positive rational}$

- (b) Yes, $\frac{\text{integer}}{\text{integer}}$ has reciprocal $\frac{\text{integer}}{\text{integer}}$ which is rational

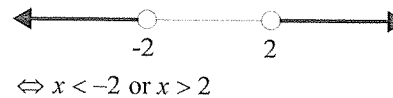
33. (a) x is a positive number located to the right of 0.

- (b) x is a negative number located to the left of -4 .

34. (a) $|x| < 1$ describes



- (b) $|x| > 2$ describes



35. If $x > 1$, then $\frac{1}{x}$ is a positive number less than 1. (x is between 0 and 1)

36. For $x < 0$, $|x| > 0$

37. $a+bj = a+b\sqrt{-1}$ which for $b=0$ is $a+bj = a$, a real number. The complex number $a+bj$ is a real number for all values of a and $b=0$.

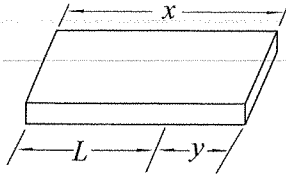
38. variables: w, t
 constants: $0.1, 1, c$

$$\begin{aligned} 39. \frac{1}{C_T} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{0.0040} + \frac{1}{0.0010} \\ &= \frac{0.0010 + 0.0040}{(0.0040)(0.0010)} \\ C_T &= \frac{(0.0040)(0.0010)}{0.0010 + 0.0040} = 0.0008 \text{ F} \end{aligned}$$

40. $|100 \text{ V}| = 100 \text{ V}$, $|-200 \text{ V}| = 200 \text{ V}$
 -200 V has the greater absolute value.

$$41. N = \frac{a \text{ bits}}{\text{byte}} \cdot \frac{1000 \text{ bytes}}{\text{kilobytes}} \cdot n \text{ kilobytes} \\ = 1000 \text{ an bits}$$

42.

 x = length of base in ft $12x$ = length of base in inches $y + L = 12x$, all dimensions in inches $L = 12x - y$

43. Yes, this statement is correct since -20 is larger than -30 .

44. For $I < 4A$, $V > 12 V$

1.2 Fundamental Operations of Algebra

$$1. 16 - 2 \times (-3) = 16 - (-6) = 22$$

$$2. \frac{-18}{-6} + 5 - (-3) = 3 + 5 - (-6) = 8 - (-6) = 14$$

$$3. \frac{-12}{8-2} + \frac{5-1}{2(-1)} = \frac{-12}{6} + \frac{4}{-2} = -2 + (-2) = -4$$

$$4. \frac{7 \times 6}{0 \times 6} = \frac{42}{0} \text{ is undefined}$$

$$5. 8 + (-4) = 8 - 4 = 4 \qquad 6. -4 + (-7) = -11$$

$$7. -3 + 9 = -(3-9) = -(-6) = 6$$

$$8. 18 - 21 = -3$$

$$9. -19 - (-16) = -19 + 16 = -3$$

$$10. 8 - (-4) = 8 + 4 = 12$$

$$11. 8(-3) = -(8 \times 3) = -24$$

$$12. -9(3) = -27$$

$$13. -7(-5) = 35$$

$$14. \frac{-9}{3} = -3$$

$$15. \frac{-6(20-10)}{-3} = \frac{-6(10)}{-3} = \frac{-60}{-3} = 20$$

$$16. \frac{28}{-7(6-5)} = \frac{28}{-7(1)} = \frac{28}{-7} = -4$$

$$17. -2(4)(-5) = (-8)(-5) = 40$$

$$18. 3(-4)(6) = -12(6) = -72$$

$$19. 2(2-7) \div 10 = 2(-5) \div 10 = -10 \div 10 = -1$$

$$20. \frac{-64}{-2|4-8|} = \frac{-64}{-2|-4|} = \frac{-64}{(-2)(4)} = \frac{-64}{-8} = 8$$

$$21. -9 - |2-10| = -9 - |-8| = -9 - 8 = -17$$

$$22. (7-7) \div (5-7) = 0 \div (-2) = 0$$

$$23. \frac{17-7}{7-7} = \frac{10}{0} \text{ is undefined}$$

$$24. \frac{7-7}{7-7} = \frac{0}{0} \text{ is indeterminate}$$

$$25. 8 - 3(-4) = 8 - (-12) = 8 + 12 = 20$$

$$26. 20 + 8 \div 4 = 20 + 2 = 22$$

$$27. -2(-6) + \left| \frac{8}{-2} \right| = 12 + |-4| = 12 + 4 \\ = 16$$

$$28. -10 - (-6)(-8) = -10 - (48) = -58$$

$$29. 30(-6)(-2) \div (0-40) = 360 \div (-40) = -9$$

$$30. \frac{7 - |-5|}{-1(-2)} = \frac{7-5}{2} = \frac{2}{2} = 1$$

$$\begin{aligned}
 31. \quad \frac{24}{3+(-5)} - 4(-9) &= \frac{24}{-(-3+5)} + (4 \times 9) \\
 &= -\left(\frac{24}{2}\right) + 36 \\
 &= -12 + 36 = 24
 \end{aligned}$$

$$32. \quad \frac{-18}{3} - \frac{4-6}{-1} = -6 - \frac{-2}{-1} = -6 - 2 = -8$$

$$\begin{aligned}
 33. \quad -7 - \frac{|-14|}{2(2-3)} - 3|6-8| &= -7 - \frac{14}{2(-1)} - 3|-2| \\
 &= -7 - \frac{14}{-2} - 3(2) \\
 &= -7 - (-7) - 6 \\
 &= 0 - 6 \\
 &= -6
 \end{aligned}$$

$$\begin{aligned}
 34. \quad -7(-3) + \frac{6}{-3} - (-9) &= 21 + (-2) + 9 \\
 &= 19 + 9 = 28
 \end{aligned}$$

$$\begin{aligned}
 35. \quad \frac{3(-9) - 2(-3)}{3-10} &= \frac{-(3 \times 9) + (2 \times 3)}{-7} \\
 &= \frac{-27+6}{-7} \\
 &= +\left(\frac{27-6}{7}\right) \\
 &= \frac{21}{7} = 3
 \end{aligned}$$

$$\begin{aligned}
 36. \quad \frac{20(-12) - 40(-15)}{98 - |-98|} &= \frac{-240 - (-600)}{98 - 98} \\
 &= \frac{360}{0} \text{ is undefined}
 \end{aligned}$$

$$37. \quad 6(7) = 7(6) \text{ demonstrates the commutative law of multiplication.}$$

$$38. \quad 6+8 = 8+6 = 14 \text{ demonstrates the commutative law of addition.}$$

$$39. \quad 6(3+1) = 6(3) + 6(1) \text{ demonstrates the distributive law.}$$

$$40. \quad 4(5 \times \pi) = (4 \times 5)(\pi) \text{ demonstrates the associative law of multiplication.}$$

$$41. \quad 3 + (5+9) = (3+5) + 9 \text{ demonstrates the associative law of addition.}$$

$$42. \quad 8(3-2) = 8(3) - 8(2) \text{ demonstrates the distributive law.}$$

$$43. \quad (\sqrt{5} \times 3) \times 9 = \sqrt{5} \times (3 \times 9) \text{ demonstrates the associative law of multiplication.}$$

$$44. \quad (3 \times 6) \times 7 = 7 \times (3 \times 6) \text{ demonstrates the commutative law of multiplication.}$$

$$45. \quad -a + (-b) = -a - b \text{ which is expression (d).}$$

$$46. \quad b - (-a) = b + a = a + b, \text{ expression (a).}$$

$$47. \quad -b - (-a) = -b + (a) = -b + a = a - b \text{ expression (b).}$$

$$48. \quad -a - (-b) = -a + b = b - a, \text{ expression (c).}$$

$$49. \quad \text{(a) The product of an even number of negative numbers is positive.}$$

$$\text{(b) The product of an odd number of negative numbers is negative.}$$

$$50. \quad \text{Subtraction is not commutative. } 7-5 \neq 5-7: \text{ counterexample.}$$

$$51. \quad \text{The definition } |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

is correct because it is equivalent to the definition on page 3 where $|x|$ is defined as x , the number itself for $x = 0$ or $x > 0$ (positive) and $|x|$ is the corresponding positive number ($= -x$) for $x < 0$ (negative).

$$52. \quad \text{The expression was evaluated as}$$

$$24 - 6 \div 2 \cdot 3 = 18 \div 2 \cdot 3$$

$$= 9 \cdot 3$$

$$= 27,$$

the error was doing the subtraction before the division and multiplication in violation of the order of operations. The correct order of operations is,

$$\begin{aligned} 24 - 6 \div 2 \cdot 3 &= 24 - 3 \cdot 3 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

53. (a) $-xy = 1 \Rightarrow y = -\frac{1}{x}$ which shows the numbers are negative reciprocals.

(b) $\frac{x-y}{x-y} = 1$ provided $x \neq y$ (to prevent division by zero). The numbers cannot be equal.

54. (a) Consider $x, y \geq 0 \Rightarrow x + y \geq 0 \Rightarrow |x| = x, |y| = y, |x + y| = x + y \Rightarrow |x + y| = |x| + |y|$ holds for $x, y \geq 0$.

Now consider

$$\begin{aligned} x, y < 0 \Rightarrow x + y < 0 \Rightarrow |x| = -x, |y| = -y, |x + y| &= -(x + y) = -x + (-y) = |x| + |y| \Rightarrow |x + y| = \\ |x| + |y| \text{ holds for } x, y < 0. \end{aligned}$$

In other words, if one or both is zero, or they have the same sign, the equality $|x + y| = |x| + |y|$ holds.

Now suppose one, say $x > 0$, and the other, $y < 0$.

$x > 0 \Rightarrow |x| = x, y < 0 \Rightarrow |y| = -y$ while $x + y$ could be (i) positive and (ii) negative.

(i) $x + y > 0 \Rightarrow |x + y| = x + y \neq x - y = |x| + |y|$, so the equality $|x + y| = |x| + |y|$ does not hold.

(ii) $x + y < 0 \Rightarrow |x + y| = -(x + y) = -x - y \neq x - y = |x| + |y|$, so the equality $|x + y| = |x| + |y|$ does not hold.

Summary: $|x + y| = |x| + |y|$ holds when x and y have the same sign, or one or both is zero.

(b) An argument similar to (a) shows x and y must have opposite signs, or one or both is zero.

55. $2.1 - 1.5(3) = -2.4 \text{ kW} \cdot \text{h}$

56. Since batting average $= \frac{\# \text{ of hits}}{\# \text{ of times at bat}} = \frac{0}{0}$ before the season starts, the batting average is indeterminate.

57. $\frac{7 + 3 + (-2) + (-3) + (-1) + 4 + 6}{7} = 2^\circ \text{C}$

58. $(70)(5) + (-16)(5) = 350 + (-80)$
 $= 270 \text{ ft above flare gun}$

59. Sum of voltages $= (6) + (-2) + (8) + (-5) + (3)$
 $= 4 + 8 + (-5) + 3$
 $= 12 + (-5) + 3$
 $= 7 + 3$
 $= 10 \text{ V}$

60. (a) Change = 2nd reading --- 1st reading
 $= -0.2 - 0.7 = -0.9 \text{ mA}$

(b) Change = 2nd reading --- 1st reading
 $= -0.9 - (-0.2) = -0.7 \text{ mA}$

(c) Change = 2nd reading --- 1st reading
 $= -0.6 - (-0.9) = 0.3 \text{ mA}$

61. $100 \text{ m} + 200 \text{ m} = 200 \text{ m} + 100 \text{ m}$ illustrates the commutative law of addition.

62. Tank 1: total amount leaked $= \frac{12 \text{ gal}}{\text{h}} \cdot 7 \text{ h} = 84 \text{ gal}$

Tank 2: $\frac{7 \text{ gal}}{\text{h}} \cdot 12 \text{ h} = 84 \text{ gal}$

The commutative property of multiplication is the fundamental law of algebra illustrated.

63. Total time spent browsing = time spent browsing first site + time spent browsing second site
 $= 4 \text{ persons} \cdot \frac{8 \text{ min}}{\text{person}} + 4 \text{ persons} \cdot \frac{6 \text{ min}}{\text{person}}$
 $= 4(8 + 6) \text{ min}$
 $= 56 \text{ min}$ which illustrates the distributive law

$$\begin{aligned}
 64. \text{ Distance} &= \text{rate} \times \text{time} \\
 &= (600 + 50) \times 3 \\
 &= 600 \times 3 + 50 \times 3, \text{ distributive law} \\
 &= 1950 \text{ km}
 \end{aligned}$$

1.3 Calculators and Approximate Numbers

1. Yes, 0.390 has three significant digits since the 0 after the 9 is not needed to locate the decimal.
2. 35.303 rounded to four significant digits is 35.30.
3. In finding the product of the approximate numbers 2.483 and 30.5 on a calculator the display shows 75.7315 which rounds to 75.7 since 30.5 has three significant digits.
4. $38.3 - 21.9(-3.58) = 116.702$ using exact numbers.
Estimating $40 - 20(-4) = 40 + 80 = 120$.
5. 8 cylinders is exact because they can be counted.
55 mi/h is approximate since it is measured.
6. 0.002 mm, approximate; \$7.50, exact
7. 24 and 1440 are exact.
8. 50 keys, exact; 50 h, approximate
9. 107 has 3 significant digits. 3004 has 4 significant digits.
10. 3600 has 2 significant digits
730 has 2 significant digits
11. 6.80 has 3 significant digits; the zero indicates precision.
6.08 has 3 significant digits; the zero is not used for decimal location, and is not a place-holder only.
12. 0.8735 has 4 significant digits
0.0075 has 2 significant digits
13. 3000 has 1 significant digit. 3000.1 has 5 significant digits.
14. 1.00 has 3 significant digits
0.01 has 1 significant digit
15. (a) 0.01 is more precise (more decimal places).
(b) 30.8 is more accurate (more significant digits).
16. (a) 0.041 and 7.673 have the same precision
(b) 7.673 is more accurate than 0.041
17. (a) Both numbers have the same precision with digits in the tenths place.
(b) 78.0 with 3 significant digits is more accurate than 0.1 with 1 significant digit.
18. (a) 0.004 is more precise than 7040
(b) 7040 is more accurate than 0.004
19. (a) 0.004 is more precise (more decimal places).
(b) Both have the same accuracy
20. (a) 50.060 and 8.914 have the same precision
(b) 50.060 is more accurate than 8.914
21. (a) $4.936 = 4.94$ rounded to 3 significant digits.
(b) $4.936 = 4.9$ rounded to 2 significant digits.
22. (a) 3 significant digits: $80.53 \rightarrow 80.5$
(b) 2 significant digits: $80.53 \rightarrow 81$
23. (a) $50,893 = 50,900$ (3 significant digits)
(b) $50,893 = 51,000$ (2 significant digits)
24. (a) $7.004 = 7.00$ to 3 significant digits
(b) $7.004 = 7.0$ to 2 significant digits
25. (a) $9549 = 9550$ rounded to 3 significant digits.
(b) $9549 = 9500$ rounded to 2 significant digits.
26. (a) 3 significant digits: $30.96 \rightarrow 31.0$
(b) 2 significant digits: $30.96 \rightarrow 31$

27. (a) $0.9449 = 0.945$ (3 significant digits)
 (b) $0.9949 = 0.94$ (2 significant digits)
28. (a) 3 significant digits: $0.9999 \rightarrow 1.00$
 (b) 2 significant digits: $0.9999 \rightarrow 1.0$
29. (a) Estimate: $13 + 1 - 2 = 12$
 (b) Calculator: $12.78 + 1.0495 - 1.633 = 12.1965$
 $= 12.20$
30. (a) Estimate: $4 \times 17 = 68$
 (b) Calculator: $3.64 \times 17.06 = 62.0984 = 62.1$
31. (a) Estimate: $0.04 - \frac{0.05}{2} = 0.015$
 (b) Calculator: $0.0350 - \frac{0.0450}{1.909} = 0.0114$
32. (a) Estimate: $\frac{0.3}{1 \times 0.5} = 0.6$
 (b) Calculator: $\frac{0.3275}{1.096 \times 0.50085} = 0.5966134943$
 $= 0.5966$
33. (a) Estimate: $\frac{20 \times 0.02}{10 - 8} = 0.2$
 (b) Calculator: $\frac{23.962 \times 0.01537}{10.965 - 8.249} = 0.1356$
34. (a) Estimate: $\frac{0.7 + 0.05}{300 \times 3} = 0.00083...$
 (b) Calculator: $\frac{0.69378 + 0.04997}{257.4 \times 3.216} = 0.0008984675496$
 $= 0.0008985$
35. (a) Estimate: $\frac{4000}{500} - \frac{2 \times 300}{400} = 6.5$
 (b) Calculator: $\frac{3872}{503.1} - \frac{2.056 \times 309.6}{395.2} = 6.085610980$
 $= 6.086$
36. (a) Estimate: $\frac{1}{0.6} + \frac{4}{3-1} = 3.6...$
 (b) Calculator: $\frac{1}{0.5926} + \frac{3.6957}{2.935 - 1.054} = 3.652231698$
 $= 3.6522$
37. $0.9788 + 14.9 = 15.8788$ since 4 is the number of decimal places in the least precise number.
38. $17.311 - 22.98 = -5.669$
39. $3.142(65) = 204.2$ (The product has the accuracy of the approximate number).
40. $8.62 \div 1728 = 0.00499$
41. 2.745 MHz and 2.755 MHz are the least possible and greatest possible frequencies respectively.
42. least: 2350 cm^3 , greatest: 2450 cm^3 .
43. There are too many significant digits; time only has 2 significant digits.
44. $4.2s - 2.72s = 1.68s$
 $= 1.7s$ to two significant digits
 since $4.2s$ has the least number of significant digits ($= 2$).
45. (a) $2.2 + 3.8 \times 4.5 = 19.3$
 (b) $(2.2 + 3.8) \times 4.5 = 27$
46. (a) $6.03 \div 2.25 + 1.77 = 2.68 + 1.77 = 4.45$
 (b) $6.03 \div (2.25 + 1.77) = 6.03 \div 4.02 = 1.5$

47. (a) $2+0$ 2
 (b) $2-0$ 2
 (c) $0-2$ -2

```
ERR:DIVIDE BY 0
Quit
2:Goto
```

(d) $2*0$ 0

```
0.0001/0.0001 1
0/0
```

(e) $2/0$

```
ERR:DIVIDE BY 0
Quit
2:Goto
```

(e2)

```
ERR:DIVIDE BY 0
Quit
2:Goto
```

49. $61812311 \rightarrow X$
 $11381216 \rightarrow Y$
 $(X-Y)/9$ 5603455

48. (a) $2 \div 0.0001 = 20,000$
 $2 \div 0$, error
 (b) $0.0001 \div 0.0001 = 1$
 $0 \div 0$, error
 (c) calculator will not divide by zero.

```
2/0.0001 20000
2/0
```

50. $9*8+7+6+5+4+3+2+$
 $1+0$ 100

51. $\frac{22}{7} = 3.142857\dots$ which differs from π starting in the thousandths place ($\pi = 3.14159265\dots$).

(a) $3.1416 > \pi$ (b) $\frac{22}{7} > \pi$

52. (a) $\frac{8}{33} = 0.242424\dots = 0.\overline{24}$

(b) $\pi = 3.14159265358973\dots$

53. (a) $\frac{1}{3} = 0.333\dots$

(b) $\frac{5}{11} = 0.454545\dots$

(c) $\frac{2}{5} = 0.4000\dots$

0 is the repeating part

54. $\frac{124}{990} = 0.1252525\dots = 0.1\overline{25}$

$= 0.1252525253$, calculator last digit different because of calculator round-off

55. $32.4 + 26.704 + 36.23 = 95.3 \text{ MJ}$

56. $1450 - 938 = 512 \rightarrow 510 \text{ km/hr}$

57. $1 \text{ K} = 1024 \text{ bytes}$

$256 \text{ K} = 256 \times 1024 = 262,144 \text{ bytes}$

58. $V = 3.55(15.2 + 5.64 + 101.23) = 433 \text{ V}$

59. $\frac{100(40.63 + 52.96)}{105.30 + 52.96} = 59.14\%$

60. $\frac{50.45(9.80)}{1 + \frac{100.9}{23}} = 92 \text{ N}$

3. $\left(\frac{b^2t}{a^3x}\right)^{-2} = \frac{(b^2t)^{-2}}{(a^3x)^{-2}} = \frac{(a^3x)^2}{(b^2t)^2} = \frac{a^6x^2}{b^4t^2}$

4. $8 - (-1)^3 - 2(-3)^2 = 8 - (-1) - 2(9)$
 $= 9 - 18$
 $= -9$

5. $x^3 \cdot x^4 = x^{3+4} = x^7$

6. $y^2y^7 = y^{2+7} = y^9$

7. $2b^4b^2 = 2b^{4+2} = 2b^6$

8. $3k^5(k) = 3k^{5+1} = 3k^6$

9. $\frac{m^5}{m^3} = m^{5-3} = m^2$

10. $\frac{x^6}{x} = 2x^{6-1} = 2x^5$

11. $\frac{n^5}{7n^9} = \frac{n^{5-9}}{7} = \frac{n^{-4}}{7} = \frac{1}{7n^4}$

12. $\frac{3s}{s^4} = 3s^{1-4} = 3s^{-3} = \frac{3}{s^3}$

13. $(P^2)^4 = P^{2 \cdot 4} = P^8$

14. $(x^8)^3 = x^{8 \cdot 3} = x^{24}$

15. $(2\pi)^3 = 2^3 \cdot \pi^3 = 8\pi^3$

16. $(ax)^5 = a^5x^5$

17. $(aT^2)^{30} = a^{30}T^{2(30)} = a^{30}T^{60}$

18. $(3r^2)^3 = 3^3r^{2 \cdot 3} = 27r^6$

19. $\left(\frac{2}{b}\right)^3 = \frac{2^3}{b^3} = \frac{8}{b^3}$

20. $\left(\frac{F}{t}\right)^{20} = \frac{F^{20}}{t^{20}}$

21. $\left(\frac{x^2}{2}\right)^4 = \frac{x^{2 \cdot 4}}{2^4} = \frac{x^8}{16}$

22. $\left(\frac{3}{n^3}\right)^3 = \frac{3^3}{(n^3)^3} = \frac{27}{n^{3 \cdot 3}} = \frac{27}{n^9}$

23. $(8a)^0 = 1$

24. $6v^0 = 6 \cdot 1 = 6$

25. $-3x^0 = (-3)(1) = -3$

26. $-(-2)^0 = -1$

1.4 Exponents

1. $(-x^3)^2 = [(-1)x^3]^2 = (-1)^2(x^3)^2 = x^{3(2)} = x^6$

2. $2x^0 = 2(1) = 2$

$$27. 6^{-1} = \frac{1}{6^1} = \frac{1}{6}$$

$$28. {}^5w^{-} = -\frac{1}{w^5}$$

$$29. \frac{1}{R^{-2}} = R^2$$

$$30. \frac{1}{t^{-48}} = t^{48}$$

$$31. (-t^2)^7 = -t^{2 \cdot 7} = -t^{14}$$

$$32. (-y^3)^5 = -y^{3 \cdot 5} = -y^{15}$$

$$33. (2v^2)^{-6} = (2^{-6})(v^{-2 \cdot 6}) = \frac{1}{64v^{12}}$$

$$\begin{aligned} 34. -(-c^4)^{-4} &= -((-1)^{-4}(c^4)^{-4}) = -c^{-4(4)} \\ &= -c^{-16} \\ &= -\frac{1}{c^{16}} \end{aligned}$$

$$35. \frac{L^{-3}}{L^{-5}} = \frac{L^5}{L^3} = -L^2$$

$$36. 2i^{40}i^{-70} = 2i^{-30} = \frac{2}{i^{30}}$$

$$37. \frac{2v^4}{(2v)^4} = \frac{2v^4}{2^4v^4} = \frac{1}{2^3} = \frac{1}{8}$$

$$38. \frac{x^2x^3}{(x^2)^3} = \frac{x^5}{x^6} = \frac{1}{x}$$

$$39. \frac{(n^2)^4}{(n^4)^2} = \frac{n^{2 \cdot 4}}{n^{4 \cdot 2}} = 1 \text{ (since } 2 \cdot 4 = 4 \cdot 2)$$

$$40. \frac{(3t)^{-1}}{3t^{-1}} = \frac{\frac{1}{3t}}{3 \cdot \frac{1}{t}} = \frac{1}{9}$$

$$\begin{aligned} 41. (\pi^0 x^2 a^{-1})^{-1} &= (1 \cdot x^2 \cdot a^{-1})^{-1} = x^{-2(1)}(a^{-1(-1)}) \\ &= \frac{a}{x^2} \end{aligned}$$

$$42. (3m^{-2}n^4)^{-2} = 3^{-2}m^4n^{-8} = \frac{m^4}{9n^8}$$

$$43. (-8g^{-1}s^3)^2 = (-8)^2 \cdot g^{-2} \cdot (s^3)^2 = \frac{64s^{3 \cdot 2}}{g^2} = \frac{64s^6}{g^2}$$

$$44. ax^{-2}(-a^2x)^2 = ax^{-2}a^4x^2 = a^5$$

$$45. \left(\frac{4x^{-1}}{a^{-1}}\right)^{-3} = \frac{4^{-3}x^{-1(-3)}}{a^{-1(-3)}} = \frac{x^3}{4^3a^3} = \frac{x^3}{64a^3}$$

$$46. \left(\frac{2b^2}{y^5}\right)^{-2} = \frac{2^{-2}b^{-4}}{y^{-10}} = \frac{\frac{1}{4} \cdot \frac{1}{b^4}}{\frac{1}{y^{10}}} = \frac{y^{10}}{4b^4}$$

$$47. \frac{15n^2T^5}{3n^{-1}T^6} = \frac{5n^{2-(-1)}}{T} = \frac{5n^3}{T}$$

$$48. \frac{(nRT^{-2})^{32}}{R^{-2}T^{32}} = \frac{n^{32}R^{32}T^{-64}}{R^{-2}T^{32}} = \frac{n^{32}R^{34}}{T^{96}}$$

$$49. 7(-4) - (-5)^2 = -28 - 25 = -53$$

$$50. 6 + (-2)^5 - (-2)(8) = -10$$

$$51. -(-26.5)^2 - (-9.85)^3 = 253 \text{ (second term is precise to ones).}$$

$$52. -0.711^2 - (-0.809)^6 = -0.786$$

$$53. \frac{3.07(-1.86)}{(-1.86)^4 + 1.596} = \frac{-5.71}{11.97 + 1.596} = \frac{-5.71}{13.57} = -0.421$$

$$54. \frac{15.66^2 - (-4.017)^4}{1.044(-3.68)} = 3.94$$

$$\begin{aligned} 55. 2.38(-60.7)^2 - \frac{2540}{1.17^3} + 0.806^5(26.1^3 - 9.88^4) \\ = 9990 \end{aligned}$$

(all three terms are precise to tens).

$$56. 0.513(-2.778) - (-3.67)^3 + \frac{0.889^4}{1.89 - 1.09^2} = 48.90$$

$$57. \left(\frac{1}{x^{-1}}\right)^{-1} = \frac{1^{-1}}{x^{-1(-1)}} = \frac{1}{x} = \text{reciprocal of } x, \text{ yes.}$$

$$58. \left(\frac{0.2 - 5^{-1}}{10^{-2}} \right)^0 = \left(\frac{0}{0.01} \right)^0 = (0)^0 \neq 1$$

since $a^0 = 1$ requires $a \neq 0$.

$$59. a^3 = 5$$

$$(a^3)^4 = 5^4$$

$$a^{3(4)} = 625$$

$$a^{12} = 625$$

$$60. \text{ If } a^{-2} < a^{-1} \text{ is true, then}$$

$$\frac{1}{a^2} < \frac{1}{a}$$

$$1 < \frac{a^2}{a} = a$$

$$a > 1 > 0$$

$$a > 0, \text{ therefore}$$

$$a^{-2} < a^{-1} \text{ is not true for any negative value of } a.$$

$$61. (x^a \cdot x^{-a})^5 = (x^{a+(-a)})^5 = (x^0)^5$$

$$= (1)^5, \quad x \neq 0$$

$$= 1$$

$$62. (y^{a-b} \cdot y^{a+b})^2 = (y^{a-b+a+b})^2 = (y^{2a})^2$$

$$= y^{2a \cdot 2}$$

$$= y^{4a}$$

$$63. \left(\frac{kT}{hc} \right)^3 (GkT\hbar c)^2 c = \frac{k^3 T^3}{h^3 c^3} G^2 k^2 T^2 \hbar^2 c^2 c$$

$$= \frac{G^2 k^5 T^5}{h}$$

$$64. (GmM)(mr)^{-1}(r^{-2}) = (GmM) \cdot \frac{1}{mr} \cdot \frac{1}{r^2}$$

$$= \frac{GM}{r^3}$$

$$65. \pi \left(\frac{r}{2} \right)^3 \left(\frac{4}{3\pi r^2} \right) = \pi \cdot \frac{r^3}{8} \cdot \frac{4}{3\pi r^2} = \frac{r}{6}$$

$$66. \frac{gM}{2\pi fC(2\pi fM)^2} = \frac{gM}{2\pi fC4\pi^2 f^2 M^2}$$

$$= \frac{g}{8\pi^3 f^3 CM}$$

$$67. 2500(1 + 0.042/4)^{24} = \$3212.27$$

$$68. \frac{x(1000 - 20x^2 + x^3)}{1850}$$

$$= \frac{6.85(1000 - 20 \times 6.85^2 + 6.85^3)}{1850}$$

$$= 1.42 \text{ cm}$$

1.5 Scientific Notation

$$1. 8.06 \times 10^3 = 8060$$

$$2. 750,000,000,000^{-1} = (7.50 \times 10^{11})^{-1}$$

$$= 7.50^{-1} \times 10^{-11}$$

$$= 1.33 \times 10^{-12}$$

$$3. 4.5 \times 10^4 = 45,000 \quad 4. 6.8 \times 10^7 = 68,000,000$$

$$5. 2.01 \times 10^{-3} = 0.00201; \text{ move decimal point 3 places to the left by adding 2 zeros.}$$

$$6. 9.61 \times 10^{-5} = 0.0000961$$

$$7. 3.23 \times 10^0 = 3.23 \quad 8. 8 \times 10^0 = 8 \times 1 = 8$$

$$9. 1.86 \times 10 = 18.6; \text{ move decimal point 1 place to the right.}$$

$$10. 1 \times 10^{-1} = 0.1 \quad 11. 4000 = 4 \times 10^3$$

$$12. 56,000 = 5.6 \times 10^4$$

$$13. 0.0087 = 8.7 \times 10^{-3}; \text{ move decimal point to the right 3 places.}$$

$$14. 0.7 = 7 \times 10^{-1} \quad 15. 6.09 = 6.09 \times 10^0$$

$$16. 100 = 1 \times 10^2$$

17. $0.063 = 6.3 \times 10^{-2}$; move decimal point 2 places to the right.

18. $0.0000908 = 9.08 \times 10^{-5}$

19. $1 = 1 \times 10^0$

20. $10 = 1 \times 10^1$

21. $28,000(2,000,000,000) = 2.8 \times 10^4 (2 \times 10^9)$
 $= 5.6 \times 10^{13}$

22. $50,000(0.006) = 300 = 3 \times 10^2$

23. $\frac{88,000}{0.0004} = \frac{8.8 \times 10^4}{4.0 \times 10^{-4}} = 2.2 \times 10^8$

24. $\frac{0.00003}{6,000,000} = 5 \times 10^{-12}$

25. $2 \times 10^{-35} + 3 \times 10^{-34} = 0.2 \times 10^{-34} + 3 \times 10^{-34}$
 $= 3.2 \times 10^{-34}$

26. $5.3 \times 10^{12} - 3.7 \times 10^{10} = 530 \times 10^{10} - 3.7 \times 10^{10}$
 $= 526.3 \times 10^{10}$
 $= 5.23 \times 10^{12}$

27. $(1.2 \times 10^{29})^3 = 1.2^3 \times 10^{29(3)}$
 $= 1.728 \times 10^{87}$

28. $(2 \times 10^{-16})^{-5} = 2^{-5} \times 10^{-16(-5)}$
 $= 0.03125 \times 10^{80}$
 $= 3.125 \times 10^{78}$

29. $1280(865,000)(43.8) = 4.85 \times 10^{10}$

30. $0.0000569(3,190,000) = 1.82 \times 10^2$

31. $\frac{0.0732(6710)}{0.00134(0.0231)} = \frac{7.32 \times 10^{-2} \times 6.71 \times 10^3}{1.34 \times 10^{-3} \times 2.31 \times 10^{-2}}$
 $= 1.59 \times 10^7$

32. $\frac{0.00452}{2430(97,100)} = 1.92 \times 10^{-11}$

33. $(3.642 \times 10^{-8})(2.736 \times 10^5) = 9.965 \times 10^{-3}$

34. $\frac{(7.309 \times 10^{-1})^2}{5.9843(2.5036 \times 10^{-20})} = 3.566 \times 10^{18}$

35. $\frac{(3.69 \times 10^{-7})(4.61 \times 10^{21})}{0.0504} = 3.40 \times 10^{16}$

36. $\frac{(9.907 \times 10^7)(1.08 \times 10^{12})^2}{(3.603 \times 10^{-5})(2054)} = 1.56 \times 10^{33}$

37. $6,500,000 \text{ kW} = 6.5 \times 10^6 \text{ kW}$

38. $85,000,000,000 \text{ bytes} = 8.5 \times 10^{10} \text{ bytes}$

39. $0.000003 \text{ W} = 3 \times 10^{-6} \text{ W}$

40. $0.0075 \text{ mm} = 7.5 \times 10^{-3} \text{ mm}$

41. $2,000,000,000 \text{ Hz} = 2 \times 10^9 \text{ Hz}$

42. $3.086 \times 10^{16} = 30,860,000,000,000,000 \text{ m}$

43. $1.6 \times 10^{-12} \text{ W} = 0.0000000000016 \text{ W}$

44. $2.4 \times 10^{43} = 0.000 \dots 00024(42 \text{ zeros})$

45. (a) $2300 = 2.3 \times 10^3$
 (b) $0.23 = 230 \times 10^{-3}$
 (c) $23 = 23 \times 10^0$

46. (a) $8,090,000 = 8.09 \times 10^6$
 (b) $809,000 = 809 \times 10^3$
 (c) $0.0809 = 80.9 \times 10^{-3}$

47. (a) $\text{google} = 1 \times 10^{100} = 10^{100}$
 (b) $\text{googleplex} = 10^{\text{google}} = 10^{10^{100}}$

48. $\text{google} = N \cdot 10^{79}$
 $10^{100} = N \cdot 10^{79}$
 $N = \frac{10^{100}}{10^{79}} = 10^{21}$

A google is 10^{21} times greater than the number of electrons in the universe.

$$49. \frac{7.5 \times 10^{-15} \text{ s}}{\text{addition}} \cdot 5.6 \times 10^6 \text{ additions} = 4.2 \times 10^{-8} \text{ s}$$

$$50. 0.085(0.00000000039) = 3.3 \times 10^{-11} \text{ mg}$$

$$51. \frac{4.57 \times 10^4}{1.86 \times 10^5} = 2.46 \times 10^{-1} \text{ s}$$

$$52. 1 \text{ day} = 1 \text{ day} \left(\frac{24 \text{ h}}{\text{day}} \right) \left(\frac{60 \text{ min}}{\text{h}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) \\ = 8.64 \times 10^4 \text{ s}$$

$$53. \text{ mass}$$

$$= 125,000,000 \text{ atoms} \left(\frac{16 \text{ amu}}{\text{atom}} \right) \left(\frac{1.66 \times 10^{-27}}{\text{amu}} \right) \\ = 3.32 \times 10^{-18} \text{ kg}$$

$$54. kT^4 = 0.000000057 \cdot 303^4 = 4.8 \times 10^2 \text{ W}$$

$$55. R = k/d^2;$$

$$R = 0.00000002196 \div 0.00007998^2 \\ = 2196 \times 10^8 \div (7.998 \times 10^{-5})^2 \\ = 3.433 \Omega$$

$$56. \text{ speed} = \frac{\text{distance}}{\text{time}} = \frac{149,600,000 \text{ km}}{499.0 \text{ s}} \\ = 2.998 \times 10^5 \text{ km/s}$$

$$5. \sqrt{81} = 9$$

$$6. \sqrt{225} = 15$$

$$7. -\sqrt{121} = -\sqrt{11^2} = -11$$

$$8. -\sqrt{36} = -6$$

$$9. -\sqrt{49} = -7$$

$$10. \sqrt{0.25} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 0.5$$

$$11. \sqrt{0.09} = \sqrt{\frac{9}{100}} = \frac{3}{10} = 0.3$$

$$12. -\sqrt{900} = -30$$

$$13. \sqrt[3]{125} = 5$$

$$14. \sqrt[4]{16} = 2$$

$$15. \sqrt[3]{-126} = -6$$

$$16. \sqrt[3]{-32} = -2$$

$$17. (\sqrt{5})^2 = 5$$

$$18. (\sqrt[3]{31})^3 = \sqrt[3]{31} \sqrt[3]{31} \sqrt[3]{31} = 31$$

$$19. (-\sqrt[3]{-47})^3 = (-1)^3 (\sqrt[3]{-47})^3 = -1(-47) = 47$$

$$20. (\sqrt[3]{-23})^5 = -23$$

$$21. (-\sqrt[4]{53})^4 = 53$$

$$22. -\sqrt{32} = -\sqrt{16 \cdot 2} = -4\sqrt{2}$$

$$23. \sqrt{1200} = \sqrt{400(3)} = 20\sqrt{3}$$

$$24. \sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$

$$25. 2\sqrt{84} = 2\sqrt{4 \cdot 21} = 2 \cdot 2\sqrt{21} = 4\sqrt{21}$$

$$26. \sqrt{\frac{108}{2}} = \sqrt{\frac{36 \cdot 3}{2}} = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

$$27. \sqrt{\frac{80}{7-3}} = \sqrt{\frac{80}{4}} = \sqrt{20} = \sqrt{(4)(5)} = 2\sqrt{5}$$

$$28. \sqrt{81 \times 10^2} = \sqrt{81} \sqrt{10^2} = 9(10) = 90$$

$$29. \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

1.6 Roots and Radicals

$$1. -\sqrt[3]{64} = -4 \text{ since } (-4)^3 = -64$$

$$2. \text{ Changing } \sqrt{(25)(3)} \text{ to } \sqrt{(15)(5)} \text{ in } \sqrt{75} \\ = \sqrt{(25)(3)} \text{ is not better since neither 15 nor} \\ 5 \text{ is a perfect square.}$$

$$3. \sqrt{16 \times 9} = \sqrt{16} \times \sqrt{9} = 4 \times 3 = 12$$

$$4. -\sqrt{-64} \text{ is imaginary, because the index is even.} \\ \text{No change required.}$$

$$30. \sqrt[4]{9^2} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$$

$$31. \frac{7^2 \sqrt{81}}{3^2 \sqrt{49}} = \frac{7^2 \times 9}{3^2 \times 7} = 7$$

$$32. \frac{2^5 \cdot \sqrt[5]{243}}{3 \sqrt{144}} = \frac{32 \cdot 3}{3 \cdot 12} = \frac{8}{3}$$

$$33. \sqrt{36+64} = \sqrt{100} = 10$$

$$34. \sqrt{25+144} = \sqrt{169} = 13$$

$$35. \sqrt{3^2+9^2} = \sqrt{9+81} = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$$

$$36. \sqrt{8^2-4^2} = \sqrt{64-16} = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

$$37. \sqrt{85.4} = 9.24$$

$$38. \sqrt{3762} = 61.34$$

$$39. \sqrt{0.4729} = 0.6877$$

$$40. \sqrt{0.0627} = 0.250$$

$$41. (a) \sqrt{1296+2304} = \sqrt{3600} = 60.00$$

$$(b) \sqrt{1296} = \sqrt{2304} = 36.00 + 48.00 = 84.00$$

$$42. (a) \sqrt{10.6276+2.1609} = \sqrt{12.7885} = 3.57610$$

$$(b) \sqrt{10.6276} + \sqrt{2.1609} = 3.26 + 1.47 = 4.7300$$

$$43. (a) \sqrt{0.0429^2 - 0.183^2} = \sqrt{0.0150552} = 0.0388$$

$$(b) \sqrt{0.0429^2} - \sqrt{0.183^2} = 0.0429 - 0.0183 = 0.0246$$

$$44. (a) \sqrt{3.625^2 + 0.614^2} = 3.677$$

$$(b) \sqrt{3.625^2} + \sqrt{0.614^2} = 4.239$$

$$45. \sqrt{24s} = \sqrt{(24)(150)} = \sqrt{3600} = 60 \text{ mi/h}$$

$$46. \sqrt{Z^2 - X^2} = \sqrt{5.362^2 - 2.875^2} = 4.526\Omega$$

$$47. \sqrt{B/d} = \sqrt{2.18 \times 10^9 / 1.03 \times 10^3} = 1450 \text{ m/s}$$

$$48. \sqrt{40 \text{ m}} = \sqrt{(40)(75)} = \sqrt{3000} \approx 55 \text{ m/s}$$

$$49. d = \sqrt{w^2 + h^2} = \sqrt{3.6 + 20.5} = 42.0 \text{ in.}$$

$$50. \text{ARD} = 1000 \left(1 - \frac{\sqrt{V}}{C} \right) = 100 \left(1 - \sqrt{\frac{15,000}{22,000}} \right) = 17\%$$

51. No, it is not true if $a < 0$. It is always true

$$\text{that } \sqrt{a^2} = |a|.$$

52. (a) \sqrt{x} requires $x \geq 0$

$$x > \sqrt{x} \geq 0 \Rightarrow x^2 > x \Rightarrow x(x-1) > 0 \Rightarrow x > 1$$

$$x > \sqrt{x} \text{ for } x > 1$$

$$(b) x = \sqrt{x} \Rightarrow x^2 = x \Rightarrow x(x-1) = 0 \Rightarrow x = 0, x = 1$$

$$x = \sqrt{x} \text{ for } x = 0, x = 1$$

$$(c) 0 \leq x < \sqrt{x} \Rightarrow x^2 < x \Rightarrow x(x-1) < 0 \Rightarrow x < 1$$

$$x < \sqrt{x} \text{ for } 0 < x < 1$$

53.

54.

55. (a) imaginary (b) real

56. (a) real (b) imaginary

1.7 Addition and Subtraction of Algebraic Expressions

1. $3x + 2y - 5y = 3x - 3y$
2. $3c - (2b - c) = 3c - 2b + c = -2b + 4c$
3. $3ax - [(ax - 5s) - 2ax] = 3ax - [ax - 5s - 2ax]$
 $= 3ax - [-ax - 5s]$
 $= 3ax + ax + 5s$
 $= 4ax + 5s$
4. $3a^2b - \{a - [2a^2b - (a + 2b)]\}$
 $= 3a^2b - \{a - [2a^2b - a - 2b]\}$
 $= 3a^2b - \{a - 2a^2b + a + 2b\}$
 $= 3a^2b - \{2a - 2a^2b + 2b\}$
 $= 3a^2b - 2a + 2a^2b - 2b$
 $= 5a^2b - 2a - 2b$
5. $5x + 7x - 4x = 12x - 4x = 8x$
6. $6t - 3t - 4t = -t$
7. $2y - y + 4x = y(2 - 1) + 4x = y + 4x$
8. $4C + L - 6C = -2C + L$
9. $2F - 2T - 2 + 3F - T = 5F - 3T - 2$
10. $x - 2y + 3x - y + z = 4x - 3y + z$
11. $a^2b - a^2b^2 - 2a^2b = a^2b - 2a^2b - a^2b^2 = -a^2b - a^2b^2$
12. $xy^2 - 3x^2y^2 + 2xy^2 = 3xy^2 - 3x^2y^2$
13. $s + 3a - 4 - 2 = s + (2a - 4) = 3s - 4$
14. $5 + (3 - 4n + p) = 5 + 3 - 4n + p = -4n + p + 8$
15. $v - (4 - 5x + 2v) = v - 4 + 5x - 2v = 5x - v - 4$
16. $2a - (b - a) = 2a - b + a = 3a - b$
17. $2 - 3 - (4 - 5a) = -1 - 4 + 5a = -5 + 5a = 5a - 5$
18. $\sqrt{A} + (h - 2\sqrt{A}) - 3\sqrt{A} = \sqrt{A} + h - 2\sqrt{A} - 3\sqrt{A}$
 $= h - 4\sqrt{A}$
19. $(a - 3) + (5 - 6a) = a - 3 + 5 - 6a = -5a + 2$
20. $(4x - y) - (-2x - 4y) = 4x - y + 2x + 4y = 6x + 3y$
21. $-(t - 2u) + (3u - t) = -t + 2u + 3u - t = 5u - 2t$
22. $2(x - 2y) + (5x - y) = 2x - 4y + 5x - y = 7x - 5y$
23. $3(2r + s) - (-5s - r) = 6r + 3s + 5s + r = 7r + 8s$
24. $3(a - b) - 2(a - 2b) = 3a - 3b - 2a + 4b = a + b$
25. $-7(6 - 3j) - 2(j + 4) = -42 + 21j - 2j - 8$
 $= 19j - 50$
26. $-(5t + a^2) - 2(3a^2 - 2st) = -5t - a^2 - 6a^2 + 4st$
 $= -7a^2 + 4st - 5t$
27. $-[(6 - n) - (2n - 3)] = -[6 - n - 2n + 3]$
 $= -6 + n + 2n - 3$
 $= -9 + 3n$
28. $-[(A - B) - (B - A)] = -[A - B - B + A]$
 $= -[2A - 2B]$
 $= -2A + 2B$
29. $2[4 - (t^2 - 5)] = 2[4 - t^2 + 5] = 2[9 - t^2] = 18 - 2t^2$
30. $3[-3 - (a - 4)] = 3[-3 - a + 4] = 3[-a + 1] = -3a + 3$
31. $-2[-x - 2a - (a - x)] = -2[-x - 2a - a + x]$
 $= 2x + 4a + 2a - 2x$
 $= 6a$
32. $-2[-3(x - 2y) + 4y] = -2[-3x + 6y + 4y]$
 $= -2[-3x + 10y]$
 $= 6x - 20y$

$$\begin{aligned}
 33. \quad aZ - [3 - (aZ + 4)] &= aZ - [3 - aZ - 4] \\
 aZ - [-1 - aZ] &= aZ + 1 + aZ \\
 &= 2aZ + 1
 \end{aligned}$$

$$\begin{aligned}
 34. \quad 9v - [6 - (v - 4) + 4v] &= 9v - [6 - v + 4 + 4v] \\
 &= 9v - [3v + 10] = 9v - 3v - 10 \\
 &= 6v - 10
 \end{aligned}$$

$$\begin{aligned}
 35. \quad 8c - \{5 - [2 - (3 + 4c)]\} &= 8c - \{5 - [2 - 3 - 4c]\} \\
 &= 8c - \{5 - 2 + 3 + 4c\} \\
 &= 8c - 5 + 2 - 3 - 4c = 4c - 6
 \end{aligned}$$

$$\begin{aligned}
 36. \quad 7y - \{y - [2y - (x - y)]\} &= 7y - \{y - [2y - x + y]\} \\
 &= 7y - \{y - 2y + x - y\} \\
 &= 7y + 2y - x \\
 &= 9y - x
 \end{aligned}$$

$$\begin{aligned}
 37. \quad 5p - (q - 2p) - [3q - (p - q)] \\
 = 5p - q + 2p - [3q - p + q] \\
 = 7p - q - [4q - p] \\
 = 7p - q - 4q + p \\
 = 8p - 5q
 \end{aligned}$$

$$\begin{aligned}
 38. \quad -(4 - LC) - [(5LC - 7) - (6LC + 2)] \\
 = -4 + LC - [5LC - 7 - 6LC - 2] \\
 = -4 + LC - [-LC - 9] \\
 = -4 + LC + LC + 9 \\
 = 2LC + 5
 \end{aligned}$$

$$\begin{aligned}
 39. \quad -2\{-[4 - x^2] - [3 + (4 - x^2)]\} \\
 = -2\{-4 + x^2 - 3 - 4 + x^2\} \\
 = 8 - 2x^2 + 6 + 8 - 2x^2 \\
 = -4x^2 + 22
 \end{aligned}$$

$$\begin{aligned}
 40. \quad -\{-[-(x - 2a) - b] - (a - x)\} \\
 = -\{-[-x + 2a - b] - a + x\} \\
 = -\{x - 2a + b - a + x\} \\
 = -\{2x - 3a + b\} \\
 = -2x + 3a + b
 \end{aligned}$$

$$\begin{aligned}
 41. \quad 5V^2 - (6 - (2V^2 + 3)) &= 5V^2 - (6 - 2V^2 - 3) \\
 &= 5V^2 - (3 - 2V^2) \\
 &= 5V^2 - 3 + 2V^2 \\
 &= 7V^2 - 3
 \end{aligned}$$

$$\begin{aligned}
 42. \quad -2F + 2((2F - 1) - 5) &= -2F + 2(2F - 1 - 5) \\
 &= -2F + 2(2F - 6) \\
 &= -2F + 4F - 12 \\
 &= 2F - 12
 \end{aligned}$$

$$\begin{aligned}
 43. \quad -(3t - (7 + 2t - (5t - 6))) &= -(3t - 7 - 2t + 5t - 6) \\
 &= -3t + 7 + 2t - 5t + 6 \\
 &= -6t + 13
 \end{aligned}$$

$$\begin{aligned}
 44. \quad a^2 - 2(x - 5 - (7 - 2(a^2 - 2x) - 3x)) \\
 = a^2 - 2(x - 5 - (7 - 2a^2 + 4x - 3x)) \\
 = a^2 - 2(x - 5 - (7 - 2a^2 + x)) \\
 = a^2 - 2(x - 5 - 7 + 2a^2 - x) \\
 = a^2 - 2(2a^2 - 12) \\
 = a^2 - 4a^2 + 24 \\
 = -3a^2 + 24
 \end{aligned}$$

$$\begin{aligned}
 45. \quad -4[4R - 2.5(Z - 2R) - 1.5(2R - Z)] \\
 = -4[4R - 2.5Z + 5R - 3R + 1.5Z] \\
 = -4[6R - Z] \\
 = -24R + 4Z \\
 = 4Z - 24R
 \end{aligned}$$

$$\begin{aligned}
 46. \quad 3\{2.1e - 1.3[f - 2(e - 5f)]\} \\
 = 3\{2.1e - 1.3[f - 2e + 10f]\} \\
 = 3\{2.1e - 1.3[11f - 2e]\} \\
 = 3\{2.1e - 14.3f + 2.6e\} \\
 = 3\{4.7e - 14.3f\} \\
 = 14.1e - 42.9f
 \end{aligned}$$

$$47. \quad 3D - (D - d) = 3D - D + d = 2D + d$$

$$48. \quad i_1 - (2 - 3i_2) + i_2 = i_1 - 2 + 3i_2 + i_2 = i_1 + 4i_2 - 2$$

$$\begin{aligned}
 49. & \left[\left(B + \frac{4}{3}\alpha \right) + 2 \left(B - \frac{2}{3}\alpha \right) \right] \\
 & - \left[\left(B + \frac{4}{3}\alpha \right) - \left(B - \frac{2}{3}\alpha \right) \right] \\
 & = \left[B + \frac{4}{3}\alpha + 2B - \frac{4}{3}\alpha \right] - \left[B + \frac{4}{3}\alpha - B + \frac{2}{3}\alpha \right] \\
 & = [3B] - \left[\frac{6}{3}\alpha \right] = 3B - 2\alpha
 \end{aligned}$$

$$\begin{aligned}
 50. \text{ distance} &= 30(t-1) + 40(t+2) \\
 &= 30t - 30 + 40t + 80 \\
 &= 70t + 50
 \end{aligned}$$

$$\begin{aligned}
 51. 15x + 25(x+10) &= 15x + 25x + 250 \\
 &= 40x + 250
 \end{aligned}$$

$$\begin{aligned}
 52. 2(2n+1)(3) - 2(n-2)(2) &= 6(2n+1) - 4(n-2) \\
 &= 12n + 6 - 4n + 8 \\
 &= 8n + 14 \text{ dollars}
 \end{aligned}$$

$$\begin{aligned}
 53. (a) 2x^2 - y + 2a + 3y - x^2 - b &= x^2 + 2y + 2a - b \\
 (b) 2x^2 - y + 2a - (3y - x^2 - b) & \\
 &= 2x^2 - y + 2a - 3y + x^2 + b \\
 &= 3x^2 - 4y + 2a + b
 \end{aligned}$$

$$\begin{aligned}
 54. 3a^2 + b - c^2 + 20c^3 - 2b - a^2 - (4c^3 - 4b + 3) & \\
 &= 2a^2 - b - c^2 + 2c^3 - 4c^3 + 4b - 3 \\
 &= 2a^2 + 3b - c^2 - 2c^3 - 3
 \end{aligned}$$

$$\begin{aligned}
 55. |a-b| &= | -(-a+b) | = | -(b-a) | \\
 &= | (-1)(b-a) | \text{ and since } |ab| = |a| \cdot |b| \\
 &= |-1| \cdot |b-a| = 1 \cdot |b-a| \\
 &= |b-a|
 \end{aligned}$$

Yes, it is true that $|a-b| = |b-a|$.

$$\begin{aligned}
 56. \text{ Subtraction is not associative as shown by the} \\
 \text{counterexample } (5-4)-3 &\neq 5-(4-3) \\
 1-3 &\neq 5-1 \\
 -2 &\neq 4
 \end{aligned}$$

1.8 Multiplication of Algebraic Expressions

$$\begin{aligned}
 1. 2s^3(-st^4)^3(4s^2t) &= 2s^3(-s)^3(t^4)^3(4s^2t) \\
 &= -2s^6t^{12}(4s^2t) \\
 &= -8s^8t^{13}
 \end{aligned}$$

$$\begin{aligned}
 2. -2ax(3ax^2 - 4yz) &= (-2ax)(3ax^2) - (-2ax)(4yz) \\
 &= -6a^2x^3 + 8axyz
 \end{aligned}$$

$$\begin{aligned}
 3. (x-2)(x-3) &= x^2 - 3x - 2x + 6 \\
 &= x^2 - 5x + 6
 \end{aligned}$$

$$\begin{aligned}
 4. (2a-b)^2 &= (2a-b)(2a-b) \\
 &= 4a^2 - 2ab - 2ab + b^2 \\
 &= 4a^2 - 4ab + b^2
 \end{aligned}$$

$$5. (a^2)(ax) = a^{2+1}x = a^3x$$

$$6. (2xy)(x^2y^3) = 2x^3y^4$$

$$\begin{aligned}
 7. -ac^2(acz^3) &= (-1)(a^1)(c^2)(a^1)(c^1)(z^3) \\
 &= (-1)a^2c^3z^3 \\
 &= -a^2c^3z^3
 \end{aligned}$$

$$8. -2cs^2(-4cs)^2 = -2cs^2(16c^2s^2) = -32s^4c^3$$

$$9. (2ax^2)^2(-2ax) = 4a^2x^4(-2ax) = -8a^3x^5$$

$$10. 6pq^3(3pq^2)^2 = 6pq^3(9p^2q^4) = 54p^3q^7$$

$$11. i^2(R+2r) = i^2R + 2i^2r$$

$$12. 2x(p-q) = 2px - 2qx$$

$$13. -3s(s^2-5t) = -3s(s^2) + 3s(5t) = -3s^3 + 15st$$

$$14. -3b(2b^2-b) = -6b^3 + 3b^2$$

$$15. 5m(m^2n+3mn) = 5m^3n + 15m^2n$$

16. $a^2bc(2ac-3b^2c) = 2a^3bc^2 - 3a^2b^3c^2$
17. $3M(-M-N+2) = 3M(-M) + 3M(-N) + 3M(2)$
 $= -3M^2 - 3MN + 6M$
18. $-4c^2(-9gc-2c+g^2) = 36gc^3 + 8c^3 - 4g^2c^2$
19. $ax(cx^2)(x+y^3) = ax(cx^2(x) + cx^2(y^3))$
 $= ax(cx^3) + ax(cx^2y^3)$
 $= acx^4 + acx^3y^3$
20. $-2(-3st^3)(3s-4t) = 6st^3(3s-4t) = 18s^2t^3 - 24st^4$
21. $(x-3)(x+5) = x^2 + 5x - 3x - 15 = x^2 + 2x - 15$
22. $(a+7)(a+1) = a^2 + 8a + 7$
23. $(x+5)(2x-1) = x(2x) + x(-1) + 5(2x) + 5(-1)$
 $= 2x^2 - x + 10x - 5$
 $= 2x^2 + 9x - 5$
24. $(4t_1+t_2)(2t_1-3t_2) = 8t_1^2 - 10t_1t_2 + 3t_2^2$
25. $(2a-b)(3a-2b) = 6a^2 - 4ab - 3ab + 2b^2$
 $= 6a^2 - 7ab + 2b^2$
26. $(4w^2-3)(3w^2-1) = 12w^4 - 13w^2 + 3$
27. $(2s+7t)(3s-5t)$
 $= 2s(3s) + 2s(-5t) + 7t(3s) + 7t(-5t)$
 $= 6s^2 - 10st + 21st - 35t^2$
 $= 6s^2 + 11st - 35t^2$
28. $(5p-2q)(p+8q) = 5p^2 + 38pq - 16q^2$
29. $(x^2-1)(2x+5) = 2x^3 + 5x^2 - 2x - 5$
30. $(3y^2+2)(2y-9) = 6y^3 - 27y^2 + 4y - 18$
31. $(x-2y-4)(x-2y+4)$
 $= x^2 - 2xy + 4x - 2xy + 4y^2 - 8y - 4x + 8y - 16$
 $= x^2 + 4y^2 - 4xy - 16$
32. $(2a+3b+1)(2a+3b-1)$
 $= 4a^2 + 6ab - 2a + 6ab + 9b^2 - 3b + 2a + 3b - 1$
 $= 4a^2 + 9b^2 + 12ab - 1$
33. $2(a+1)(a-9) = 2(a^2 - 8a - 9) = 2a^2 - 16a - 18$
34. $-5(y-3)(y+6) = -5(y^2 + 3y - 18)$
 $= -5y^2 - 15y + 90$
35. $-3(3-2T)(3T+2) = (-9+6T)(3T+2)$
 $= -27T - 18 + 18T^2 + 12T$
 $= 18T^2 - 15T - 18$
36. $2n(5-n)(6n+5) = (10n-2n^2)(6n+5)$
 $= 60n^2 + 50n - 12n^3 - 10n^2$
 $= -12n^3 + 50n^2 + 50n$
37. $2L(L+1)(4-L) = 2L(4L-L^2+4-L)$
 $= 2L(-L^2+3L+4)$
 $= -2L^3 + 6L^2 + 8L$
38. $ax(x+4)(7-x^2) = ax(7x-x^3+28-4x^2)$
 $= 7ax^2 - ax^4 + 28ax - 4ax^3$
39. $(2x-5)^2 = 4x^2 - 20x + 25$
40. $(x-3y)^2 = x^2 - 6xy + 9y^2$
41. $(x_1+3x_2)^2 = (x_1+3x_2)(x_1+3x_2)$
 $= x_1^2 + 3x_1x_2 + 3x_1x_2 + 9x_2^2$
 $= x_1^2 + 6x_1x_2 + 9x_2^2$
42. $(7m+1)^2 = 49m^2 + 14m + 1$
43. $(xyz-2)^2 = x^2y^2z^2 - 4xyz + 4$
44. $(b-6x^2)^2 = b^2 - 2(b)(6x^2) + (6x^2)^2$
 $= b^2 - 12bx^2 + 36x^4$
45. $2(x+8)^2 = 2(x^2 + 16x + 64) = 2x^2 + 32x + 128$

$$46. 3(3R+4)^2 = 3(9R^2 + 24R + 16) = 27R^2 + 72R + 48$$

$$\begin{aligned} 47. (2+x)(3-x)(x-1) &= (6+x-x^2)(x-1) \\ &= 6x - 6 + x^2 - x - x^3 + x^2 \\ &= -x^3 + 2x^2 + 5x - 6 \end{aligned}$$

$$\begin{aligned} 48. (3x-c^2)^3 &= (9x^2-6xc^2+c^4) \cdot (3x-c^2) \\ &= 27x^3 - 9x^2c^2 - 18x^2c^2 + 6xc^4 + 3xc^4 - c^6 \\ &= 27x^3 - 27x^2c^2 + 9xc^4 - c^6 \end{aligned}$$

$$\begin{aligned} 49. 3T(T+2)(2T-1) &= (3T^2+6T)(2T-1) \\ &= 6T^3 - 3T^2 + 12T^2 - 6T \\ &= 6T^3 + 9T^2 - 6T \end{aligned}$$

$$\begin{aligned} 50. [(x-2)^2(x+2)]^2 &= [(x^2-4x+4)(x+2)]^2 \\ &= [x^3+2x^2-4x^2-8x+4x+8]^2 \\ &= (x^3-2x^2-4x+8)(x^3-2x^2-4x+8) \\ &= x^6-2x^5-4x^4+8x^3-2x^5+4x^4+8x^3-16 \\ &\quad -4x^4+8x^3+16x^2-32x+8x^3-16x^2-32x+64 \\ &= x^6-4x^5-4x^4+32x^3-16x^2-64x+64 \end{aligned}$$

$$\begin{aligned} 51. (a) \text{ For } x=3, y=4, \\ (x+y)^2 &= (3+4)^2 = 7^2 = 49 \\ x^2+y^2 &= 3^2+4^2 = 9+16 = 25 \\ \text{which shows } (x+y)^2 &\neq x^2+y^2. \end{aligned}$$

$$\begin{aligned} (b) \text{ For } x=3, y=4, \\ (x-y)^2 &= (3-4)^2 = (-1)^2 = 1 \\ x^2-y^2 &= 3^2-4^2 = 9-16 = -7 \\ \text{which shows } (x-y)^2 &\neq x^2-y^2. \end{aligned}$$

$$\begin{aligned} 52. (98)(102) &= (100-2)(100+2) = 100^2 - 2^2 \\ &= 10,000 - 4 = 9996 \end{aligned}$$

53. Let $1 < n < 9$, $n^2 - 1 = (n-1)(n+1)$ which shows the square of the integer minus 1 = product of the integer before n and the integer after n .

$$\begin{aligned} 54. (x-2)(x+3)(x+2)(x-3) \\ &= (x-2)(x+2)(x+3)(x-3) \\ &= (x^2-4)(x^2-9) \\ &= x^4 - 9x^2 - 4x^2 + 36 \\ &= x^4 - 13x^2 + 36 \end{aligned}$$

$$\begin{aligned} 55. (x+y)^3 &= (x+y)(x+y)(x+y) \\ &= (x^2+2xy+y^2)(x+y) \\ &= x^3+x^2y+2x^2y+2xy^2+xy^2+y^3 \\ &= x^3+3x^2y+3xy^2+y^3 \neq x^3+y^3 \end{aligned}$$

$$\begin{aligned} 56. (x+y)(x^2-xy+y^2) \\ &= x^3-x^2y+xy^2+x^2y-xy^2+y^3 \\ &= x^3+y^3 \end{aligned}$$

$$\begin{aligned} 57. P(1+0.01r)^2 &= P(1+0.02r+0.0001r^2) \\ &= P+0.02Pr+0.0001Pr^2 \end{aligned}$$

$$\begin{aligned} 58. w(1-x)(4-x^2) &= w(4-x^2-4x+x^3) \\ &= 4w-wx^2-4wx+wx^3 \end{aligned}$$

$$\begin{aligned} 59. (2R-X)^2 - (R^2+X^2) \\ &= 4R^2-4RX+X^2-R^2-X^2 \\ &= 3R^2-4RX \end{aligned}$$

$$\begin{aligned} 60. (2T^3+3)(T^2-T-3) \\ &= 2T^5-2T^4-6T^3+3T^2-3T-9 \end{aligned}$$

$$\begin{aligned} 61. (n+100)^2 &= n^2+200n+100^2 \\ &= n^2+200n+10,000 \end{aligned}$$

$$\begin{aligned} 62. (T^2-100)(T-10)(T+10) \\ &= (T^2-100)(T^2-100) \\ &= T^4-100T^2-100T^2+100^2 \\ &= T^4-200T^2+10,000 \end{aligned}$$

$$\begin{aligned} 63. (R_1+R_2)^2 - 2R_2(R_1+R_2) \\ &= R_1^2+2R_1R_2+R_2^2-2R_1R_2-2R_2^2 \\ &= R_1^2-R_2^2 \end{aligned}$$

$$\begin{aligned}
 64. \quad & 27x^2 - 24(x-6)^2 - (x-12)^3 \\
 &= 27x^2 - 24(x^2 - 12x + 36) \\
 &\quad - (x^3 - 36x^2 + 432x - 1728) \\
 &= 27x^2 - 24x^2 + 288x - 864 - x^3 + 36x^2 \\
 &\quad - 432x + 1728 \\
 &= -x^3 + 39x^2 - 144x + 864
 \end{aligned}$$

$$7. \quad \frac{-16r^3t^5}{-4r^5t} = \frac{4t^{5-1}}{r^{5-3}} = \frac{4t^4}{r^2}$$

$$8. \quad \frac{51mn^5}{17m^2n^2} = \frac{3n^3}{m}$$

$$9. \quad \frac{(15x^2)(4bx)(2y)}{30bxy} = 4x^2$$

$$10. \quad \frac{(5sT)(8s^2T^3)}{10s^3T^2} = \frac{40s^3T^4}{10s^3T^2} = 4T^2$$

$$11. \quad \frac{6(ax)^2}{-ax^2} = \frac{6a^2x^2}{-ax^2} = -6a^{2-1}x^{2-2} = -6a$$

$$12. \quad \frac{12a^2b}{(3ab^2)} = \frac{12a^2b}{9a^2b^4} = \frac{4}{3b^3}$$

$$13. \quad \frac{3a^2x + 6xy}{3x} = \frac{3a^2x}{3x} + \frac{6xy}{3x} = a^2 + 2y$$

$$14. \quad \frac{2m^2n - 6mn}{2m} = \frac{2m^2n}{2m} - \frac{6mn}{2m} = mn - 3n$$

$$15. \quad \frac{3rst - 6r^2st^2}{3rs} = \frac{3rst}{3rs} - \frac{6r^2st^2}{3rs} = t - 2rt^2$$

$$16. \quad \frac{-5a^2n - 10an^2}{5an} = \frac{-5a^2n}{5ab} - \frac{10an^2}{5an} = -a - 2n$$

$$\begin{aligned}
 17. \quad & \frac{4pq^3 + 8p^2q^2 - 16pq^5}{4pq^2} = \frac{4pq^3}{4pq^2} + \frac{8p^2q^2}{4pq^2} - \frac{16pq^5}{4pq^2} \\
 &= q + 2p - 4q^3
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{a^2x_1x_2^2 + ax_1^3 - ax_1}{ax_1} = \frac{a^2x_1x_2^2}{ax_1} + \frac{ax_1^3}{ax_1} - \frac{ax_1}{ax_1} \\
 &= ax_2^2 + x_1^2 - 1
 \end{aligned}$$

$$19. \quad \frac{2\pi fL - \pi fR^2}{\pi fR} = \frac{2\pi fL}{\pi fR} - \frac{\pi fR^2}{\pi fR} = \frac{2L}{R} - R$$

$$20. \quad \frac{9(aB)^4 - 6aB^4}{3aB^3} = \frac{9a^4B^4}{3aB^3} - \frac{6aB^4}{3aB^3} = 3a^3B - 2B$$

1.9 Division of Algebraic Expressions

$$1. \quad \frac{-6a^2xy^2}{-2a^2xy^5} = 3y^{2-5} = 3y^{-3} = \frac{3}{y^3}$$

$$\begin{aligned}
 2. \quad & \frac{4x^3y - 8x^3y^2 + 2x^2y}{2xy^2} = \frac{4x^3y}{2xy^2} - \frac{8x^3y^2}{2xy^2} + \frac{2x^2y}{2xy^2} \\
 &= \frac{2x^2}{y} - 4x^2 + \frac{x}{y}
 \end{aligned}$$

$$\begin{array}{r}
 3. \quad 2x-1 \overline{) 6x^2 - 7x + 2} \\
 \underline{6x^2 - 3x} \\
 -4x + 2 \\
 \underline{-4x + 2} \\
 0
 \end{array}$$

$$\begin{array}{r}
 4. \quad 4x^2-1 \overline{) 8x^3 - 4x^2 + 0x + 3} \\
 \underline{8x^3 - 2x} \\
 -4x^2 + 2x + 3 \\
 \underline{-4x^2 + 1} \\
 2x + 2
 \end{array}$$

$$\frac{8x^3 - 4x^2 + 3}{4x^2 - 1} = 2x - 1 + \frac{2x + 2}{4x^2 - 1}$$

$$5. \quad \frac{8x^3y^2}{-2xy} = -4x^2y$$

$$6. \quad \frac{-18b^7c^3}{bc^2} = -18b^6c$$

$$21. \frac{3ab^2 - 6ab^3 + 9a^2b^2}{9a^2b^2} = \frac{3ab^2}{9a^2b^2} - \frac{6ab^3}{9a^2b^2} + \frac{9a^2b^2}{9a^2b^2}$$

$$= \frac{1}{3a} - \frac{2b}{3a} + 1$$

$$22. \frac{2x^{n+2} + 4ax^n}{2x^n} = \frac{2x^n x^2}{2x^n} + \frac{4ax^n}{2x^n} = x^2 + 2a$$

$$23. \frac{6y^{2n} - 4ay^{n+1}}{2y^n} = \frac{6y^{2n}}{2y^n} - \frac{4ay^{n+1}}{2y^n} = 3y^{2n-n} - 2ay^{n+1-n}$$

$$= 3y^n - 2ay$$

$$24. \frac{3a(F+T)b^2 - (F+T)}{a(F+T)} = \frac{3a(F+T)b^2}{a(F+T)} - \frac{(F+T)}{a(F+T)}$$

$$= 3b^2 - \frac{1}{a}$$

$$25. \begin{array}{r} 2x+1 \\ x+3 \overline{) 2x^2+7x+3} \\ \underline{2x^2+6x} \\ x+3 \\ \underline{x+3} \\ 0 \end{array}$$

$$26. \begin{array}{r} 3t-4 \\ t-1 \overline{) 3t^2-7t+4} \\ \underline{3t^2-3t} \\ -4t+4 \\ \underline{-4t+4} \\ 0 \end{array}$$

$$27. \begin{array}{r} x-1 \\ x-2 \overline{) x^2-3x+2} \\ \underline{x^2-2x} \\ -x+2 \\ \underline{-x+2} \\ 0 \end{array}$$

$$28. \begin{array}{r} 2x-7 \\ x+1 \overline{) 2x^2-5x-7} \\ \underline{2x^2+2x} \\ -7x-7 \\ \underline{-7x-7} \\ 0 \end{array}$$

$$29. \begin{array}{r} 4x^2-x-1 \\ 2x-3 \overline{) 8x^3-14x^2+x+0} \\ \underline{8x^3-12x^2} \\ -2x^2+x \\ \underline{-2x^2+3x} \\ -2x+0 \\ \underline{-2x+3} \\ -3 \end{array}$$

$$30. \begin{array}{r} 3y+2 \\ 2y+1 \overline{) 6y^2+7y+6} \\ \underline{6y^2+3y} \\ 4y+6 \\ \underline{4y+2} \\ 4 \end{array}$$

$$(6+7y+6y^2) \div (2y+1) = 3y+2 + \frac{4}{2y+1}$$

$$31. \begin{array}{r} Z-2 \\ 4Z+3 \overline{) 4Z^2-5Z-7} \\ \underline{4Z^2+3Z} \\ -8Z-7 \\ \underline{-8Z-6} \\ -1 \end{array}$$

$$(4Z^2-5Z-7) \div (4Z+3) = Z-2 + \frac{-1}{4Z+3}$$

$$32. \begin{array}{r} 2x+1 \\ 3x-4 \overline{) 6x^2-5x-9} \\ \underline{6x^2-8x} \\ 3x-9 \\ \underline{3x-4} \\ -5 \end{array}$$

$$(6x^2-5x-9) \div (3x-4) = 2x+1 + \frac{-5}{3x-4}$$

$$\begin{array}{r}
 x^2 + x - 6 \\
 33. \ x + 2 \overline{) x^3 + 3x^2 - 4x - 12} \\
 \underline{x^3 + 2x^2} \\
 x^2 - 4x \\
 \underline{x^2 + 2x} \\
 -6x - 12 \\
 \underline{-6x - 12} \\
 0
 \end{array}$$

$$\begin{array}{r}
 x^2 + 7x + 9 \\
 34. \ 3x - 2 \overline{) 3x^3 + 19x^2 + 13x - 20} \\
 \underline{3x^3 - 2x^2} \\
 21x^2 + 13x \\
 \underline{21x^2 - 14x} \\
 27x - 20 \\
 \underline{27x - 18} \\
 -2
 \end{array}$$

$$\frac{3x^3 - 19x^2 + 13x - 20}{3x - 2} = x^2 + 7x + 9 - \frac{2}{3x - 2}$$

$$\begin{array}{r}
 2a^2 + 8 \\
 35. \ a^2 - 2 \overline{) 2a^4 + 0a^3 + 4a^2 + 0a - 16} \\
 \underline{2a^4 - 4a^2} \\
 +8a^2 \\
 \underline{8a^2 - 16} \\
 0
 \end{array}$$

$$\frac{2a^4 + 4a^2 - 16}{a^2 - 2} = 2a^2 + 8 + \frac{0}{a^2 - 2}$$

$$\begin{array}{r}
 2T + 1 \\
 36. \ 3T^2 - T + 2 \overline{) 6T^3 + T^2 + 0T + 2} \\
 \underline{6T^3 - 2T^2 + 4T} \\
 3T^2 - 4T + 2 \\
 \underline{3T^2 - T + 2} \\
 -3T
 \end{array}$$

$$\frac{6T^3 + T^2 + 2}{3T^2 - T + 2} = 2T + 1 + \frac{-3T}{3T^2 - T + 2}$$

$$\begin{array}{r}
 x^2 - 2x + 4 \\
 37. \ x + 2 \overline{) x^3 + 0x^2 + 0x + 8} \\
 \underline{x^3 + 2x^2} \\
 -2x^2 + 0x \\
 \underline{-2x^2 - 4x} \\
 4x + 8 \\
 \underline{4x + 8} \\
 0
 \end{array}$$

$$\begin{array}{r}
 D^2 + D + 1 \\
 38. \ D - 1 \overline{) D^3 + 0D^2 + 0D - 1} \\
 \underline{D^3 - D^2} \\
 D^2 + 0D \\
 \underline{D^2 - D} \\
 D - 1 \\
 \underline{D - 1} \\
 0
 \end{array}$$

$$\begin{array}{r}
 x - y \\
 39. \ x - y \overline{) x^2 - 2xy + y^2} \\
 \underline{x^2 - xy} \\
 -xy + y^2 \\
 \underline{-xy + y^2} \\
 0
 \end{array}$$

$$\begin{array}{r}
 3r + 4R \\
 40. \ r - 3R \overline{) 3r^2 - 5rR + 2R^2} \\
 \underline{3r^2 - 9rR} \\
 4rR + 2R^2 \\
 \underline{4rR - 12R^2} \\
 14R^2
 \end{array}$$

$$\begin{array}{r}
 x - y + z \\
 41. \ x + y - z \overline{) x^2 + 0xy + 0xz - y^2 + 2yz - z^2} \\
 \underline{x^2 + xy - xz} \\
 -xy + xz - y^2 + 2yz - z^2 \\
 \underline{-xy - y^2 + yz} \\
 +xz + yz - z^2 \\
 \underline{+xz + yz - z^2} \\
 0
 \end{array}$$

$$\frac{x^2 - y^2 + 2yz - z^2}{x + y - z} = x - y + z + \frac{0}{x + y - z}$$

$$\begin{array}{r} 42. \quad a^2 - 2ab + 2b^2 \overline{) a^4} \qquad \qquad \qquad \frac{a^2 + 2ab + 2b^2}{+ b^4} \\ \underline{a^4 - 2a^3b + 2a^2b^2} \\ + 2a^3b - 2a^2b^2 \\ \underline{+ 2a^3b - 4a^2b^2 + 4ab^3} \\ + 2a^2b^2 - 4ab^3 \\ \underline{+ 2a^2b^2 - 4ab^3 + 4b^4} \\ - 3b^4 \\ \hline \frac{a^4 + b^4}{a^2 - 2ab + 2b^2} = (a^2 + 2ab + 2b^2) + \frac{-3b^4}{a^2 - 2ab + 2b^2} \end{array}$$

$$\begin{array}{r} 43. \quad x + c \overline{) 2x^2 - 9x - 5} \qquad \frac{2x + (-9 - 2c)}{2x^2 + 2cx} \\ \underline{2x^2 + 2cx} \\ (-9 - 2c)x - 5 \\ \underline{(-9 - 2c)x + c(-9 - 2c)} \\ -5 - c(-9 - 2c) \end{array}$$

The required value of c must make $(-9 - 2c) = 1 \Rightarrow$

$c = -5$ and must make $-5 - c(-9 - 2c) = 0$.

$$-5 - c(-9 - 2c) = 0 \Rightarrow 2c^2 + 9c - 5 = 0$$

$$(2c - 1)(c + 5) = 0$$

$$2c - 1 = 0 \quad c + 5 = 0$$

$$c = \frac{1}{2}, \quad c = -5$$

$c = -5$ is the required value.

$$\begin{array}{r} 44. \quad 3x + 4 \overline{) 6x^2 - x + k} \qquad \frac{2x - 3}{6x^2 + 18x} \\ \underline{6x^2 + 18x} \\ -9x + k \\ \underline{-9x - 12} \\ k + 12 \end{array}$$

$$k + 12 = 0$$

$$k = -12$$

$$\begin{array}{r} 45. \quad x + 1 \overline{) x^4} \qquad \frac{x^3 - x^2 + x - 1}{+ 1} \\ \underline{x^4 + x^3} \\ -x^3 \\ \underline{-x^3 - x^2} \\ x^2 \\ \underline{x^2 + x} \\ -x + 1 \\ \underline{-x - 1} \\ 2 \end{array}$$

$$\frac{x^4 + 1}{x + 1} = x^3 - x^2 + x - 1 + \frac{2}{x + 1} \neq x^3$$

$$\begin{array}{r} 46. \quad x + y \overline{) x^3} \qquad \frac{x^2 - xy + y^2}{+ y^3} \\ \underline{x^3 + x^2y} \\ -x^2y \\ \underline{-x^2y - xy^2} \\ xy^2 + y^3 \\ \underline{xy^2 + y^3} \\ 0 \end{array}$$

$$\frac{x^3 + y^3}{x + y} = x^2 - xy + y^2 \neq x^2 + y^2$$

$$\begin{aligned} 47. \quad & \frac{8A^5 + 4A^3\mu^2E^2 - A\mu^4E^4}{8A^4} \\ &= \frac{8A^5}{8A^4} + \frac{4A^3\mu^2E^2}{8A^4} - \frac{A\mu^4E^4}{8A^4} \\ &= A + \frac{\mu^2E^2}{2A} - \frac{\mu^4E^4}{8A^3} \end{aligned}$$

$$\begin{aligned} 48. \quad \frac{6R_1 + 6R_2 + R_1R_2}{6R_1R_2} &= \frac{6R_1}{6R_1R_2} + \frac{6R_2}{6R_1R_2} + \frac{R_1R_2}{6R_1R_2} \\ &= \frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{6} \end{aligned}$$

$$\begin{aligned} 49. \quad \frac{GMm[(R+r)-(R-r)]}{2rR} &= \frac{GMm[R+r-R+r]}{2rR} \\ &= \frac{GMm[2r]}{2rR} = \frac{GMm}{R} \end{aligned}$$

$$\begin{array}{r}
 3T^2 - 2T - 4 \\
 50. \quad T - 2 \overline{) 3T^3 - 8T^2 + 0T + 8} \\
 \underline{3T^3 - 6T^2} \\
 -2T^2 \\
 \underline{-2T^2 + 4T} \\
 -4T + 8 \\
 \underline{-4T + 8} \\
 0
 \end{array}$$

$$\begin{array}{r}
 s^2 + 2s + 6 \\
 51. \quad \left(\frac{s^2 - 2s - 2}{s^4 + 4} \right)^{-1} = \frac{s^4 + 4}{s^2 - 2s - 2} \\
 s^2 - 2s - 2 \overline{) s^4 + 0s^3 + 0s^2 + 0s + 4} \\
 \underline{s^4 - 2s^3 - 2s^2} \\
 2s^3 + 2s^2 + 0s \\
 \underline{2s^3 - 4s^2 - 4s} \\
 6s^2 + 4s + 4 \\
 \underline{6s^2 - 12s - 12} \\
 16s + 16
 \end{array}$$

$$\begin{array}{r}
 t^2 - 3t + 5 \\
 52. \quad 2t + 100 \overline{) 2t^3 + 94t^2 - 290t + 500} \\
 \underline{2t^3 + 100t^2} \\
 -6t^2 - 290t \\
 \underline{-6t^2 - 300t} \\
 10t + 500 \\
 \underline{10t + 500} \\
 0
 \end{array}$$

1.10 Solving Equations

1. (a) $x - 3 = -12$

$$\begin{array}{l}
 x - 3 + 3 = -12 + 3 \\
 x = -9
 \end{array}$$

(b) $x + 3 = -12$

$$\begin{array}{l}
 x + 3 - 3 = -12 - 3 \\
 x = -15
 \end{array}$$

(c) $\frac{x}{3} = -12$

$$\begin{array}{l}
 3\left(\frac{x}{3}\right) = 3(-12) \\
 x = -36
 \end{array}$$

(d) $3x = -12$

$$\begin{array}{l}
 \frac{3x}{3} = \frac{-12}{3} \\
 x = -4
 \end{array}$$

2. $7 - 2t = 9$

$$\begin{array}{l}
 -2t = 9 - 7 \\
 -2t = 2 \\
 t = -1
 \end{array}$$

Check:

$$\begin{array}{l}
 7 - (2)(-1) \stackrel{?}{=} 9 \\
 7 - (-2) \stackrel{?}{=} 9 \\
 7 + 2 \stackrel{?}{=} 9 \\
 9 = 9
 \end{array}$$

3. $x - 7 = 3x - (8 - 6x)$

$$\begin{array}{l}
 x - 7 = 3x - 8 + 6x \\
 x - 7 = 9x - 8 \\
 -8x = -1 \\
 x = \frac{1}{8}
 \end{array}$$

4. $\frac{1.52}{60.0} = \frac{I}{48.0}$

$$\begin{array}{l}
 48.0\left(\frac{1.52}{60.0}\right) = 48\left(\frac{I}{48.0}\right) \\
 1.22A = I \\
 I = 1.22A
 \end{array}$$

5. $x - 2 = 7$

$$\begin{array}{l}
 x = 7 + 2 \\
 x = 9
 \end{array}$$

6. $x - 4 = -1$

$$x = 3$$

7. $x + 5 = 4$

$$\begin{array}{l}
 x = 4 - 5 \\
 x = -1
 \end{array}$$

8. $s + 6 = -3$

$$s = -9$$

9. $\frac{t}{2} = -5$

$$\begin{array}{l}
 t = 2(-5) \\
 t = -10
 \end{array}$$

10. $\frac{x}{-4} = 2$

$$\begin{array}{l}
 x = (-4)(2) \\
 x = -8
 \end{array}$$

11. $4E = -20$

$$E = \frac{-20}{4} = -5$$

12. $2x = 12$

$$x = 6$$

26. $1.5x - 0.3(x - 4) = 6$

$$15x - 3(x - 4) = 60$$

$$15x - 3x + 12 = 60$$

$$12x = 48$$

$$x = 4$$

13. $3t + 5 = -4$

$$3t = -4 - 5$$

$$3t = -9$$

$$t = -3$$

14. $5D - 2 = 13$

$$5D = 15$$

$$D = 3$$

27. $7 - 3(1 - 2p) = 4 + 2p$

$$7 - 3 + 6p = 4 + 2p$$

$$6p - 2p = 4 - 7 + 3$$

$$4p = 0$$

$$p = \frac{0}{4} = 0$$

15. $5 - 2y = -3$

$$-2y = -8$$

$$y = \frac{-8}{-2} = 4$$

16. $8 - 5t = 18$

$$-5t = 10$$

$$t = -2$$

17. $3x + 7 = x$

$$3x - x = -7$$

$$2x = -7$$

$$x = \frac{-7}{2}$$

18. $6 + 4L = 5 - 3L$

$$7L = -1$$

$$L = \frac{-1}{7}$$

28. $3 - 6(2 - 3t) = t - 5$

$$3 - 12 + 18t = t - 5$$

$$17t = 4$$

$$t = \frac{4}{17}$$

19. $2(s - 4) = s$

$$2s - 8 = s$$

$$s - 8 = 0$$

$$s = 8$$

20. $3(4 - n) = -n$

$$12 - 3n = -n$$

$$-2n = -12$$

$$n = 6$$

29. $\frac{4x - 2(x - 4)}{3} = 8$

$$4x - 2(x - 4) = 24$$

$$4x - 2x + 8 = 24$$

$$2x = 16$$

$$x = 8$$

21. $6 - (r - 4) = 2r$

$$6 - r + 4 = 2r$$

$$-3r = -10$$

$$r = \frac{10}{3}$$

22. $5 - (x + 2) = 5x$

$$5 - x - 2 = 5x$$

$$-6x = -3$$

$$x = \frac{1}{2}$$

30. $2x = \frac{3 - 5(7 - 3x)}{4}$

$$8x = 3 - 5(7 - 3x)$$

$$8x = 3 - 35 + 15x$$

$$-7x = -32$$

$$x = \frac{32}{7}$$

23. $2(x - 3) = -x$

$$2x - 6 = -x$$

$$2x + x = 6$$

$$3x = 6$$

$$x = 2$$

24. $4(7 - F) = -7$

$$28 - 4F = -7$$

$$-4F = -35$$

$$F = \frac{35}{4}$$

31. $|x| - 1 = 8$

$$|x| = 9$$

$$x = -9 \text{ or } x = 9$$

32. $2 - |x| = 4$

$$-|x| = 2$$

$$|x| = -2, \text{ no solution}$$

25. $0.1x - 0.5(x - 2) = 2$

$$x - 5(x - 2) = 20$$

$$x - 5x + 10 = 20$$

$$-4x = 10$$

$$x = -2.5$$

33. $5.8 - 0.3(x - 6.0) = 0.5x$

$$5.8 - 0.3x + 1.8 = 0.5x$$

$$7.6 = 0.8x$$

$$x = 9.5$$

$$34. \quad 1.9t = 0.5(4.0 - t) - 0.8$$

$$1.9t = 2.0 - 0.5t - 0.8$$

$$2.4t = 1.2$$

$$t = 0.5$$

$$35. \quad -0.24(C - 0.50) = 0.63$$

$$-0.24C + 0.12 = 0.63$$

$$-0.24C = 0.63 - 0.12$$

$$-0.24C = 0.51$$

$$C = -2.1$$

$$36. \quad 27.5(5.17 - 1.44x) = 73.4$$

$$142.2 - 39.6x = 73.4$$

$$-39.6x = -68.8$$

$$x = 1.74$$

$$37. \quad \frac{x}{2.0} = \frac{17}{6.0}$$

$$x = \frac{34}{6.0}$$

$$x = 5.7$$

$$38. \quad \frac{3.0}{7.0} = \frac{x}{42}$$

$$\frac{126}{7.0} = x$$

$$x = 18$$

$$39. \quad \frac{165}{223} = \frac{13V}{15}$$

$$\frac{15}{13} \left(\frac{165}{223} \right) = \frac{15}{13} \left(\frac{13V}{15} \right)$$

$$V = \frac{2475}{2899} = 0.85$$

$$40. \quad \frac{276x}{17.0} = \frac{1360}{46.4}$$

$$x = \frac{1360}{46.4} \cdot \frac{17.0}{276}$$

$$x = 1.81$$

$$41. \quad (a) \quad 2x + 3 = 3 + 2x$$

$$2x + 3 = 2x + 3, \text{ identity}$$

$$(b) \quad 2x - 3 = 3 - 2x$$

$$2x - 3 + 3 = 3 + 3 - 2x$$

$$2x = 6 - 2x$$

$$4x = 6$$

$$x = \frac{3}{2}, \text{ conditional}$$

$$42. \quad \text{For } a = 0, \quad 2x + a = 2x \text{ because } 2x + 0 = 2x$$

$$2x = 2x$$

an identity. For $a \neq 0$, $2x + a = 2x$

$$a = 2x - 2x$$

$$a = 0 \text{ but } a \neq 0$$

$\Rightarrow 2x + a = 2x$ is a contradiction for $a \neq 0$.

Thus, for all values of a ($a = 0$ or $a \neq 0$) $2x + a = 2x$

is either an identity or a contradiction. So there are

no values for a for which $2x + a = 2x$ is a conditional

equation.

43. EQUATION SOLVER
eqn: 0=X-7-3X+(6X-8)
-8)

X-7-3X+(6X-8)=0
X=3.75
bound=(-1E99,1...

3.75 Frac
15/4

44. EQUATION SOLVER
eqn: 0=0.0595-0.525X-0.525X-8.85(X+0.00316)

0.0595-0.525X-0.525X-8.85(X+0.00316)=0
X=.00336362666
bound=(-1E99,1...

45. $2.0v + 40 = 2.5(v + 5.0)$

$2.0v + 40 = 2.5v + 12.5$

$-0.5v = -27.5$

$v = 55 \text{ mi/h}$

46. $15(5.5 + v) = 24(5.5 - v)$

$82.5 + 15v = 132 - 24v$

$39v = 49.5$

$v = 1.3 \text{ km/h}$

47. $1.1 = \frac{T - 76}{40}$

$44 = T - 76$

$T = 120^\circ\text{C}$

48. $1.12V - 0.67V(10.5 - V) = 0$

$1.12V - 7.035 + 0.67V = 0$

$1.79V = 7.035$

$V = 3.9V$

49. $0.14n + 0.06(2000 - n) = 0.09(2000)$

$0.14n + 120 - 0.06n = 180$

$0.08n = 180 - 120$

$n = 750 \text{ gal}$

50. $210(3x) = 55.3x + 38.5(8.25 - 3x)$

$630x = 55.3x + 317.625 - 115.5x$

$690.2x = 317.625$

$x = 0.46 \text{ m}$

51. $\frac{x}{5.5} = \frac{1250}{15}$

$x = \frac{1250(5.5)}{15}$

$x = 460 \text{ mi}$

52. $\frac{1.8}{20} = \frac{x}{16}$

$x = \frac{1.8(16)}{20}$

$x = 1.4 \text{ m}$

1.11 Formulas and Literal Equations

1. $v = v_0 + at$

$v - v_0 = at$

$a = \frac{v - v_0}{t}$

2. $W = \frac{L(wL + 2P)}{8}$

$8W = L(wL + 2P)$

$8W = L^2w + 2LP$

$L^2w = 8W - 2LP$

$w = \frac{8W - 2LP}{L^2}$

3. $V = V_0[1 + b(T - T_0)]$

$V = V_0[1 + bT - bT_0]$

$V = V_0 + bV_0T - bV_0T_0$

$bV_0T_0 = V_0 + bV_0T - V$

$T_0 = \frac{V_0 + bV_0T - V}{bV_0}$

4. $R = R_0 + R_0\alpha T$

$R_0\alpha T = R - R_0$

$\alpha = \frac{R - R_0}{R_0T}$

5. $E = IR$

$\frac{E}{I} = \frac{IR}{I}$

$R = \frac{E}{I}$

6. $PV = nRT$

$T = \frac{PV}{nR}$

7. $rL = g_2 - g_1$

$g_1 = g_2 - rL$

8. $W = S_dT - Q$

$Q = S_dT - W$

9. $Q = SLd^2$

$L = \frac{Q}{Sd^2}$

10. $P = 2\pi Tf$

$T = \frac{P}{2\pi f}$

$$\begin{aligned}
 11. \quad p &= p_a + dgh \\
 p - p_a &= dgh \\
 \frac{p - p_a}{dg} &= \frac{dgh}{dg} \\
 h &= \frac{p - p_a}{dg}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad 2Q &= 2I + A + S \\
 \frac{2Q - A - S}{2} &= I
 \end{aligned}$$

$$\begin{aligned}
 13. \quad A &= \frac{Rt}{PV} \\
 Rt &= APV \\
 t &= \frac{APV}{R}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad ct^2 &= 0.3t - ac \\
 ac &= 0.3t - ct^2 \\
 a &= \frac{0.3t - ct^2}{c}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad 2p + dv^2 &= 2d(C - W) \\
 2p + dv^2 &= 2dC - 2dW \\
 2dC &= 2p + dv^2 + 2dW \\
 C &= \frac{2p + dv^2 + 2dW}{2d}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad T &= \frac{c+d}{v} \\
 Tv &= c+d \\
 d &= Tv - c
 \end{aligned}$$

$$\begin{aligned}
 19. \quad \frac{K_1}{K_2} &= \frac{m_1 + m_2}{m_1} \\
 K_1 m_1 &= K_2 m_1 + K_2 m_2 \\
 K_2 m_2 &= K_1 m_1 - K_2 m_1 \\
 m_2 &= \frac{K_1 m_1 - K_2 m_1}{K_2}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad u &= -\frac{eL}{2u} \\
 L &= -\frac{2um}{e}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad L &= \frac{N\Phi}{i} \\
 Li &= N\Phi \\
 \Phi &= \frac{Li}{N}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad f &= \frac{F}{d - F} \\
 fd - fF &= F \\
 fd &= F + fF \\
 d &= \frac{F + fF}{f}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad v &= \frac{V(m + M)}{m} \\
 mv &= Vm + VM \\
 VM &= mv - mV \\
 M &= \frac{mv - mV}{V}
 \end{aligned}$$

$$\begin{aligned}
 24. \quad A_1 &= A(M + 1) \\
 A_1 &= AM + A \\
 M &= \frac{A_1 - A}{A}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad T &= 3(T_2 - T_1) \\
 T &= 3T_2 - 3T_1 \\
 T_1 &= \frac{3T_2 - T}{3}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad p_2 &= p_1 + rp_1(1 - p_1) \\
 p_2 - p_1 &= rp_1(1 - p_1) \\
 r &= \frac{p_2 - p_1}{p_1(1 - p_1)}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad Q_1 &= P(Q_2 - Q_1) \\
 Q_1 &= PQ_2 - PQ_1 \\
 Q_1(1 + P) &= PQ_2 \\
 Q_2 &= \frac{Q_1 + PQ_1}{P}
 \end{aligned}$$

$$\begin{aligned}
 30. \quad p - p_a &= dg(y_2 - y_1) \\
 y_2 - y_1 &= \frac{p - p_a}{dg} \\
 y_2 &= \frac{p - p_a}{dg} + y_1
 \end{aligned}$$

$$\begin{aligned}
 21. \quad a &= \frac{2mg}{M + 2m} \\
 aM + 2ma &= 2mg \\
 aM &= 2mg - 2ma \\
 M &= \frac{2mg - 2ma}{a}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad C_0^2 &= C_1^2(1 + 2V) \\
 C_0^2 &= C_1^2 + 2C_1^2V \\
 V &= \frac{C_0^2 - C_1^2}{2C_1^2}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad N &= r(A - s) \\
 N &= rA - rs \\
 rs &= rA - N \\
 s &= \frac{rA - N}{r}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad T_2 &= T_1 - \frac{h}{100} \\
 \frac{h}{100} &= T_1 - T_2 \\
 h &= 100(T_1 - T_2)
 \end{aligned}$$

$$31. \quad N = N_1T - N_2(1-T)$$

$$N + N_2(1-T) = N_1T$$

$$N_1 = \frac{N + N_2 - N_2T}{T}$$

$$32. \quad t_a = t_c + (1-h)t_m$$

$$\frac{t_a - t_c}{t_m} = 1 - h$$

$$h = 1 - \frac{t_a - t_c}{t_m}$$

$$33. \quad L = \pi(r_1 + r_2) + 2x_1 + x_2$$

$$L = \pi r_1 + \pi r_2 + 2x_1 + x_2$$

$$\pi r_1 = L - \pi r_2 - 2x_1 - x_2$$

$$r_1 = \frac{L - \pi r_2 - 2x_1 - x_2}{\pi}$$

$$34. \quad I = \frac{VR_2 + VR_1(1+\mu)}{R_1R_2}$$

$$\frac{IR_1R_2 - VR_2}{VR_1} = 1 + \mu$$

$$\mu = \frac{IR_1R_2 - VR_2}{VR_1} - 1$$

$$35. \quad P = \frac{V_1(V_2 - V_1)}{gJ}$$

$$gJP = V_1V_2 - V_1^2$$

$$gJP + V_1^2 = V_1V_2$$

$$V_2 = \frac{gJP + V_1^2}{V_1}$$

$$36. \quad W = T(S_1 - S_2) - Q$$

$$\frac{W + Q}{T} = S_1 - S_2$$

$$S_2 = S_1 - \frac{W + Q}{T}$$

$$37. \quad C = \frac{2eAk_1k_2}{d(k_1 + k_2)}$$

$$Cd(k_1 + k_2) = 2eAk_1k_2$$

$$e = \frac{Cd(k_1 + k_2)}{2Ak_1k_2}$$

$$38. \quad d = \frac{3LPx^2 - Px^3}{GEI}$$

$$GEId = 3LPx^2 - Px^3$$

$$3LPx^2 = GEId + Px^3$$

$$L = \frac{GEId + Px^3}{3Px^2}$$

$$39. \quad V = C\left(1 - \frac{n}{N}\right)$$

$$V = C - \frac{C}{N} \cdot n$$

$$\frac{C}{N} \cdot n = C - V$$

$$n = \frac{N}{C}(C - V) = N - \frac{NV}{C}$$

$$40. \quad \frac{p}{P} = \frac{AI}{B + AI}$$

$$p(B + AI) = PAI$$

$$pB + pAI = PAI$$

$$pB = PAI - pAI$$

$$B = \frac{PAI - pAI}{p}$$

$$41. \quad e = \frac{T_1}{T_1 + T_2}$$

$$T_1 + T_2 = \frac{T_2}{e}$$

$$T_1 = \frac{T_2}{e} - T_2 = \frac{850}{0.45} - 850$$

$$T_1 = 1040 \text{ K}$$

$$42. \quad P_t = P_c(1 + 0.500m^2)$$

$$P_c = \frac{P_t}{1 + 0.500m^2} = \frac{685}{1 + 0.500(0.925)^2}$$

$$P_c = 480 \text{ W}$$

$$43. \quad F = \frac{9}{5}C + 32$$

$$90.2 = \frac{9}{5}C + 32$$

$$\frac{5}{9}(90.2 - 32) = C$$

$$C = \frac{5}{9} \times 58.2$$

$$C = 32.3^\circ \text{C}$$

$$44. \quad V = \frac{1}{2}L(B + b)$$

$$b = \frac{2V}{L} - B = \frac{2(38.6)}{16.1} - 2.63$$

$$b = 2.17 \text{ ft}^2$$

$$45. \quad V_1 = \frac{VR_1}{R_1 + R_2}$$

$$R_1 + R_2 = \frac{VR_1}{V_1}$$

$$R_2 = \frac{VR_1}{V_1} - R_1 = \frac{12.0(3.56)}{6.30} - 3.56$$

$$R_2 = 3.22\Omega$$

$$46. \quad E = \frac{1}{q + p(1 - q)}$$

$$qE + Ep(1 - q) = 1$$

$$Ep(1 - q) = 1 - qE$$

$$p = \frac{1 - qE}{E(1 - q)} = \frac{1 - 0.83(0.66)}{0.66(1 - 0.83)}$$

$$p = 4$$

$$47. \quad d = v_2 \cdot 4 + v_1(t + 2)$$

$$d = 4v_2 + v_1t + 2v_1$$

$$v_1t = d - 4v_2 - 2v_1$$

$$t = \frac{d - 4v_2 - 2v_1}{v_1} \text{ h}$$

$$48. \quad x + 15y = C$$

$$15y = C - x$$

$$y = \frac{C - x}{15}$$

1.12 Applied Word Problems

1. x = number of 1.5Ω resistors

$34 - x$ = number of 2.5Ω resistors

$$1.5x + 2.5(34 - x) = 56$$

$$1.5x + 85 - 2.5x = 56$$

$$-1.0x = -29$$

$$x = 29$$

$$34 - x = 5$$

29 1.5Ω resistors 5 2.5Ω resistors

2. x = number of slides with 5 mg

$x - 3$ = number of slides with 6 mg

$$5x = 6(x - 3)$$

$$5x = 6x - 18$$

$$-x = -18$$

$$x = 18$$

18 slides with 5 mg

3. $29,500t = 6000 + 27,500t$

$$2000t = 6000$$

$$t = 3.000 \text{ h}$$

4. $0.0500(7600) + 0.500x = 0.100(7600 + x)$

$$380 + 0.500x = 760 + 0.100x$$

$$0.400x = 380$$

$$x = 950 \text{ L}$$

5. x = cost 6 years ago

$x + 5000$ = cost today

$$x + (x + 5000) = 49,000$$

$$2x + 5000 = 49,000$$

$$2x = 44,000$$

$$x = 22,000$$

$$x + 5000 = 27,000$$

\$22,000 six years ago \$27,000 is the cost today

6. Let x = flow rate of first stream in ft^3/s ;

$x + 1700$ = flow rate of second stream in ft^3/s

$$(x + x + 1700) \cdot 3600 = 1.98 \times 10^7$$

$$x = 1900 \text{ ft}^3/\text{s}$$

$$x + 1700 = 3600 \text{ ft}^3/\text{s}$$

7. Let x = recycled cars the first year;
 $x + 700\,000$ = recycled cars the second year
 $x + x + 700\,000 = 4.5 \times 10^6$
 $2x = 3\,800\,000$
 $x = 1.9$ million the first year;

2.6 million the second year

8. Let x = number of times accessed on 1st day
 $\frac{x}{4} + 4000$ = number of times accessed on 2nd day
 $\frac{x}{4}$ = number of times accessed on 3rd day
 $x = \frac{x}{4} + 4000 + \frac{x}{4}$ from which
 $x = 8000$, 1st day
 $\frac{x}{4} + 4000 = 6000$, 2nd day
 $\frac{x}{4} = 2000$, 3rd day

9. Let x = number of acres @ \$200
 $200 \cdot x + 300 \cdot (140 - x) = 37,000$
 $2x + 420 - 3x = 370$
 $x = 50$ acres @ \$200
 $140 - x = 90$ acres @ \$300

10. Let x = mg in first dose
 $x + 660$ = mg in second dose
 $x + x + 660 = 2000$
 $x = 670$ mg in first dose
 $x + 660 = 1330$ mg in second dose

11. x = amount after ppm/h
 $5x = 3(150)$
 $x = 90$
 $150 - 90 = 60$ ppm/h reduction

12. Let x = number of teeth in 1st gear
 $x + 13$ = number of teeth in 2nd gear
 $(x + 13) + 15 = x + 28$ = number of teeth in 3rd gear
 $x + x + 13 + x + 28 = 107$
 $3x + 41 = 107$

$$3x = 66$$

$$x = 22$$

$$x + 13 = 35$$

$$x + 28 = 50$$

The three gears have 22, 35, and 50 teeth.

13. Let x = 15 m girders
 $x - 4$ = 18 m girders
 $15x = 18(x - 4)$
 $3x = 72$
 $x = 24$,
 so there are twenty 18 m girders needed.

14. Let x = number of gal used per week to use up
 original supply in eight weeks.
 $8x = 6(x + 5000)$ from which
 $x = 15,000$
 $8x = 120,000$ gal in original supply

15. $x + 2x + (x + 9.2) = 0$
 $4x = -9.2 \Rightarrow x = -2.3 \mu A$ for the first current
 $2x = -4.6 \mu A$ for the second current
 $x + 9.2 = 6.9 \mu A$ for the third current

16. Let x = number of trucks in first fleet
 $x + 5$ = number of trucks in second fleet
 $8x + 6(x + 5) = 198$ from which
 $x = 12$ trucks in first fleet
 $x + 5 = 17$ trucks in second fleet

17. Let x = the main pipeline
 $x + 2.6$ = the smaller pipeline
 $3(x + 2.6) + x = 35.4$
 $3x + 7.8 + x = 35.4$
 $4x = 27.6$
 $x = 6.9$ km for the main pipeline;
 9.5 km for the smaller pipeline

$$\begin{aligned}
 18. \quad G_1 + G_2 &= 750 \Rightarrow G_2 = 750 - G_1 \\
 0.65G_1 + 0.75G_2 &= 530 \\
 0.65G_1 + 0.75(750 - G_1) &= 530 \\
 0.65G_1 + 562.5 - 0.75G_1 &= 530 \\
 -0.10G_1 &= -32.5 \\
 G_1 &= 325 \text{ MW} \\
 G_2 &= 750 - G_1 = 425 \text{ MW}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad C + D &= 54 \Rightarrow D = 54 - C \\
 15C + 18D &= 876 \\
 15C + 18(54 - C) &= 876 \\
 15C + 972 - 18C &= 876 \\
 -3C &= -96 \\
 C &= 32 \text{ CD's} \\
 D &= 54 - C = 22 \text{ DVD's}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad 0.75(20,000) &= 15,000 \text{ after taxes} \\
 x + y &= 15,000 \Rightarrow y = 15,000 - x \\
 0.40x - 0.10y &= 2000 \\
 0.40x - 0.10(15,000 - x) &= 2000 \\
 0.4x - 1500 + 0.1x &= 2000 \\
 0.5x &= 3500 \\
 x &= \$7000 \text{ at 40\% gain} \\
 y &= 15,000 - x = \$8000 \text{ at 10\% gain}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \text{Let } d &= \text{slope length} \\
 d &= vt, t = \frac{d}{v}, \frac{d}{v_1} = 24 = \frac{d}{v_2} \\
 \frac{d}{50} &= 24 - \frac{d}{140}; \frac{d}{50} + \frac{d}{140} = 24 \\
 0.0267d &= 24; d = 900 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad \text{Let } x &= \text{speed of sound} \\
 (x - 120)(1.0) + (x + 410)(3.0) &= 3990 \text{ from which} \\
 x &= 720 \text{ mi/h,} \\
 &\text{speed of sound}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \text{Let } v &= \text{speed of French train} \\
 v\left(\frac{17}{60}\right) + (v - 8)\left(\frac{17}{60}\right) &= 50 \Rightarrow 17v + 17v - 136 \\
 3000 &\Rightarrow 34v = 3136 \Rightarrow v = 92.2 \\
 v - 8 &= 84.2
 \end{aligned}$$

The speed of the French train is 92.2 km/h. The speed of the English train is 84.2 km/h.

$$\begin{aligned}
 24. \quad \text{Let } t &= \text{time until appointment} \\
 60.0(t - 10) &= 45.0(t - 5.0) \text{ from which} \\
 t &= 25 \text{ min until the appointment}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \text{Let } x - 30 &= \text{time first car started race;} \\
 x &= \text{time second car started race} \\
 260(x - 30) &= 240(x) \\
 260x - 7800 &= 240x \\
 20x &= 7800 \\
 x &= 390 \text{ s}
 \end{aligned}$$

8 laps = $250 \times 8 = 20$ mi, 105,600 ft, total distance
 $d = vt = 260 \times 390 = 101,400 \text{ ft} < 105,600 \text{ ft}$
 So, the first car is ahead after 8 laps.

$$\begin{aligned}
 26. \quad \text{Let } x &= \# \text{ of 1st type, tested 0.5\% defective} \\
 6100 - x &= \# \text{ of 2nd type tested, 0.8\% defective} \\
 0.005x + 0.008(6100 - x) &= 38 \text{ from which} \\
 x &= 3600 \text{ of first type were tested} \\
 6100 - x &= 2500 \text{ of second type were tested}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \text{Assume the customer is located between A and B} \\
 \text{at a distance } x \text{ from A.} \\
 3.40 + 0.0002x &= 3.20 + 0.0002(228 - x) \\
 x &= 64 \text{ mi from A}
 \end{aligned}$$

$$28. \text{ Let } x = \text{number of L of gasoline added}$$

$$\frac{15}{16} = 93.75\% \text{ gasoline} \left\{ \begin{array}{c} x \\ 100\% \text{ gasoline} \\ 8 - x \\ 75\% \text{ gasoline} \end{array} \right\} 8.0 \text{ L}$$

$$\begin{aligned}
 0.9375(8.0) &= x + 0.75(8.0 - x) \text{ from which} \\
 x &= 6.0 \text{ L of gasoline added}
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \text{Let } x &= \text{number of liters of pure antifreeze added} \\
 x &= \text{number of liters of 50\% antifreeze drained} \\
 0.25(12 - x) + x &= 0.50(12) \\
 3.0 - 0.25x + x &= 6.0 \\
 0.75x &= 3.0 \\
 x &= 4.0 \text{ L}
 \end{aligned}$$

30. Let x = number of lb of sand

$$25\% \text{ sand} \left\{ \begin{array}{|c|} \hline 100\% \ x \text{ sand} \\ \hline 250 - x \\ \hline 22\% \text{ sand} \\ \hline \end{array} \right\} 250 \text{ lbs}$$

$$0.25(250) = x + 0.22(250 - x) \text{ from which}$$

$$x = 9.6 \text{ lbs of sand}$$

31. v = speed of car when it overtakes semitrailer

v = speed of car as it passes semitrailer

In passing, 25 m are covered in 10 s at a relative speed of $v - 70$. Changing 25 m to kilometers, 10 s to hours, and using $d = rt$ gives

$$\frac{25}{1000} = (v - 70) \left(\frac{10}{3600} \right) \text{ from which}$$

$$v = \frac{25(3600)}{1000(10)} + 70$$

$$v = 79 \text{ km/h}$$

32. t = time for primary wave to reach station

$t + 2$ = time for secondary wave to reach station

$$8.0t = 5.0(t + 2) \text{ from which}$$

$$t = \frac{10}{3.0} \text{ min}$$

$$\text{distance} = 8.0t = 8.0 \frac{10}{3.0} \left(\frac{60 \text{ s}}{\text{min}} \right) = 1600 \text{ km}$$

Chapter 1 Review Exercises

1. $(-2) + (-5) - 3 = -7 - 3 = -10$
2. $6 - 8 - (-4) = -2 + 4 = 2$
3. $\frac{(-5)(6)(-4)}{(-2)(3)} = \frac{(-30)(-4)}{-6} = \frac{120}{-6} = -20$
4. $\frac{(-9)(-12)(-4)}{24} = \frac{108(-4)}{24} = \frac{-432}{24} = -18$
5. $-5 - |2(-6)| + \frac{-15}{3} = -5 - |-12| + (-5)$
 $= -5 - 12 - 5$
 $= -17 - 5$
 $= -22$
6. $3 - 5|-3 - 2| - \frac{12}{-4} = 3 - 5|-5| - (-3)$
 $= 3 - 5(5) + 3$
 $= 3 - 25 + 3$
 $= -19$
7. $\frac{18}{3-5} - (-4)^2 = \frac{18}{-2} - 16 = -9 - 16 = -25$
8. $-3(-3)^2 - \frac{-8}{(-2) - |-4|} = -9 - \frac{-8}{-2-4} = -9 - \frac{-8}{-6}$
 $= -9 - \frac{4}{3} = -\frac{31}{3}$
9. $\sqrt{16} - \sqrt{64} = 4 - 8 = -4$
10. $-\sqrt{81+144} = -\sqrt{225} = -15$
11. $(\sqrt{7})^2 - \sqrt[3]{8} = 7 - 2 = 5$
12. $-\sqrt[4]{16} + (\sqrt{6})^2 = -2 + 6 = 4$
13. $(-2rt^2)^2 = 4r^2(t^2)^2 = 4r^2t^4$
14. $(3a^0b^{-2})^3 = (3b^{-2})^3 = 3^3(b^{-2})^3 = 27b^{-6} = \frac{27}{b^6}$
15. $-3mn^{-5}t(8m^{-3}n^4) = -24m^{1-3}n^{-5+4}t = -24m^{-2}n^{-1}t$
 $= -\frac{24t}{m^2n}$
16. $\frac{15p^4q^2r}{5pq^5r} = \frac{3p^3}{q^3}$
17. $\frac{-16N^{-2}(NT^2)}{-2N^0T^{-1}} = 8N^{-2+1}T^{2-(-1)}$
 $= 8N^{-1}T^3$
 $= \frac{8T^3}{N}$
18. $\frac{-35x^{-1}y(x^2y)}{5xy^{-1}} = \frac{-7y^2(x^2y)}{x^2} = -7y^3$
19. $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$
20. $\sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$
21. (a) 8840 has 3 significant digits
 (b) 8840 rounded to 2 significant digits is 8800
22. (a) 21,450 has 3 significant digits
 (b) 21,450 rounded to 2 significant digits is 21,000
23. (a) 9.040 has 4 significant digits
 (b) 9.040 rounded to 2 significant digits is 9.0
24. (a) 0.700 has 3 significant digits
 (b) 0.700 rounded to 2 significant digits is 0.70
25. $37.3 - 16.92(1.067)^2 = 18.03676612$
 on a calculator; 18.0
26. $\frac{8.896 \times 10^{-12}}{3.5954 + 6.0449} = 9.22792859 \times 10^{-13}$, calculator
 $= 9.228 \times 10^{-13}$, 4 significant digits

$$27. \frac{\sqrt{0.1958+2.844}}{3.142(65)^2} = 1.313377619 \times 10^{-4}, \text{ calculator}$$

$$= 1.3 \times 10^{-4}, 2 \text{ significant digits}$$

$$28. \frac{1}{0.03568} + \frac{37.466}{(29.63)^2} = 70.70195457, \text{ calculator}$$

$$= 70.70, 4 \text{ significant digits}$$

$$29. a - 3ab - 2a + ab = a - 2a - 3ab + ab = -a - 2ab$$

$$30. xy - y - 5y - 4xy = xy - 4xy - 6y = -3xy - 6y$$

$$31. 6LC - (3 - LC) = 6LC - 3 + LG = 7LC - 3$$

$$32. -(2x - b) - 3(-x - 5b) = -2x + b + 3x + 15b$$

$$= x + 16b$$

$$33. (2x - 1)(x + 5) = 2x^2 + 10x - x - 5$$

$$= 2x^2 + 9x - 5$$

$$34. (C - 4D)(2C - D) = 2C^2 - CD - 8CD + 4D^2$$

$$= 2C^2 - 9CD + 4D^2$$

$$35. (x + 8)^2 = (x + 8)(x + 8) = x^2 + 8x + 8x + 64$$

$$= x^2 + 16x + 64$$

$$36. (2r - 9x)^2$$

$$= (2r - 9x)(2r - 9x) = 4r^2 - 9rs - 9rs + 81s^2$$

$$= 4r^2 - 18rs + 81s^2$$

$$37. \frac{2h^3k^2 - 6h^4k^5}{2h^2k} = \frac{2h^3k^2}{2h^2k} - \frac{6h^4k^5}{2h^2k}$$

$$= hk - 3h^2k^4$$

$$38. \frac{4a^2x^3 - 8ax^4}{-2ax^2} = \frac{4a^2x^3}{-2ax^2} - \frac{8ax^4}{-2ax^2} = -2ax + 4x^2$$

$$39. 4R - [2r - (3R - 4r)] = 4R - [2r - 3R + 4r]$$

$$= 4R - [6r - 3R]$$

$$= 4R - 6r + 3R$$

$$= 7R - 6r$$

$$40. 3b - [3a - (a - 3b)] + 4a = 3b - [3a - a + 3b] + 4a$$

$$= 3b - [2a + 3b] + 4a$$

$$= 3b - 2a - 3b + 4a$$

$$= -2a + 4a$$

$$= 2a$$

$$41. 2xy - \{3z - [5xy - (7z - 6xy)]\}$$

$$= 2xy - \{3z - [5xy - 7z + 6xy]\}$$

$$= 2xy - \{3z - [11xy - 7z]\}$$

$$= 2xy - \{3z - 11xy + 7z\}$$

$$= 2xy - \{10 - 11xy\}$$

$$= 2xy - 10z + 11xy$$

$$= 13xy - 10z$$

$$42. x^2 + 3b + [(b - y) - 3(2b - y + z)]$$

$$= x^2 + 3b + [b - y - 6b + 3y - 3z]$$

$$= x^2 + 3b + [-5b + 2y - 3z]$$

$$= x^2 - 2b + 2y - 3z$$

$$43. (2x + 1)(x^2 - x - 3) = 2x^3 - 2x^2 - 6x + x^2 - x - 3$$

$$= 2x^3 - x^2 - 7x - 3$$

$$44. (x - 3)(2x^2 - 3x + 1) = 2x^3 - 3x^2 + x - 6x^2 + 9x - 3$$

$$= 2x^3 - 9x^2 + 10x - 3$$

$$45. -3y(x - 4y)^2 = -3y(x^2 - 8xy + 16y^2)$$

$$= -3x^2y + 24xy^2 - 48y^3$$

$$46. -s(4s - 3t)^2 = -s(16s^2 - 24st + 9t^2)$$

$$47. 3p[(q - p) - 2p(1 - 3q)] = 3p[q - p - 2p + 6pq]$$

$$= 3p[q - 3p + 6pq]$$

$$= 3pq - 9p^2 + 18p^2q$$

$$48. 3x[2y - r - 4(s - 2r)] = 3x[2y - r - 4s + 8x]$$

$$= 3x[2y + 7r - 4s]$$

$$= 6xy + 21xr - 12xs$$

$$49. \frac{12p^3q^2 - 4p^4q + 6pq^5}{2p^4q} = \frac{12p^3q^2}{2p^4q} - \frac{4p^4q}{2p^4q} + \frac{6pq^5}{2p^4q}$$

$$= \frac{6q}{p} - 2 + \frac{3q^4}{p^3}$$

$$50. \frac{27s^3t^2 - 18s^4t + 9s^2t}{9s^2t} = \frac{27s^3t^2}{9s^2t} - \frac{18s^4t}{9s^2t} + \frac{9s^2t}{9s^2t}$$

$$= 3st - 2s^2 + 1$$

$$51. \begin{array}{r} 2x-5 \\ x+6 \overline{) 2x^2 + 7x - 30} \\ \underline{2x^2 + 12x} \\ -5x - 30 \\ \underline{-5x - 30} \end{array}$$

$$52. \begin{array}{r} 2x-7 \\ 2x+7 \overline{) 4x^2 + 0x - 41} \\ \underline{2x^2 + 14x} \\ -14x - 41 \\ \underline{-14x - 49} \\ 8 \end{array}$$

$$53. \begin{array}{r} x^2 - 2x + 3 \\ 3x-1 \overline{) 3x^3 - 7x^2 + 11x - 3} \\ \underline{3x^3 - x^2} \\ -6x^2 + 11x \\ \underline{-6x^2 + 2x} \\ 9x - 3 \\ \underline{9x - 3} \end{array}$$

$$54. \begin{array}{r} w^2 - w + 4 \\ w-3 \overline{) w^3 - 4w^2 + 7x - 12} \\ \underline{w^3 - 3w^2} \\ -w^2 + 7w \\ \underline{-w^2 + 3w} \\ 4w - 12 \\ \underline{4w - 12} \end{array}$$

$$55. \begin{array}{r} 4x^3 - 2x^2 + 6x \\ x+3 \overline{) 4x^4 + 10x^3 + 0x^2 + 18x - 1} \\ \underline{4x^4 + 12x^3} \\ -2x^3 + 0x^2 \\ \underline{-2x^3 - 6x^2} \\ 6x^2 + 18x \\ \underline{6x^2 + 18x} \\ -1 \end{array}$$

$$56. \begin{array}{r} 4x^2 - 6x + 2 \\ 2x+3 \overline{) 8x^3 + 0x^2 - 14x + 3} \\ \underline{8x^3 + 12x^2} \\ -12x^2 - 14x \\ \underline{-12x^2 - 18x} \\ 4x + 3 \\ \underline{4x + 6} \\ -3 \end{array}$$

$$57. \begin{aligned} & -3\{(r+s-t) - 2[(3r-2s) - (t-2s)]\} \\ &= -3\{(r+s-t) - 2[3r-2s-t+2s]\} \\ &= -3\{r+s-t-2[3r-t]\} \\ &= -3\{r+s-t-6r+2t\} \\ &= -3\{-5r+s+t\} \\ &= 15r-3s-3t \end{aligned}$$

$$58. \begin{aligned} & (1-2x)(x-3) - (x+4)(4-3x) \\ &= x-3-2x^2+6x-4x+3x^2-16+12x \\ &= x^2+15x-19 \end{aligned}$$

$$59. \begin{array}{r} y^2 + 5y - 1 \\ 2y-1 \overline{) 2y^3 + 9y^2 - 7y + 5} \\ \underline{2y^3 - y^2} \\ 10y^2 - 7y \\ \underline{10y^2 - 5y} \\ -2y + 5 \\ \underline{-2y + 1} \\ 4 \end{array}$$

$$\begin{array}{r}
 3x+4y \\
 60. \quad 2x-y \overline{) 6x^2+5xy-4y^2} \\
 \underline{6x^2-3xy} \\
 8xy-4y^2 \\
 \underline{8xy-4y^2} \\
 0
 \end{array}$$

$$\begin{array}{l}
 61. \quad 3x+1=x-8 \\
 3x-x=-8-1 \\
 2x=-9 \\
 x=\frac{-9}{2}
 \end{array}$$

$$\begin{array}{l}
 62. \quad 4y-3=5y+7 \\
 -3-7=5y-4y \\
 y=-10
 \end{array}$$

$$\begin{array}{l}
 63. \quad \frac{5x}{7} = \frac{3}{2} \\
 2(5x) = 3(7) \\
 10x = 21 \\
 x = \frac{21}{10}
 \end{array}$$

$$\begin{array}{l}
 64. \quad \frac{2(N-4)}{3} = \frac{5}{4} \\
 8(N-4) = 15 \\
 8N-32 = 15 \\
 8N = 47 \\
 N = \frac{47}{8}
 \end{array}$$

$$\begin{array}{l}
 65. \quad 6x-5=3(x-4) \\
 6x-5=3x-12 \\
 3x=-7 \\
 x=\frac{-7}{3}
 \end{array}$$

$$\begin{array}{l}
 66. \quad -2(-4-y)=3y \\
 8+26=3y \\
 y=8
 \end{array}$$

$$\begin{array}{l}
 67. \quad 2s+4(3-s)=6 \\
 2s+12-4s=6 \\
 -2s=-6 \\
 s=3
 \end{array}$$

$$\begin{array}{l}
 68. \quad 2|x|-1=3 \\
 2|x|=4 \\
 |x|=2 \\
 x=\pm 2
 \end{array}$$

$$\begin{array}{l}
 69. \quad 3t-2(7-t)=5(2t+1) \\
 3t-14+2t=10t+5 \\
 5t-14=10t+5 \\
 -5t=19 \\
 t=\frac{-19}{5}
 \end{array}$$

$$\begin{array}{l}
 70. \quad -(8-x)=x-2(2-x) \\
 -8+x=x-4+2x \\
 2x=-4 \\
 x=-2
 \end{array}$$

$$\begin{array}{l}
 71. \quad 2.7+2.0(2.1x-3.4)=0.1 \\
 2.7+2.0(2.1x)-2.0(3.4)=0.1 \\
 2.0(2.1x)=0.1-2.7+2.0(3.4) \\
 x=\frac{0.1-2.7+2.0(3.4)}{2.0(2.1)} \\
 x=1, \text{ calculator} \\
 x=1.0, 2 \text{ significant digits}
 \end{array}$$

$$\begin{array}{l}
 72. \quad 0.250(6.721-2.44x)=2.08 \\
 0.250(6.721)-0.250(2.44x)=2.08 \\
 0.250(2.44x)=2.08-(0.250)(6.721) \\
 x=\frac{2.08-0.250(6.721)}{0.250(2.44)} \\
 x=0.6553278689, \text{ calculator} \\
 x=0.655, 3 \text{ significant digits}
 \end{array}$$

$$73. \quad 60,000,000,000 = 6 \times 10^{10} \text{ bytes}$$

$$74. \quad 25,000 \text{ mi/h} = 2.5 \times 10^4 \text{ mi/h}$$

$$75. 192,000,000 \text{ km} = 1.92 \times 10^8 \text{ km}$$

$$76. 1.02 \times 10^9 \text{ Hz} = 1,020,000,000 \text{ Hz}$$

$$77. 2.53 \times 10^{13} \text{ mi} = 25,300,000,000,000 \text{ mi}$$

$$78. 10^7 \text{ ft}^2 = 10,000,000 \text{ ft}^2$$

$$79. 10^{-12} \text{ W/m}^2 = 0.000000000001 \text{ W/m}^2$$

$$80. 0.00000015 \text{ m} = 1.5 \times 10^{-7} \text{ m}$$

$$81. 1.5 \times 10^{-1} \text{ Bq/L} = 0.15 \text{ Bq/L}$$

$$82. 0.00000018 \text{ m} = 1.8 \times 10^{-7} \text{ m}$$

$$83. R = n^2 z \Rightarrow z = \frac{R}{n^2}$$

$$84. R = \frac{2GM}{c^2} \Rightarrow 2GM = Rc^2 \Rightarrow G = \frac{Rc^2}{2M}$$

$$85. P = \frac{\pi^2 EI}{L^2}$$

$$E = \frac{PL^2}{\pi^2 I}$$

$$86. f = p(c-1) - c(p-1)$$

$$f = pc - p - pc + c$$

$$p = c - f$$

$$87. Pp + Qq = Rr$$

$$Qq = Rr - Pp$$

$$q = \frac{Rr - Pp}{Q}$$

$$88. V = IR + Ir \Rightarrow Ir = V - IR$$

$$r = \frac{V - IR}{I}$$

$$89. d = (n-1)A$$

$$d = nA - A$$

$$na = d + A$$

$$n = \frac{d + A}{A}$$

$$90. mu = (m + M)v \Rightarrow m + M = \frac{mu}{v} \Rightarrow M = \frac{mu}{v} - m$$

$$91. N_1 = T(N_2 - N_3) + N_3 = TN_2 - TN_3 + N_3$$

$$TN_2 = N_1 + TN_3 - N_3$$

$$N_2 = \frac{N_1 + TN_3 - N_3}{T}$$

$$92. q = \frac{KA(B-C)}{L} \Rightarrow Lq = KAB - KAC$$

$$KAB = Lq + KAC$$

$$B = \frac{Lq}{KA} + C$$

$$93. R = \frac{A(T_2 - T_1)}{H}$$

$$RH = AT_2 - AT_1$$

$$AT_2 = RH + AT_1$$

$$T_2 = \frac{RH + AT_1}{A}$$

$$94. Z^2 \left(1 - \frac{\lambda}{2a} \right) = k$$

$$1 - \frac{\lambda}{2a} = \frac{k}{z^2}$$

$$\frac{\lambda}{2a} = 1 - \frac{k}{z^2}$$

$$\lambda = 2a \left(1 - \frac{k}{z^2} \right)$$

$$95. d = kx^2 [3(a+b) - x] = kx^2 [3a + 3b - x]$$

$$3a + 3b - x = \frac{d}{kx^2} \Rightarrow 3a = \frac{d}{kx^2} + x - 3b$$

$$a = \frac{d}{3kx^2} + \frac{x}{3} - b$$

$$96. V = V_o [1 + 3a(T_2 - T_1)] \Rightarrow \frac{V}{V_o} = 1 + 3aT_2 - 3aT_1$$

$$3aT_2 = \frac{V}{V_o} - 1 + 3aT_1$$

$$T_2 = \frac{V}{3aV_o} - \frac{1}{3a} + T_1$$

$$97. \frac{5.25 \times 10^{10}}{6.4 \times 10^4} = 8.2 \times 10^5$$

$$98. t = 0.25\sqrt{h} = 0.25\sqrt{66} = 2.03s$$

$$99. 0.553 \text{ km} = 553 \text{ m}; 553\text{m} - 443\text{m} = 110\text{m}$$

$$100. t = \left(\frac{u}{2650}\right)^2 = \left(\frac{48}{2650}\right)^2 = 3.28 \times 10^{-4} s$$

$$101. \frac{R_1 R_2}{R_1 + R_2} = \frac{0.0275(0.0590)}{0.0275 + 0.0590} = 0.0188\Omega$$

$$102. d = 1.5 \times 10^{11} \sqrt{\frac{m}{M}} = 1.5 \times 10^{11} \sqrt{\frac{5.98 \times 10^{24}}{1.99 \times 10^{30}}}$$

$$d = 2.6 \times 10^8 \text{ m}$$

$$103. (x - 2a) + 3(x + 2a) = x - 2a + 3x + 6a$$

$$= 4x + 4a \text{ ft}$$

$$104. (Ai - R)(1 + i)^2 = (Ai - R)(1 + 2i + i^2)$$

$$= Ai + 2Ai^2 + Ai^3 - R - 2Ri - Ri^2$$

$$105. 4(t + h) - 2(t + h)^2 = 4t + 4h - 2(t^2 + 2th + h^2)$$

$$= 4t + 4h - 2t^2 - 4th - 2h^2$$

$$106. \frac{k^2 r - 2h^2 k + h^2 r v^2}{k^2 r} = 1 - \frac{2h^2}{kr} + \frac{h^2 v^2}{k^2}$$

$$107. 3 \times 18 \div (9 - 6) = 3 \times 18 \div 3 = 54 \div 3 = 18$$

$$3 \times 18 \div 9 - 6 = 54 \div 9 - 6 = 6 - 6 = 0,$$

yes, the value changes

$$108. (3 \times 18) \div 9 - 6 = 54 \div 9 - 6 = 6 - 6 = 0$$

$$3 \times 18 \div 9 - 6 = 54 \div 9 - 6 = 6 - 6 = 0,$$

no, the value does not change

$$109. x - (3 - x) = 2x - 3$$

$$x - 3 + x = 2x - 3$$

$$2x - 3 = 2x - 3, \text{ the equation is an identity}$$

$$110. 7 - (2 - x) = x + 2$$

$$7 - 2 + x = x + 2$$

$$5 = 2, \text{ contradiction}$$

$$111. (x - y)^3 = (x - y)(x - y)(x - y)$$

$$= (-(y - x))(-(y - x))(-(y - x))$$

$$= -(y - x)(y - x)(y - x)$$

$$= -(y - x)^3$$

112. No division is not associative as the counterexample shows.

$$(8 \div 4) \div 2 = 2 \div 2 = 1$$

$$8 \div (4 \div 2) = 8 \div 2 = 4$$

$$113. \frac{8 \times 10^{-3}}{2 \times 10^4} = 4 \times 10^{-7}$$

$$114. \frac{\sqrt{4+36}}{\sqrt{4}} = \frac{\sqrt{40}}{\sqrt{4}} = \frac{\sqrt{4(10)}}{2} = \frac{\sqrt{10}}{2} = \sqrt{10}$$

$$115. x + x + 72 = 190 \Rightarrow 2x = 118 \Rightarrow x = \$59$$

$$x + 72 = \$131$$

$$116. x + x + 1100 = 9500 \Rightarrow 2x = 8400 \Rightarrow x = \$4200$$

$$x + 1100 = \$5300$$

$$117. x + y + z = 560$$

$$x = 2y$$

$$z = 2x$$

from which

$$x = 160 \text{ cm}^3, y = 80 \text{ cm}^3, z = 320 \text{ cm}^3$$

$$118. (5.5 - w)(8.0) = (5.5 + w)(5.0)$$

$$44 - 8.0w = 5.5(5.0) + 5.0w$$

$$w = \frac{44 - 5.5(5.0)}{5.0 + 8.0}$$

$$= 1.269230769, \text{ calculator}$$

$$w = 1.3 \text{ mi/h}$$

$$119. 2.4 \times 10^{-6} R + 2.4 \times 10^{-6} (R + 1200) = 12.0 \times 10^{-3}$$

$$4.86 \times 10^{-6} R = 12.0 \times 10^{-3} - 1200 \times 2.4 \times 10^{-6}$$

$$R = 1900\Omega$$

$$R + 1200 = 3100\Omega$$

120. $x + 4x = 4.0 \Rightarrow 5x = 4.0$

$$x = 0.80 \text{ ppm}$$

$$4x = 3.2 \text{ ppm}$$

121. $\frac{12 \text{ h}}{450m} = \frac{x}{250m}$
 $x = 6.67 \text{ h}$

122. $15G + G = 6.6 \Rightarrow G = \frac{6.6}{16} = 0.4125$, calculator

$$15g = 6.1875, \text{ calculator}$$

mixture contains 6.2L oil and 0.41L gasoline

123. $17.4(t+2) + 21.8t = 1390 \Rightarrow t = \frac{1390 - 34.8}{39.2}$
 $= 34.57142857$,
 calculator

Ships pass 34.6 h after the second ship enters canal.

124. distance pond to fire = distance fire to pond

$$70t = 105\left(\frac{1}{2} - t\right)$$

$$t = 0.3h = 18 \text{ min}$$

125. $x + y = 1000$

$$0.0050x + 0.0075y = 0.0065(1000)$$

from which $x = 400 \text{ L}$

$$y = 600 \text{ L}$$

126. $0.45 = \frac{0.6(20)}{20+x} \Rightarrow 9 + 0.45x = 12$

$$0.45x = 3$$

$$x = 6.67 \text{ mL}$$

127. $\frac{\text{square ft of tile}}{\text{square ft in house}} = 0.25$

$$\frac{0.15(2200)}{2200+x} = 0.25$$

$$x = 290 \text{ ft}^2 \text{ in kitchen and entry}$$

128. $x + y = 200 \Rightarrow y = 200 - x$

$$\frac{9}{24}x + \frac{18}{24}y = \frac{14}{24}(200) \Rightarrow 9x + 18y = 2800$$

$$9x + 18(200 - x) = 2800$$

$$x = 89 \Rightarrow y = 111$$

89gm of 9-karat gold mixed with 111 gm of

18-karat gold will produce a mixture of 200 gm of 14-karat gold.

129. $P = P_0 + P_0rt$

$$P_0rt = P - P_0$$

$$r = \frac{P - P_0}{P_0t}$$

$$r = \frac{7625 - 6250}{6250(4.000)} = 0.055$$

$$r = 5.5\%$$

On a calculator $r = (7625 - 6250) / (6250(4.000))$.