

Chapter 1

Multiple choice questions

1. (c) A rolling ball is an example of a physical phenomenon. A point-like object is a simplified model of an object. Acceleration is a physical quantity for describing motion, while free fall is a model of a process.
2. (b) Average speed, path length, and clock reading are all scalar quantities. Displacement and acceleration are examples of vector quantity.
3. (b) A time interval is the difference between two times. Both statements, (2) The lesson lasted for 45 minutes and (4) An astronaut orbited Earth in 4 hours, are examples of time interval.
4. (a) The student should have said: “The distance between my dorm and the lecture hall is 1 km.” There is no indication of the direction (needed for indicating displacement). The path length depends on the path followed and that is also not indicated.
5. (b) With $x(t) = +12 - 4t + t^2$, the corresponding velocity and acceleration are $v_x(t) = -4 + 2t$ and $a_x(t) = +2$. Therefore, we see that the object is accelerating with $a_x = +2.0 \text{ m/s}^2$. The speed of the object first decreases, reaches zero at $t = 2.0 \text{ s}$, and then increases beyond that. So (b) is not true.
6. (a) The motion of the car is described by graph (a). The non-zero flat part corresponds to the car moving at a constant velocity. The car then begins to slow down (indicated by the negative slope), coming to a stop ($v = 0$, through the x -axis), and moves in the opposite direction with the same acceleration.
7. (d) The average velocity and instantaneous velocity are equal when the object moves at a constant velocity or does not move (zero velocity).
8. (c) At the instant the second ball is released, the first ball not only has traveled 3 cm, it also has acquired non-zero velocity. Therefore, the distance between the balls will increase with time. Mathematically, the positions of the two balls can be written as $y_1(t) = \frac{1}{2}g(t + \Delta t)^2$ and $y_2(t) = \frac{1}{2}gt^2$, where Δt is the time interval between dropping the first ball and the second. The distance between them is given by (taken downward direction to be positive +y) $\Delta y = y_1 - y_2(t) = (g\Delta t)t + \frac{1}{2}g(\Delta t)^2$, which shows clearly that Δy increases with t .

9. (b) The position of the car can be written as $x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, where $x_0 = +20$ m. Since the car is traveling west ($-x$ -axis), $v_{0x} = -12$ m/s. With $v_x(t) = v_{0x} + a_x t$, the acceleration a_x is positive in order to bring the velocity of the car to zero at the stoplight.

10. (c) The velocity-versus-time graph in (c) describes the motion of the car with $v_x(t) = v_{0x} + a_x t$, $v_{0x} = -12$ m/s, and $a_x > 0$.

11. (c) velocity-versus-time graph in (c) can be written as $v_x(t) = v_{0x} + a_x t$, with $v_{0x} < 0$ and $a_x > 0$. The velocity remains negative until the object comes to rest.

12. (c) At the moment the sandbag is released, it has the same upward velocity as the hot air balloon, according to the ground observer 2. Therefore, he sees the sandbag going up first then coming down. On the other hand, observer 1 in the hot air balloon sees the sandbag undergo free fall.

13. (b) The height of the tree is $h = \frac{v^2}{2g} = \frac{(5.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 1.3$ m. The closest value is (b).

14. (c) Whether you drop the ball or throw it down, the acceleration of the ball is due to gravitational force exerted by Earth, and it remains the same: $a_y = -g = -9.8 \text{ m/s}^2$ (where we have taken upward to be $+y$). So statement (c) is incorrect.

15. (c) The total flight time of the ball is $t = 2v_0 / g$, which is linear in v_0 . Thus, the second ball, with twice the initial speed of the first one, will spend twice as much time in flight. Note that the maximum height the ball reaches is given by $y_{\max} = v_0^2 / 2g$, which is quadratic in v_0 .

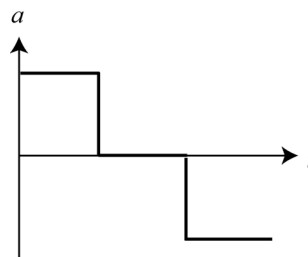
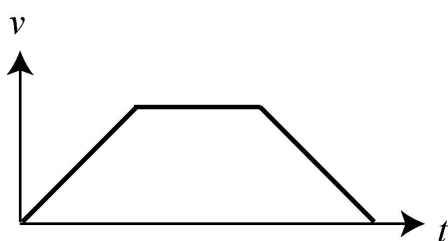
16. (a) The total flight time is given by $t = 2v_0 / g$, where v_0 is the initial speed. The fact that t is linear in v_0 means that if it takes twice as much time for the second ball to come back, the initial speed of the second ball must be twice that of the first one.

Conceptual questions

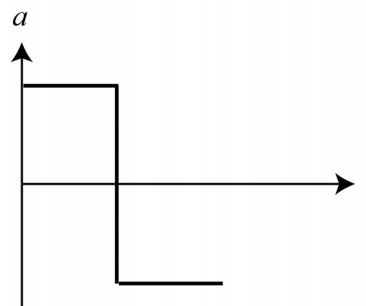
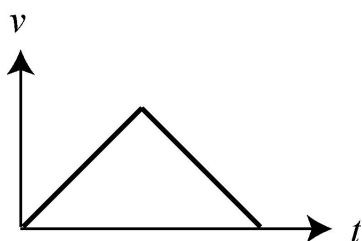
17. Both (b) and (e) correctly describe the Armstrong's motion. Graph (e) corresponds to the fact that his cycling speed is constant, and graph (b) shows that his displacement (from starting point) increases linearly.

18. One scenario is as follows: As the light turns green, the car starts to accelerate from rest. Upon reaching the appropriate speed, the driver stops accelerating and the car moves at a constant speed. Upon seeing a red light some distance ahead, the driver starts to

brake. With constant deceleration, the car comes to a complete stop at the light. The velocity-versus-time and acceleration-versus-time graphs are shown below.

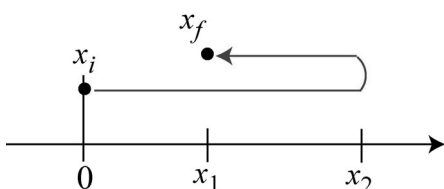


Another possibility is that the car starts from rest with constant acceleration. Seeing another traffic light a short distance ahead, the driver abruptly applies the brake and slows the car to a complete stop. The velocity-versus-time and acceleration-versus-time graphs are shown below.



19. Speed is a scalar quantity that characterizes how fast an object moves. An example of speed is 55 mi/h. On the other hand, velocity refers to the rate of change of position; it is a vector quantity that has both magnitude and direction. An example of a velocity vector is 55 mi/h, due north.

The path length is how far an object moves as it travels from its initial position to its final position. Imagine laying a string along the path the object takes; the length of the string is the path length. On the other hand, distance is the magnitude of displacement. Path length is not necessarily equal to distance. To illustrate the distinction, consider a person running from $x_i = 0$ to $x_f = x_1$ along the path shown below.



The distance the person has run is

$$\Delta x = x_f - x_i = x_1.$$

However, the path length is

$$l = x_2 + (x_2 - x_1) = 2x_2 - x_1.$$

Displacement is the difference between two positions in space; it is a vector quantity that has both magnitude and direction. For example, if an object moves from its initial position at $x_i = 0$ to $x_f = -3.0$ m, its displacement is $d_x = x_f - x_i = -3.0$ m. The displacement is negative since the object moves in the negative x -direction. The distance the object has traveled is 3.0 m, the magnitude of the displacement.

20. Listed below are some examples of physical quantities that are used to describe motion:

Physical quantity	Symbol	SI units	Scalar/vector	What it characterizes
time	t	s	scalar	reading on a clock
time interval	Δt	s	scalar	the difference between two times
position	x, y, z	m	vector	location of an object with respect to a given coordinate system
displacement	\vec{d}	m	vector	the difference between two positions in space
distance	d	m	scalar	the magnitude of displacement
path length	l	m	scalar	how far an object moves as it travels from its initial position to its final position
velocity	\vec{v}	m/s	vector	the rate of change of position
speed	v	m/s	scalar	how fast an object moves
acceleration	\vec{a}	m/s ²	vector	the rate of change of velocity

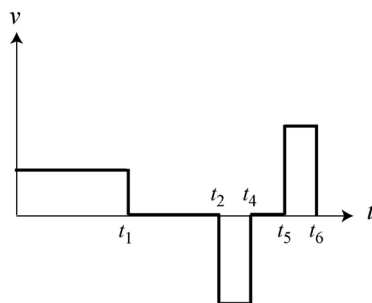
21. In all of the following, we choose a reference frame with Earth as the object of reference.

(a) As the light turns green, a car starts to accelerate from rest. Upon reaching the speed limit, the driver stops accelerating and the car moves at a constant speed. Upon seeing a red light some distance ahead, the driver starts to brake. With constant deceleration, the car comes to a complete stop at the light.

(b) A ball is projected vertically upward with an initial velocity of 40 m/s. About 4.0 seconds later, it reaches a maximum height and its instantaneous velocity is zero. The ball then undergoes free fall to return to its initial position. Its velocity before impacting the ground is -40 m/s.

(c) A ball is initially at rest on a frictionless incline plane. It begins to slide down at a constant acceleration of -7.5 m/s² (positive taken to be *upward*).

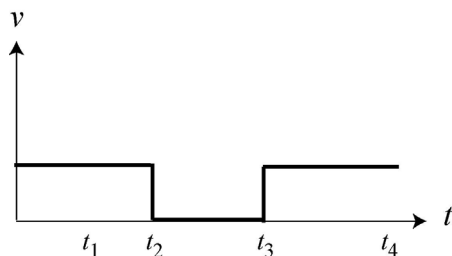
22. (a) The graphs are shown below.



The acceleration is zero throughout.

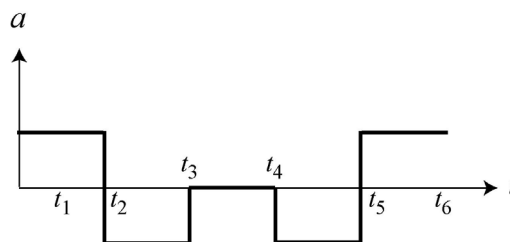
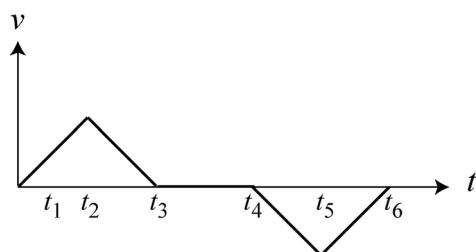
Note that technically there is non-zero acceleration at t_1 , t_2 , t_4 , t_5 , and t_6 because the velocity changes abruptly at these instants. However, we ignore the “endpoints” of each time interval.

(b)



The acceleration is zero throughout.

(c)

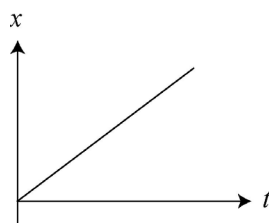


23. Yes. For example, a car driving at a constant speed of 35 mi/h due east has non-zero velocity but zero acceleration, which is defined as the rate of change of velocity.

24. Yes. For example, if you throw a ball vertically upward, when the ball reaches its maximum height, the ball comes to rest momentarily and its instantaneous velocity is zero, but its free-fall acceleration is non-zero.

25. We first measure the distance traveled by the toy truck at regular time intervals. If the truck is moving in a straight line and the distances traveled during each time interval remains unchanged, then we may conclude that the truck is moving at a constant velocity with no acceleration. In this case, we would have a straight line on the position-versus-time graph.

On the other hand, if the distance traveled varies from one time interval to another, then the truck has non-zero acceleration, with changing velocity and speed. In this case, the curve on the position-versus-time graph would not be a straight line, as shown in the figure below.

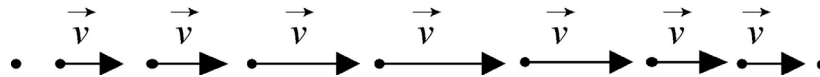


Whether the acceleration is constant or not can be deduced by looking at the velocity-versus-time graph. A straight line would imply a constant acceleration.

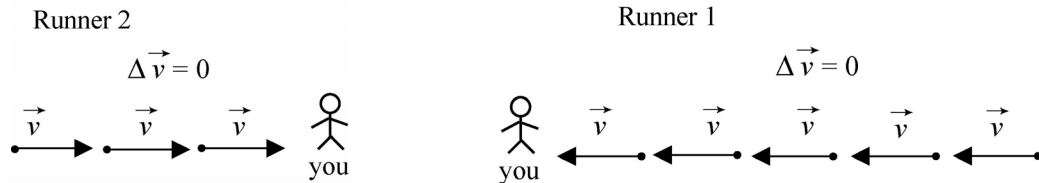
26. Disagree with your friend. At the top of the flight, while the instantaneous velocity of the ball reaches zero, its free-fall acceleration is non-zero.

Problems

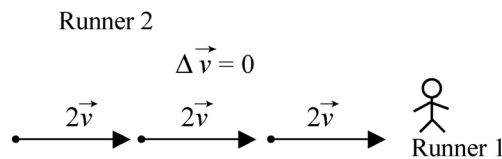
1. The motion diagram is shown below.



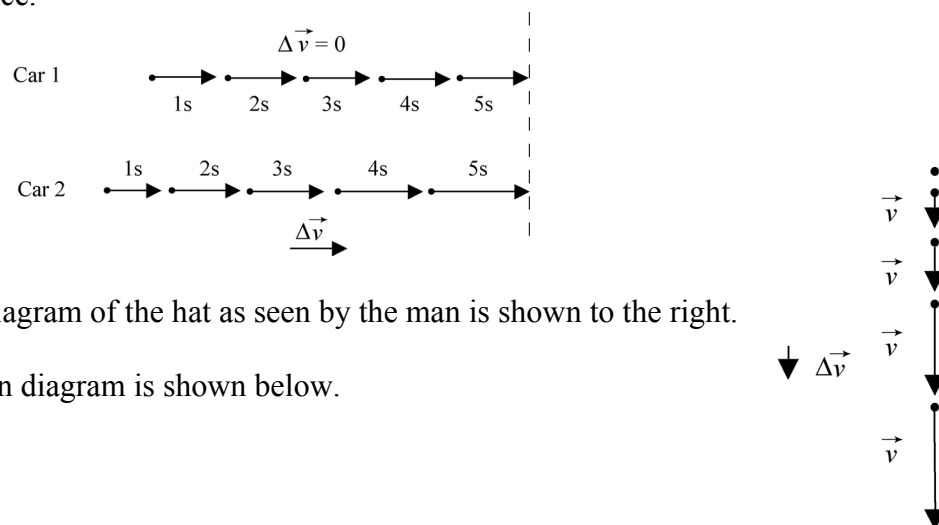
2. (a) The motion diagrams are shown below.



(b) In the reference frame of runner 1, he sees runner 2 coming toward him at a speed equal to $|v_1| + |v_2| = 2v$.

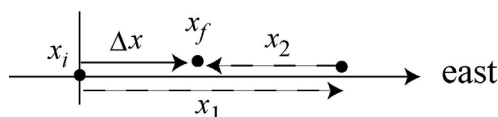


3. The motion diagram is shown below. We choose a reference frame with Earth as the object of reference.



4. The motion diagram of the hat as seen by the man is shown to the right.

5. (a) The motion diagram is shown below.



(b) We choose a reference frame with Earth as the object of reference. The origin of the coordinate system is the initial position of the car and the positive direction is toward east. The position of the car when you stop for lunch is $\Delta x = (+100 \text{ km}) - (50 \text{ km}) = 50 \text{ km}$.

(c) As in (b), we choose Earth to be the object of reference. The origin of the coordinate system is the initial position of the car and the positive direction is toward *west*. The position of the car when you stop for lunch is $\Delta x = (-100 \text{ km}) + (50 \text{ km}) = -50 \text{ km}$.

(d) We choose a reference frame with Earth as the object of reference. However, in this case, let the origin of the coordinate system be +50 km due east of the initial position of the car, and the positive direction is toward east. The position of the car when you stop for lunch then becomes $\Delta x = -(50 \text{ km}) + (+100 \text{ km}) - (50 \text{ km}) = 0$.

6. We choose a reference frame with Earth as the object of reference. For simplicity, let the motion of the two people be one dimensional, and the positive direction is to the right. Suppose the initial and final positions of the two people are

$$\text{A: } x_{Ai} = -20 \text{ m, } x_{Af} = +30 \text{ m;}$$

$$\text{B: } x_{Bi} = +20 \text{ m, } x_{Bf} = +70 \text{ m.}$$

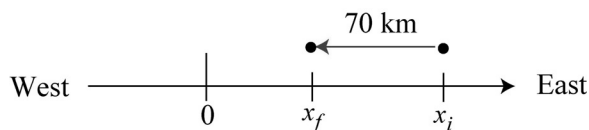
The corresponding displacements are

$$\Delta x_A = x_{Af} - x_{Ai} = (+30 \text{ m}) - (-20 \text{ m}) = +50 \text{ m;}$$

$$\Delta x_B = x_{Bf} - x_{Bi} = (+70 \text{ m}) - (+20 \text{ m}) = +50 \text{ m.}$$

Thus, we see that two people starting and ending their trips at different locations can still have the same displacement vectors.

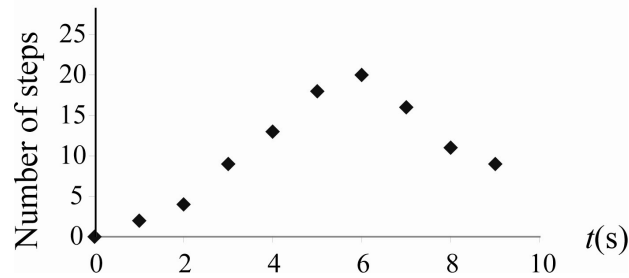
7. Choose Earth to be the object of reference. Let the displacement be positive when moving in the east direction. Thus, the x -component of the displacement vector can be written as $\Delta x = -70 \text{ km}$. On the other hand, if the displacement is defined to be positive when moving in the west direction, then we would have $\Delta x = +70 \text{ km}$.



8. (a) You can simply count your steps or use a pedometer, and a watch to record the time.

(b) There are uncertainties associated with the recording of time as well as the number of steps counted as you move from the front door. Both quantities have been rounded to the nearest integer.

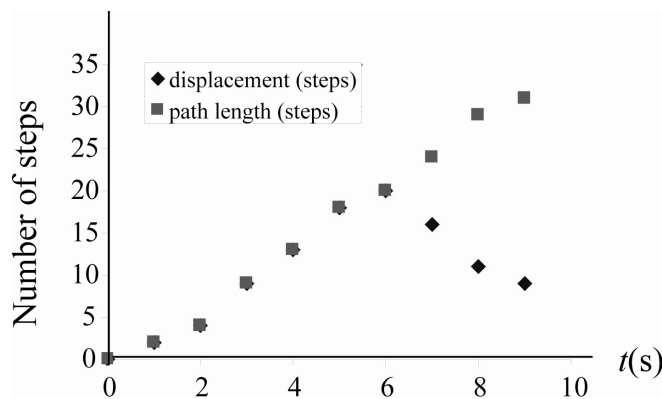
(c) Your position (measured in number of steps) relative to the front door as a function of time (in seconds) is shown below.



(d) The mailman has just delivered your mail. You've been waiting impatiently to receive the new physics textbook you'd ordered. You rushed from the front door of your house to the mailbox located 20 steps away and picked up the mail 6 seconds later. As you hurried back, you started looking through the mail, and were disappointed that the package had not yet arrived.

(e) Assuming the motion to be one-dimensional (along a straight path), the values for x given in Table 1.9 would represent the displacement (relative to the door). In this case, the distance is the same as the path length since both measure the total number of steps walked.

t (s)	1	2	3	4	5	6	7	8	9
x (steps)	2	4	9	13	18	20	16	11	9
path length (steps)	2	4	9	13	18	20	24	29	31



9. To determine the time interval it takes for light to pass through an atomic nucleus, we first need to know the size of the nucleus. The size depends on the atomic mass of the

sample you use, but generally is on the order of a few Fermi ($1 \text{ fm} = 10^{-15} \text{ m}$). We also assume that the nucleons (protons and neutrons) have their mass distributed uniformly throughout the nucleus. By further assuming that the speed of light in the nucleus is the same as that in vacuum, we estimate the time taken to pass through the nucleus to be $t \sim (5 \times 10^{-15} \text{ m}) / (3 \times 10^8 \text{ m/s}) \sim \times 10^{-23} \text{ s}$. Typically, it takes a few yoctoseconds (10^{-24} s) for light to traverse through a nucleus.

10. (a) Using the conversion factors

$$1 \text{ mi} = 5280 \text{ ft} = 1.609 \text{ km} = 1609 \text{ m}$$

$$1 \text{ h} = 60 \text{ min} = 3600 \text{ s},$$

we have

$$65 \frac{\text{mi}}{\text{h}} = 65 \frac{\text{mi}}{\text{h}} \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \cdot \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 95.3 \text{ ft/s}$$

$$65 \frac{\text{mi}}{\text{h}} = 65 \frac{\text{mi}}{\text{h}} \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \cdot \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) = 29.1 \text{ m/s}$$

$$65 \frac{\text{mi}}{\text{h}} = 65 \frac{\text{mi}}{\text{h}} \cdot \left(\frac{1.609 \text{ km}}{1 \text{ mi}} \right) = 104.6 \text{ km/h}$$

(b) The speed of the car in mi/h is

$$100 \text{ km/h} = 100 \frac{\text{km}}{\text{h}} \cdot \left(\frac{1 \text{ mi}}{1.609 \text{ km}} \right) = 62.1 \text{ mi/h}$$

11. With the conversion factors for distance and time

$$1 \text{ mi} = 5280 \text{ ft} = 1.609 \text{ km} = 1609 \text{ m}$$

$$1 \text{ h} = 60 \text{ min} = 3600 \text{ s}$$

we obtain the following conversion factors for speed:

$$1 \text{ mph} = 1 \frac{\text{mi}}{\text{h}} \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \cdot \left(\frac{1609 \text{ m}}{1 \text{ mi}} \right) = 0.447 \text{ m/s}$$

$$1 \text{ mph} = 1 \frac{\text{mi}}{\text{h}} \cdot \left(\frac{1.609 \text{ km}}{1 \text{ mi}} \right) = 1.609 \text{ km/h}$$

$$1 \text{ km/h} = 1 \frac{\text{km}}{\text{h}} \cdot \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \cdot \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) = 0.278 \text{ m/s}$$

$$(a) \ 36 \text{ mph} = (36 \text{ mph}) \cdot \left(\frac{1.609 \text{ km/h}}{1 \text{ mph}} \right) = 57.9 \text{ km/h} = (36 \text{ mph}) \cdot \left(\frac{0.447 \text{ m/s}}{1 \text{ mph}} \right) = 16.1 \text{ m/s}$$

(b)

$$349 \text{ km/h} = (349 \text{ km/h}) \cdot \left(\frac{1 \text{ mph}}{1.609 \text{ km/h}} \right) = 217 \text{ mph} = (349 \text{ km/h}) \cdot \left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}} \right) = 96.9 \text{ m/s}$$

(c)

$$980 \text{ m/s} = (980 \text{ m/s}) \cdot \left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}} \right) = 3528 \text{ km/h} = (3528 \text{ km/h}) \cdot \left(\frac{1 \text{ mph}}{1.609 \text{ km/h}} \right) = 2192 \text{ mph}$$

12. Assume that the hair grows at a rate of approximately 1.3 cm in one month, we have

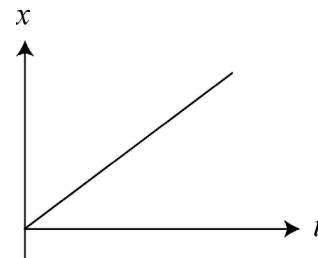
$$1.3 \frac{\text{cm}}{\text{month}} = 1.3 \frac{\text{cm}}{\text{month}} \cdot \left(\frac{0.01 \text{ m}}{1 \text{ cm}} \right) \cdot \left(\frac{1 \text{ month}}{30 \times 86400 \text{ s}} \right) = 5.0 \times 10^{-9} \text{ m/s}$$

or about 5 nm/s.

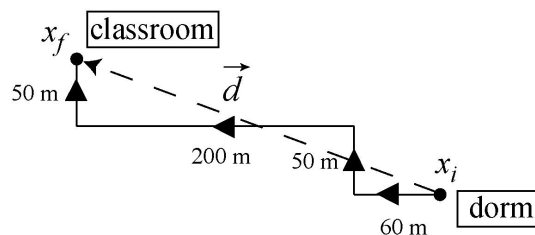
13. (a) Take the time interval for 80 heartbeats to be about 1 minute. The speed of the van is then

$$v_x = \frac{d}{\Delta t} = \frac{1 \text{ mi}}{1 \text{ min}} = \frac{1 \text{ mi}}{1 \text{ min}} \cdot \left(\frac{60 \text{ min}}{1 \text{ h}} \right) = 60 \text{ mi/h} = 60 \text{ mph}$$

(b) We choose a reference frame with Earth as the object of reference. For a person standing at the first exit, he notices that the distance of separation between him and the van increases as $x(t) = v_x t = (60 \text{ mi/h})t$. The position-versus-time graph is shown to the right.



14. (a) An example of the simplified map is shown below.



(b) For the map shown, the total path length is given by

$$l = 60 \text{ m} + 200 \text{ m} + 50 \text{ m} + 50 \text{ m} = 360 \text{ m}$$

Assuming that it takes about 5 minutes to walk from the dorm to the classroom, then the average speed would be about $v = \frac{l}{\Delta t} = \frac{360 \text{ m}}{5 \times 60 \text{ s}} = 1.2 \text{ m/s}$.

15. (a) The story can be: A hiker is 10 km away from his camping ground. He walks along a straight path at a speed of 4.0 km/h to get back to his tent to rest. The time $t = 0$ corresponds to the moment when the hiker has just begun to walk. The observer is another hiker who is already back at the camping ground.

Another story could be: A father and his child are 10 km away from home (where the mother is). The father then carries the child on his back and starts walking toward the mother at a pace of 4 km/h. However, in the reference frame of the child (the observer), it is the mother who is moving toward them at a speed of 4 km/h.

(b) The equation $x(t) = (10 \text{ km}) - (4 \text{ km/h})t$ gives $x(0) = 10 \text{ km}$. If the value 10 km has just one significant digit, the actual distance could fall between 9.5 km and 15 km. However, if 10 km has two significant digits, then the actual distance would be between 9.0 km and 11 km.

16. The average speed of the hike is

$$v = \frac{l}{\Delta t} = \frac{17,000 \text{ steps}}{2.50 \text{ h}} = 6800 \text{ steps/h} = 6.80 \times 10^3 \text{ steps/h}$$

The result above is written in three significant digits, assuming that the value 17,000 has at least three significant digits. However, if the step count by the pedometer is known to only two significant digits, then using scientific notation, we have $l = 1.7 \times 10^4 \text{ steps}$, and the speed would be $v = 6.8 \times 10^3 \text{ steps/h}$. This holds true regardless of whether time is given as 2.5 h or 2.50 h.

17. We interpret “four seconds” to be 4.0 s, with two significant digits. The distance between the clouds and the hikers is then

$$d = vt = (340 \text{ m/s})(4.0 \text{ s}) = 1360 \text{ m} = 1.36 \text{ km}$$

or 1.4 km in two significant digits. To calculate the uncertainty, we write v and t as $v = (3.4 \pm 0.1) \times 10^2 \text{ m/s}$ and $t = (4.0 \pm 0.1) \text{ s}$. Thus, we find the distance to be between $(3.3 \times 10^2 \text{ m/s})(3.9 \text{ s}) = 1287 \text{ m}$ and $(3.5 \times 10^2 \text{ m/s})(4.1 \text{ s}) = 1435 \text{ m}$. Our result should be quoted as $d = (1.4 \pm 0.1) \text{ km}$.

18. The time it takes for light to reach the Earth from the Sun is

$$\Delta t = \frac{d}{c} = \frac{150 \times 10^9 \text{ m}}{3.0 \times 10^8 \text{ m/s}} = 500 \text{ s}$$

which should be written as $5.0 \times 10^2 \text{ s}$ in two significant digits. The uncertainties can be included by reporting the result as $(5.0 \pm 0.1) \times 10^2 \text{ s}$, which means that one could be off by as much as 20 seconds. Note, however, that experimentally both the speed of light and the Sun–Earth distance (the astronomical unit) have been measured to much greater accuracy, and the travel time can be shown to be about 499.2 s.

19. In SI units, 1 light-year is

$$1 \text{ ly} = c\Delta t = (3.00 \times 10^8 \text{ m/s})(365)(86,400 \text{ s}) = 9.46 \times 10^{15} \text{ m}$$

Therefore, the distance to Proxima Centauri is

$$d = (4.22 \pm 0.01) \text{ ly} = (4.22 \pm 0.01)(9.46 \times 10^{15} \text{ m}) = (3.99 \pm 0.009) \times 10^{16} \text{ m}$$

where $0.009 \times 10^{16} \text{ m}$ (or $9 \times 10^{13} \text{ m}$) represents the uncertainty in the calculation.

20. With $1 \text{ y} = 365.25 \text{ days} = 3.16 \times 10^7 \text{ s}$, we find the distance between the Uranus and the Earth to be approximately equal to

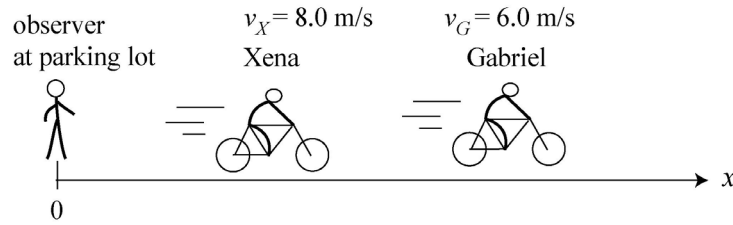
$$d = vt = (1.1 \times 10^4 \text{ m/s})(12)(3.16 \times 10^7 \text{ s}) = 4.2 \times 10^{12} \text{ m}$$

This distance is about 28 times that of the distance from the Earth to the Sun. In our calculation, we have assumed that Uranus remains stationary, and Voyager traveled a straight path. In reality, both the Earth and the Uranus orbit around the Sun, so the Earth–Uranus distance are constantly changing, depending on their relative position in the Solar system. Measurements have shown that the Earth–Uranus distance falls between $2.57 \times 10^{12} \text{ m}$ and $3.15 \times 10^{12} \text{ m}$.

21. **Sketch and translate** From Figure P1.21, we see that both Gabriele and Xena, the objects of interest, are riding their bicycles at constant speeds, as indicated by the horizontal line in the velocity-versus-time plot.

Simplify and diagram We assume that Gabriele and Xena are point-like objects. We choose a reference frame with Earth as the object of reference. We choose the origin of the coordinate system to be the position of a stationary observer at the parking lot, and

the positive direction will be toward the right, in the direction of cycling. The situation is depicted below.



Represent mathematically The positions of Gabriele and Xena with respect to the parking lot as a function of time are given by

$$x_X(t) = x_{X0} + (8.0 \text{ m/s})t$$

$$x_G(t) = x_{G0} + (6.0 \text{ m/s})t$$

where x_{X0} and x_{G0} are the distances between Xena and Gabriele and the observer at time $t = 0$.

Solve and evaluate

(a) At $t = 20 \text{ s}$, the displacements with respect to their initial positions are $\Delta x_G(20 \text{ s}) = (6.0 \text{ m/s})(20 \text{ s}) = 120 \text{ m}$ for Gabriele and $\Delta x_X(20 \text{ s}) = 160 \text{ m}$ for Xena.

(b) The relative speed between Xena and Gabriele is

$$v_{\text{rel}} = v_X(t) - v_G(t) = (8.0 \text{ m/s}) - (6.0 \text{ m/s}) = 2.0 \text{ m/s}.$$

With a separation of $d = 60 \text{ m}$, the time it takes for Xena to catch up would be

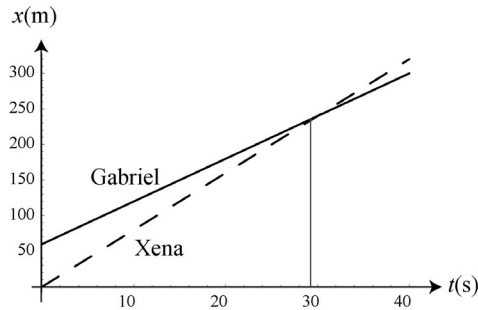
$$t = \frac{d}{v_{\text{rel}}} = \frac{60 \text{ m}}{2.0 \text{ m/s}} = 30 \text{ s}$$

The position of Gabriel as function of time can be written as $x_G(t) = 60 \text{ m} + (6.0 \text{ m/s})t$. Thus, when Xena catches up, $x_G = x_X$, or $(8.0 \text{ m/s})t = 60 \text{ m} + (6.0 \text{ m/s})t$. Solving for t , we find $t = 30 \text{ s}$, which agrees with the answer obtained above.

(c) The position of Gabriel relative to Xena as function of time can be written as

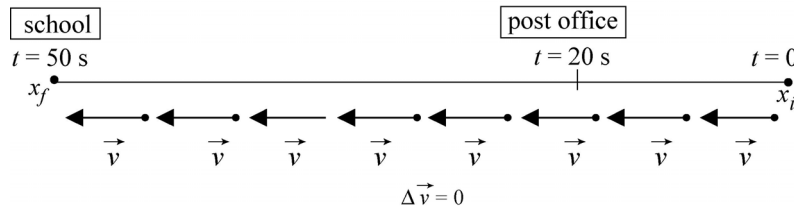
$$x_{G \text{ rel } X}(t) = x_G - x_X = [60 \text{ m} + (6.0 \text{ m/s})t] - (8.0 \text{ m/s})t = 60 \text{ m} - (2.0 \text{ m/s})t.$$

Their positions as function of time are plotted below.

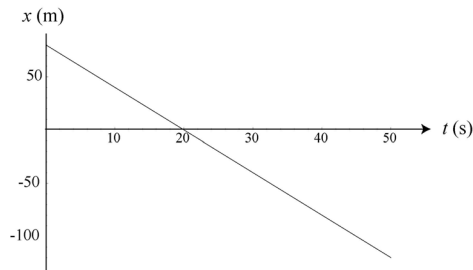


22. (a) The object of reference is the Earth. At $t = 0$, the person was 80 m east of the post office (taken to be the origin). He wanted to get to school that is located 120 m due west of the post office. He started to walk at a constant speed, passing the post office after 20 seconds, and then got to school at $t = 50$ s.

(b) The motion diagram is shown below.

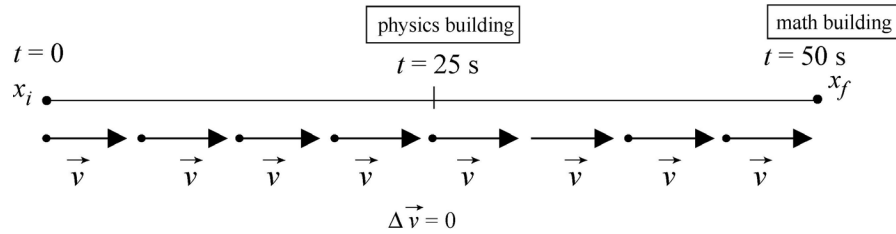


The position as a function of time can be written as $x(t) = 80 \text{ m} - (4.0 \text{ m/s})t$, and is plotted below.

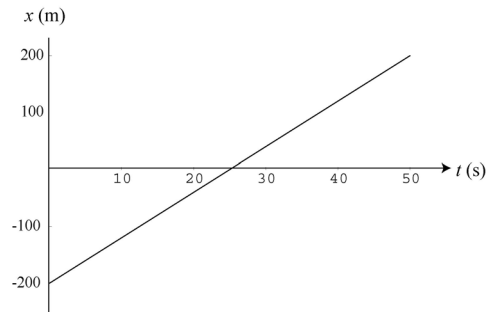


23. (a) The object of reference is Earth, with east as the positive direction. At $t = 0$, your friend Tom is 200 m due west of the physics building (taken to be the origin). He wants to get to the math building located 200 m east of the physics building. He starts to ride his bicycle at a constant speed, passing the physics building after 25 seconds, and then reaches the math building after another 25 seconds.

(b) The motion diagram is shown below.



The position as a function of time can be written as $x(t) = -200 \text{ m} + (8.0 \text{ m/s})t$, and is plotted below.



24. The relative speed between you and your friend is

$$v_{\text{rel}} = v_{\text{friend}} - v_{\text{you}} = 1.3 \text{ m/s} - 1.0 \text{ m/s} = 0.30 \text{ m/s}.$$

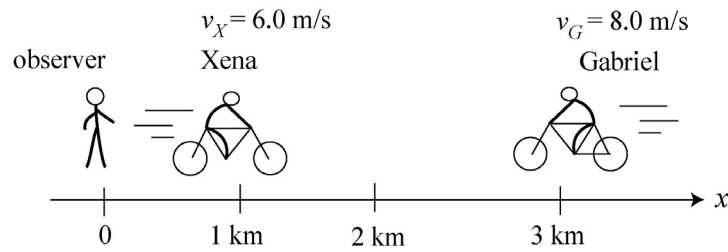
After walking for two minutes ($t_0 = 120 \text{ s}$), the initial separation between you and your friend is $d = v_{\text{you}} t_0 = (1.0 \text{ m/s})(120 \text{ s}) = 120 \text{ m}$. Therefore, the time it takes for your friend to catch up to you would be

$$t = \frac{d}{v_{\text{rel}}} = \frac{1.2 \times 10^2 \text{ m}}{0.30 \text{ m/s}} = 400 \text{ s}$$

or $4.0 \times 10^2 \text{ s}$, in two significant digits. In solving the problem, we have assumed that both you and your friends have walked a straight path at a constant speed (no stopping).

25. **Sketch and translate** Both Gabriele and Xena, the objects of interest, are riding their bicycles at constant speeds, but in opposite directions. We choose a reference frame with Earth as the object of reference.

Simplify and diagram We assume that Gabriele and Xena are point-like objects. The origin of the coordinate system will be the zero mark on the bike path, where an observer stands, and the positive direction will be toward east.



Represent mathematically The positions of Gabriele and Xena with respect to the zero mark of the bike path as a function of time are given by

$$x_G(t) = 3.0 \times 10^3 \text{ m} - (8.0 \text{ m/s})t$$

$$x_X(t) = 1.0 \times 10^3 \text{ m} + (6.0 \text{ m/s})t$$

Solve and evaluate

(a) The functions describing the positions of Gabriele and Xena with respect to Earth are:

$$x_G(t) = 3.0 \times 10^3 \text{ m} - (8.0 \text{ m/s})t$$

$$x_X(t) = 1.0 \times 10^3 \text{ m} + (6.0 \text{ m/s})t$$

(b) The two people meet when $x_G = x_X$. Solving the equation gives

$$3.0 \times 10^3 \text{ m} - (8.0 \text{ m/s})t = 1.0 \times 10^3 \text{ m} + (6.0 \text{ m/s})t,$$

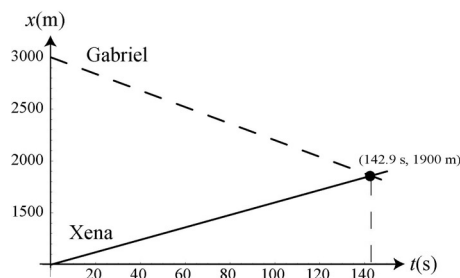
or $t = 143 \text{ s}$, which leads to $x_G(t) = x_X(t) = 3.0 \times 10^3 \text{ m} - (8.0 \text{ m/s})(143 \text{ s}) = 1.86 \times 10^3 \text{ m}$, or approximately 1.9 km, in two significant digits.

The problem can be solved in three different ways. Besides using Earth as the frame of reference, we could have used either Gabriele or Xena as our frame of reference.

(c) Xena's position with respect to Gabriele can be written as

$$\begin{aligned} x_{X \text{ rel } G} &= x_X(t) - x_G(t) = 1.0 \times 10^3 \text{ m} + (6.0 \text{ m/s})t - [3.0 \times 10^3 \text{ m} - (8.0 \text{ m/s})t] \\ &= -2.0 \times 10^3 \text{ m} + (14 \text{ m/s})t \end{aligned}$$

The positions of Gabriel and Xena as a function of time are plotted below.



26. The relative speed between Jim and the police car is

$$v_{\text{rel}} = v_{\text{police}} - v_{\text{Jim}} = (36 \text{ m/s}) - (32 \text{ m/s}) = 4.0 \text{ m/s}.$$

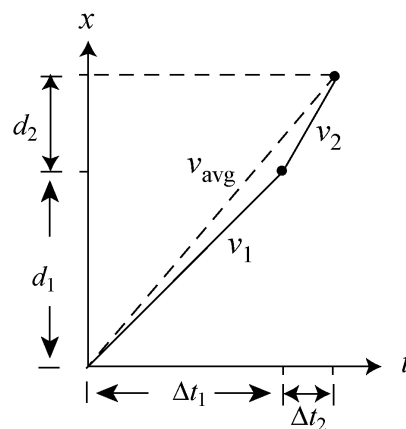
With a separation of $d = 300 \text{ m}$, the time it takes for the police car to catch up to Jim would be

$$\Delta t = \frac{d}{v_{\text{rel}}} = \frac{300 \text{ m}}{4.0 \text{ m/s}} = 75 \text{ s}$$

which implies a distance of $d_{\text{police}} = v_{\text{police}} \Delta t = (36 \text{ m/s})(75 \text{ s}) = 2700 \text{ m}$, or 2.7 km.

27. Sketch and translate The average speed is the total distance traveled divided by the total time of travel. Let the distance to the top of the hill be d . The time taken for the first part of the hike is $\Delta t_1 = d_1 / v_1$, where d_1 is the distance and v_1 is the speed. Similarly, for the second part, we have $\Delta t_2 = d_2 / v_2$, where d_2 is the distance and v_2 is the speed.

Simplify and diagram The total distance traveled is $d = d_1 + d_2$, and the total time of travel is $\Delta t = \Delta t_1 + \Delta t_2$. The average speed is given by the slope of the straight line connecting the two end points, as shown in the figure.



Represent mathematically The total time of travel can be written as

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1 v_2 + d_2 v_1}{v_1 v_2}$$

Thus, the average speed is

$$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{v_1 v_2}{d_1 v_2 + d_2 v_1} d$$

Solve and evaluate Given that $d_1 = 2d/3$, $v_1 = 3.0$ mph, $d_2 = d/3$, and $v_2 = 6.0$ mi/h, the above expression simplifies to

$$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{(3.0 \text{ mi/h})(6.0 \text{ mi/h})d}{(2d/3)(6.0 \text{ mi/h}) + (d/3)(3.0 \text{ mi/h})} = \frac{3(3.0 \text{ mi/h})(6.0 \text{ mi/h})}{2(6.0 \text{ mi/h}) + (3.0 \text{ mi/h})} = 3.6 \text{ mi/h}$$

28. Using the result from the previous problem, we find the average speed of the race to be (with $d_1 = d_2 = d/2$)

$$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{v_1 v_2 d}{d_1 v_2 + d_2 v_1} = \frac{2v_1 v_2}{v_1 + v_2}$$

Solving the above expression for v_2 , we obtain

$$v_2 = \frac{v_1 v_{\text{avg}}}{2v_1 - v_{\text{avg}}} = \frac{(2.01 \text{ m/s})(2.05 \text{ m/s})}{2(2.01 \text{ m/s}) - 2.05 \text{ m/s}} = 2.09 \text{ m/s}$$

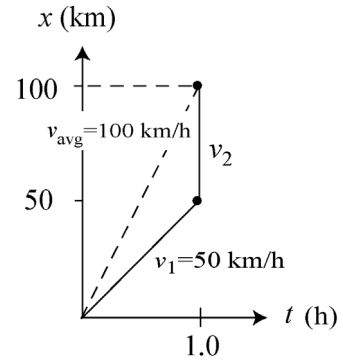
29. **Sketch and translate** Let the time taken for the first part of the trip be $\Delta t_1 = d_1 / v_1$, where d_1 is the distance traveled and v_1 is the speed. Similarly, for the second part, we have $\Delta t_2 = d_2 / v_2$, where d_2 is the distance and v_2 is the speed. The total distance traveled is $d = d_1 + d_2$, and the total time of travel is

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{d_1 v_2 + d_2 v_1}{v_1 v_2}$$

Simplify and diagram Given that $d_1 = d_2 = d/2 = 50$ km, the speed for the first 50 km is $v_1 = 50$ km/h, and we want the average speed of the 100-km trip to be 100 km/h. The sketch is shown to the right.

Represent mathematically The average speed can be written as

$$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{v_1 v_2 d}{d_1 v_2 + d_2 v_1} = \frac{2v_1 v_2}{v_1 + v_2}$$



Solving the above equation for v_2 gives

$$v_2 = \frac{v_1 v_{\text{avg}}}{2v_1 - v_{\text{avg}}}$$

Solve and evaluate With $v_1 = 50$ km/h, we see that in order to attain an average speed of 100 km/h, the speed for the second part of the trip must be infinite ($2v_1 - v_{\text{avg}} = 0$), as can

be seen from the vertical line shown in the figure above. Therefore, we conclude that it is physically impossible to reach this average speed, no matter how fast the driver goes.

30. A stationary observer on the ground sees Jane moving at a speed of 4.0 m/s and Bob at a speed of 3.0 m/s. From Jane's reference frame, Bob is moving toward her at a speed of 7.0 m/s. Similarly, from Bob's reference frame, Jane approaches him at a speed of 7.0 m/s. In all three frames, the time elapsed for Bob and Jane to meet up is

$$\Delta t = \frac{d}{v_{\text{rel}}} = \frac{100 \text{ m}}{7.0 \text{ m/s}} = 14.3 \text{ s}$$

or about 14 s in two significant digits. Jane has moved about 57 m, while Bob has moved 43 m.

31. (a) The position functions for the four cases are

$$\begin{aligned}x_1(t) &= 30 \text{ m} + (-8.33 \text{ m/s})t \\x_2(t) &= -10 \text{ m} \\x_3(t) &= -10 \text{ m} + (5.0 \text{ m/s})t \\x_4(t) &= -10 \text{ m} + (-3.33 \text{ m/s})t\end{aligned}$$

(b) For object 1, we have $x_{10} = 30 \text{ m}$ and $v_{10} = (-50 \text{ m}) / (6.0 \text{ s}) = -8.33 \text{ m/s}$. The object, with an initial displacement of +30 m with respect to a reference point which we take to be the origin, moves at a constant velocity of -8.33 m/s . Object 2 remains at rest at the position $x_2 = -10 \text{ m}$. Object 3 has an initial displacement of -10 m with respect to the origin, and moves at a constant velocity of $+5.0 \text{ m/s}$. Similarly, object 4 has an initial displacement of -10 m with respect to the origin, and moves at a constant velocity of -3.33 m/s . The acceleration is zero in all four cases.

32. The initial speed of the car is $v_{xi} = 0$ and its speed 30 s later is $v_{xf} = 10 \text{ m/s}$. The average acceleration of the car is

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{10 \text{ m/s} - 0}{30 \text{ s}} = 0.33 \text{ m/s}^2$$

33. (a) The truck has an initial velocity $v_{x0} = 16 \text{ m/s}$. With a constant acceleration $a_x = 1.0 \text{ m/s}^2$ for $t_1 = 5.0 \text{ s}$, its velocity at the end of the time interval is

$$v_{1x} = v_{0x} + a_{1x}t_1 = 16 \text{ m/s} + (1.0 \text{ m/s}^2)(5.0 \text{ s}) = 21 \text{ m/s}$$

The distance the truck has traveled while accelerating is

$$x_1 = v_{0x}t_1 + \frac{1}{2}a_{1x}t_1^2 = (16 \text{ m/s})(5.0 \text{ s}) + \frac{1}{2}(1.0 \text{ m/s}^2)(5.0 \text{ s})^2 = 92.5 \text{ m}$$

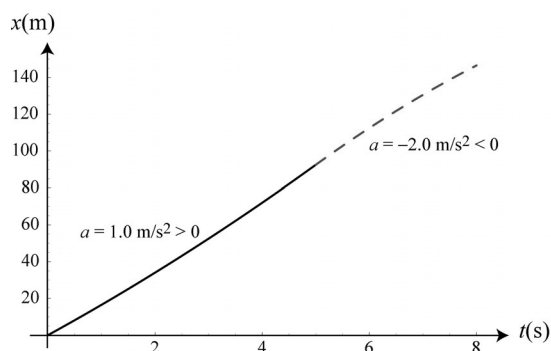
(b) With an acceleration of $a_{2x} = -2.0 \text{ m/s}^2$ for $t_2 = 3.0 \text{ s}$, the final velocity of the truck is

$$v_{2x} = v_{1x} + a_{2x}t_2 = 21 \text{ m/s} + (-2.0 \text{ m/s}^2)(3.0 \text{ s}) = 15 \text{ m/s}$$

and the displacement during this time interval is

$$x_2 = v_{1x}t_2 + \frac{1}{2}a_{2x}t_2^2 = (21 \text{ m/s})(3.0 \text{ s}) + \frac{1}{2}(-2.0 \text{ m/s}^2)(3.0 \text{ s})^2 = 54 \text{ m}$$

The distance traveled by the truck as function of time is plotted below.



The total distance traveled during the 8-second interval is $92.5 \text{ m} + 54 \text{ m} = 146.5 \text{ m}$.

34. We assume constant acceleration during the collision process. In the reference frame of a stationary observer on Earth, the velocity change for each car is $\Delta v_x = -3.2 \text{ m/s}$.

With $a_x = -28 \text{ m/s}^2$, the time required for the car to come to a complete stop is

$$t = \frac{\Delta v_x}{a_x} = \frac{-3.2 \text{ m/s}}{-28 \text{ m/s}^2} = 0.114 \text{ s}$$

and the corresponding stopping distance is

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (3.2 \text{ m/s})(0.114 \text{ s}) + \frac{1}{2}(-28 \text{ m/s}^2)(0.114 \text{ s})^2 = 0.183 \text{ m}$$

or about 18 cm.

35. We assume that the bus starts from rest ($v_{0x} = 0$) at the intersection and undergoes constant acceleration with $a_x = +2.0 \text{ m/s}^2$. At the end of a 5.0-second interval, the displacement of the bus with respect to the intersection is

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(2.0 \text{ m/s}^2)(5.0 \text{ s})^2 = 25 \text{ m}$$

On the other hand, if the bus has a non-zero initial velocity $v_{0x} \neq 0$, then its displacement after 5.0 s would be $v_{0x}(5.0 \text{ s}) + 25 \text{ m}$, which is greater than 25 m.

36. The acceleration of the bus is

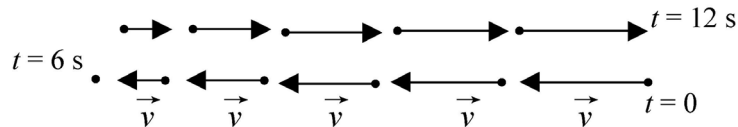
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20 \text{ m/s} - 16 \text{ m/s}}{8.0 \text{ s}} = 0.50 \text{ m/s}^2$$

Since the jogger speeds up at the same acceleration as the bus, his velocity and position as a function of time as observed by a stationary observer (standing at where the bus first passed him) are

$$v(t) = v_0 + at = (4.0 \text{ m/s}) + (0.50 \text{ m/s}^2)t$$

$$x(t) = v_0 t + \frac{1}{2}at^2 = (4.0 \text{ m/s})t + \frac{1}{2}(0.50 \text{ m/s}^2)t^2$$

37. (a) The motion diagram is shown below

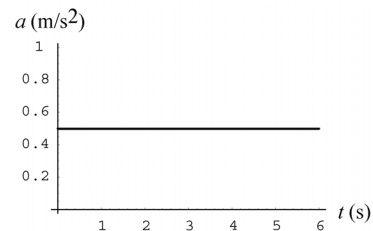
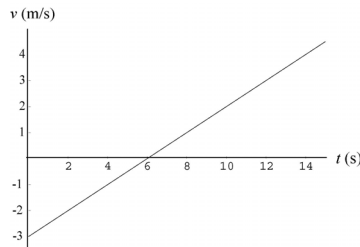
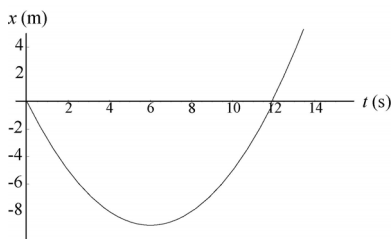


The position, velocity, and acceleration as function of time are given by

$$x(t) = (-3.0 \text{ m/s})t + \frac{1}{2}(0.50 \text{ m/s}^2)t^2$$

$$v(t) = -3.0 \text{ m/s} + (0.50 \text{ m/s}^2)t$$

$$a(t) = 0.50 \text{ m/s}^2$$

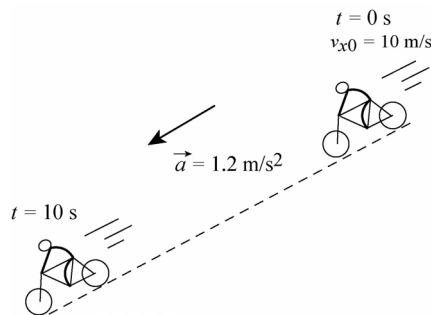


(b) A story for the motion could be as follows: At $t = 0$, a bicyclist, with an initial speed of 3.0 m/s due west (east taken to be positive), realizes that he's going in the "wrong" direction and begins to apply the brake. His acceleration is 0.50 m/s^2 (due east). He comes to a complete stop 6.0 s later at

$$x(t = 6.0 \text{ s}) = (-3.0 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2}(0.50 \text{ m/s}^2)(6.0 \text{ s})^2 = -9.0 \text{ m}$$

He then quickly turns around and pedals eastward with the same acceleration as before.

38. Sketch and translate The sketch of the process is shown below. The bicycle is the object of interest and is moving downhill, which we take to be the positive direction with respect to the chosen reference frame.



Simplify and diagram With our convention, the components of the bicycle's velocity along the axis of motion are positive at $t = 0$ and $t = 10 \text{ s}$. The speed of the bicycle (the magnitude of its velocity) increase; it is moving faster in the positive direction.

Represent mathematically The speed of the cyclist at the end of the descent is $v_x = v_{0x} + at$. Similarly, the distance traveled during this time interval is $x = v_{0x}t + \frac{1}{2}at^2$.

Solve and evaluate With $v_{0x} = 10 \text{ m/s}$ and $a = 1.2 \text{ m/s}^2$, the speed of the cyclist at the end of the descent that lasts for 10 s is

$$v_x = v_{0x} + at = (10.0 \text{ m/s}) + (1.2 \text{ m/s}^2)(10 \text{ s}) = 22 \text{ m/s}$$

The distance traveled during the 10-s interval is

$$x = v_{0x}t + \frac{1}{2}at^2 = (10 \text{ m/s})(10 \text{ s}) + \frac{1}{2}(1.2 \text{ m/s}^2)(10 \text{ s})^2 = 160 \text{ m}$$

In arriving at the results above, we have assumed a constant acceleration (ignoring factors such as the air resistance and condition of the road), and a completely straight path for our one-dimensional kinematics analyses to hold.

39. The initial speed of the truck in SI units is $v_0 = 16 \text{ km/h} = 4.44 \text{ m/s}$. With $\Delta x = 6.4 \text{ cm}$, we find the acceleration of the truck during the collision to be

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0 - (4.44 \text{ m/s})^2}{2(0.064 \text{ m})} = -154 \text{ m/s}^2$$

or $-1.5 \times 10^2 \text{ m/s}^2$ rounded to two significant digits. The negative sign indicates that the truck is slowing down.

40. (a) Using $v = v_0 + at$, the time interval required for the speed increase is

$$t = \frac{v - v_0}{a} = \frac{0.45 \text{ m/s} - 0.15 \text{ m/s}}{1.2 \text{ m/s}^2} = 0.25 \text{ s}$$

(b) Using $v^2 = v_0^2 + 2a\Delta x$, we find the distance the squid has traveled while accelerating to be

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(0.45 \text{ m/s})^2 - (0.15 \text{ m/s})^2}{2(1.2 \text{ m/s}^2)} = 0.075 \text{ m}$$

or about 7.5 cm.

41. The initial and final speeds are $v_0 = 0$ and $v = 663 \text{ km/h} = 184.2 \text{ m/s}$, respectively. The time it takes for her to reach this speed is $\Delta t = 3.22 \text{ s}$, so the average acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{184.2 \text{ m/s}}{3.22 \text{ s}} = 57.2 \text{ m/s}^2$$

On the other hand, to stop in $\Delta t' = 20 \text{ s}$, the required acceleration would be

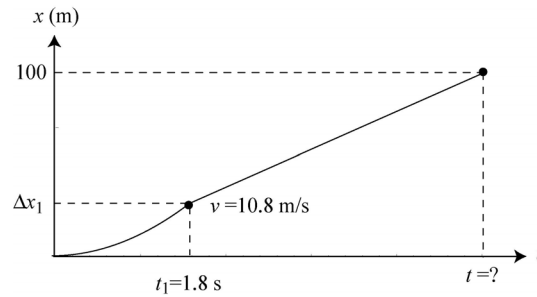
$$a' = \frac{\Delta v}{\Delta t'} = \frac{0 - 184.2 \text{ m/s}}{20 \text{ s}} = -9.2 \text{ m/s}^2$$

In our calculations, we have assumed one-dimensional motion, where the dragster is taken to be a point-like object. In addition, by ignoring factors such as air resistance and road condition, the acceleration is taken to be constant throughout.

42. **Sketch and translate** The sprinter's motion consists of two phases: an accelerating phase that lasts for 1.8 s, attaining a speed of 10.8 m/s, and a constant-speed phase, in which the sprinter maintains a speed of 10.8 m/s until he reaches the finish line.

Simplify and diagram We assume the motion to be one-dimensional, and model the sprinter as a point-like object. We also assume a constant acceleration that ends abruptly

upon reaching 10.8 m/s, the speed at which the sprinter maintains for the remaining part of the race. The situation is depicted below.



Represent mathematically The acceleration of the sprinter to attain a final speed of 10.8 m/s in 1.8 s is

$$a = \frac{v - v_0}{t_1} = \frac{10.8 \text{ m/s} - 0}{1.8 \text{ s}} = 6.0 \text{ m/s}^2$$

During this time interval, the sprinter has advanced

$$\Delta x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = 0 + \frac{1}{2} (6.0 \text{ m/s}^2) (1.8 \text{ s})^2 = 9.72 \text{ m}$$

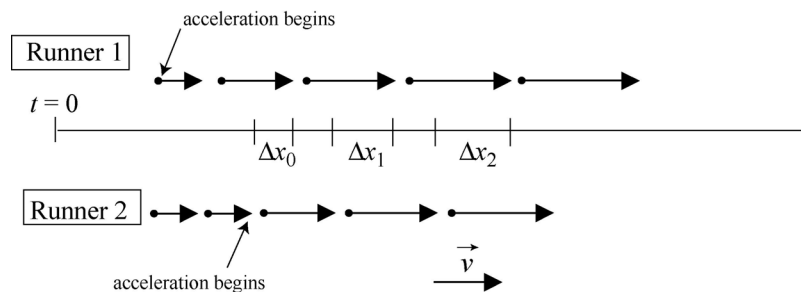
and the distance remained for the 100-m race is $\Delta x_2 = 100 - \Delta x_1 = 90.28 \text{ m}$.

Solve and evaluate The time required to finish this segment at a constant speed of 10.8 m/s is

$$t_2 = \frac{\Delta x_2}{v} = \frac{90.28 \text{ m}}{10.8 \text{ m/s}} = 8.36 \text{ s}.$$

So it took the sprinter $t = t_1 + t_2 = 1.8 \text{ s} + 8.36 \text{ s} \approx 10.2 \text{ s}$ to finish the entire race.

43. The motion diagrams for the two runners are shown below.



From the diagram, we see that the distance between the two runners will continue to increase with time.

Quantitatively, the positions of the two runners can be represented by

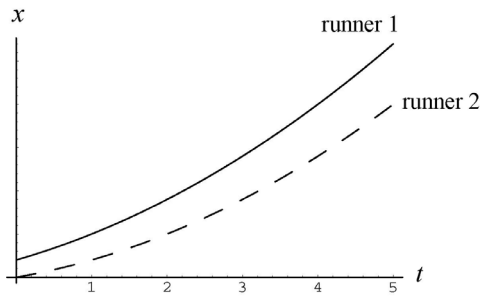
$$x_1(t) = x_0 + v_0(t + \Delta t) + \frac{1}{2}a(t + \Delta t)^2$$

$$x_2(t) = (x_0 + v_0\Delta t) + v_0t + \frac{1}{2}at^2 = x_0 + v_0(t + \Delta t) + \frac{1}{2}at^2$$

Note that the time origin in the expressions above corresponds to the moment when the second runner also begins to accelerate. The separation between the two runners is

$$\Delta x(t) = x_1 - x_2 = \frac{1}{2}a(t + \Delta t)^2 - \frac{1}{2}at^2 = (a\Delta t)t + \frac{1}{2}a(\Delta t)^2$$

Initially the distance between the two runners is $\Delta x_0 = a(\Delta t)^2 / 2$. However, the linear term $(a\Delta t)t$ tells us that Δx will increase with time, as shown in the position-versus-time graph below.



44. (a) Using $v^2 = v_0^2 + 2a\Delta x$, we find the acceleration of the meteorite to be

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{0 - (500 \text{ m/s})^2}{2(0.20 \text{ m})} = -6.25 \times 10^5 \text{ m/s}^2$$

which has a magnitude of $6.25 \times 10^5 \text{ m/s}^2$. In the above, we have taken the meteorite to be a point-like object, and applied the one-dimensional kinematics analysis.

(b) The time it takes for the meteorite to come to a complete stop is

$$t = \frac{v - v_0}{a} = \frac{0 - 500 \text{ m/s}}{-6.25 \times 10^5 \text{ m/s}^2} = 8.0 \times 10^{-4} \text{ s}$$

45. The acceleration of the frog hopper is

$$a = \frac{\Delta v}{\Delta t} = \frac{4.0 \text{ m/s} - 0}{1.0 \times 10^{-3} \text{ s}} = 4.0 \times 10^3 \text{ m/s}^2$$

The distance moved while accelerating is

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(4.0 \text{ m/s})^2 - 0}{2(4.0 \times 10^3 \text{ m/s}^2)} = 2.0 \times 10^{-3} \text{ m}$$

or about 2.0 mm.

46. The final speed of the tennis ball is $v = 209 \text{ km/h} = 58.1 \text{ m/s}$. Using $v^2 = v_0^2 + 2a\Delta x$, the average acceleration of the tennis ball is

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(58.1 \text{ m/s})^2 - 0}{2(0.10 \text{ m})} = 1.7 \times 10^4 \text{ m/s}^2$$

The time interval for the racket-ball contact is $t = \frac{v - v_0}{a} = \frac{58.1 \text{ m/s}}{1.7 \times 10^4 \text{ m/s}^2} = 3.4 \times 10^{-3} \text{ s}$, or about 3.4 ms.

47. To attain a speed of $v_1 = 80 \text{ km/h} = 22.2 \text{ m/s}$ in $t_1 = 0.40 \text{ s}$, the acceleration during launch is

$$a_1 = \frac{v_1}{t_1} = \frac{22.2 \text{ m/s} - 0}{0.40 \text{ s}} = 55.6 \text{ m/s}^2$$

After the launch, the time it takes to stop the motion using a catching net with acceleration $a_2 = -180 \text{ m/s}^2$ is

$$t_2 = \frac{0 - v_1}{a_2} = \frac{0 - 22.2 \text{ m/s}}{180 \text{ m/s}^2} = 0.12 \text{ s}$$

The distance traveled while being stopped by the net is

$$\Delta x_2 = v_1 t_2 + \frac{1}{2} a_2 t_2^2 = (22.2 \text{ m/s})(0.12 \text{ s}) + \frac{1}{2}(-180 \text{ m/s}^2)(0.12 \text{ s})^2 = 1.37 \text{ m}$$

or about 1.4 m, rounded to two significant digits.

48. With an initial speed of $v_0 = 284.4 \text{ m/s}$, the average acceleration required to stop the motion in 1.4 s is

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 284.4 \text{ m/s}}{1.4 \text{ s}} = -203 \text{ m/s}^2$$

The negative sign indicates that the velocity is decreasing. The distance traveled as the sled slows down is

$$\Delta x = v_0 t + \frac{1}{2} a t^2 = (284.4 \text{ m/s})(1.4 \text{ s}) + \frac{1}{2}(-203 \text{ m/s}^2)(1.4 \text{ s})^2 = 199 \text{ m}$$

or about 200 m.

49. (a) Using $v = v_0 + at$, we find the acceleration of the sprinter to be

$$a = \frac{\Delta v}{t_1} = \frac{11.2 \text{ m/s}}{2.0 \text{ s}} = 5.6 \text{ m/s}^2$$

(b) The distance traveled during this time interval is

$$\Delta x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = 0 + \frac{1}{2}(5.6 \text{ m/s}^2)(2.0 \text{ s})^2 = 11.2 \text{ m}$$

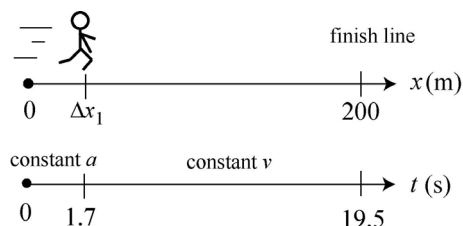
(c) In our calculations, we have assumed one-dimensional motion, where the sprinter is taken to be a point-like object and the path is a straight line. We also take the acceleration to be constant throughout the interval.

(d) The distance to the finish line is $\Delta x_2 = 100 \text{ m} - 11.2 \text{ m} = 88.8 \text{ m}$. The time required to finish this segment of the race at a constant speed of 11.2 m/s is

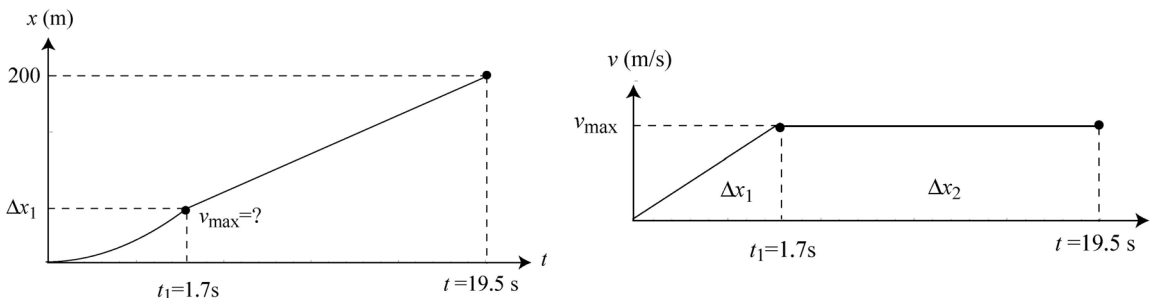
$$t_2 = \frac{\Delta x_2}{v} = \frac{88.8 \text{ m}}{11.2 \text{ m/s}} = 7.93 \text{ s}.$$

(e) It takes the sprinter $t = t_1 + t_2 = 2.0 \text{ s} + 7.9 \text{ s} = 9.9 \text{ s}$ to finish the entire race. With 2 significant digits, the uncertainty is 0.1 s.

50. **Sketch and translate** The sprinter's motion consists of two phases: an accelerating phase that lasts for 1.7 s, attaining a maximum speed v_{max} , followed by a constant-speed phase, in which the sprinter maintains his maximum speed v_{max} until he reaches the finish line.



Simplify and diagram We assume the motion to be one-dimensional, and model the sprinter as a point-like object. We also assume a constant acceleration that ends abruptly upon reaching v_{\max} . The position and velocity as a function of time are depicted below.



Represent mathematically The average acceleration needed to reach the maximum speed v_{\max} in $t_1 = 1.7$ s is $a = v_{\max} / t_1$. During this phase, the sprinter has traveled

$$\Delta x_1 = \frac{1}{2} a t_1^2 = \frac{1}{2} \left(\frac{v_{\max}}{t_1} \right) t_1^2 = \frac{1}{2} v_{\max} t_1$$

Note that Δx_1 is the area of the triangle in the velocity-versus-time graph. Thus, the distance remaining for the 200-m race is $\Delta x_2 = 200 \text{ m} - v_{\max} t_1 / 2$.

Solve and evaluate In order to tie for the 19.5-s record, he must complete this distance in $t_2 = 19.5 \text{ s} - 1.7 \text{ s} = 17.8 \text{ s}$. The two statements together imply

$$t_2 = \frac{\Delta x_2}{v_{\max}} = \frac{200 \text{ m} - v_{\max} t_1 / 2}{v_{\max}} = \frac{200 \text{ m}}{v_{\max}} - \frac{t_1}{2} \Rightarrow \frac{200 \text{ m}}{v_{\max}} = t_2 + \frac{t_1}{2} = 17.8 \text{ s} + \frac{1.7 \text{ s}}{2} = 18.65 \text{ s}$$

which can be solved to give $v_{\max} = 10.7 \text{ m/s}$.

51. The initial speed of the bus is $v_0 = 36 \text{ km/h} = 10 \text{ m/s}$. Assuming that the bus has a constant acceleration of $a = -1.2 \text{ m/s}^2$, the distance it travels before coming to a complete stop is

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (10 \text{ m/s})^2}{2(-1.2 \text{ m/s}^2)} = 41.67 \text{ m}$$

or about 42 m, in two significant digits.

52. We assume the car to be a point-like object and apply one-dimensional kinematics to analyze its motion. The information on the acceleration of the car can be extracted from one of the three formulas: $a = \Delta v / t$, or $a = (v^2 - v_0^2) / 2\Delta x$, $\Delta x = \frac{1}{2} a t^2$. For example, if we

know the time (t) it takes for the car to travel a certain distance (Δx), starting from rest, then the acceleration would be $a = 2\Delta x / t^2$. On the other hand, if we know the speed of the car at the end of the accelerating interval, then we can simply use $a = v / t$ to deduce the acceleration.

53. During the t th second, the displacement is $2t$. For the first t second, the total displacement is $2t + x_{t-1}$. Since $x_{t-1} = 2(t-1) + x_{t-2}$, . . . one can readily show that the displacement as a function of time can be written as

$$x(t) = 2 + 4 + 6 + \dots + 2t = 2 \frac{t(t+1)}{2} = t + t^2$$

Comparing with the standard form $x(t) = v_0 t + \frac{1}{2} a t^2$ for constant acceleration a , we see that the equation above indeed describes a motion with constant acceleration $a = 2.0 \text{ m/s}^2$ (and $v_0 = 1.0 \text{ m/s}$).

54. (a) Using $v = v_0 + at$, we find the acceleration of the car to be

$$a = \frac{\Delta v}{\Delta t} = \frac{-20 \text{ m/s} - (-10 \text{ m/s})}{4.0 \text{ s}} = -2.5 \text{ m/s}^2$$

(b) In this case, the acceleration is $a = \frac{\Delta v}{\Delta t} = \frac{-18 \text{ m/s} - (-20 \text{ m/s})}{2.0 \text{ s}} = +1.0 \text{ m/s}^2$.

(c) In (a), the object is moving in the $-x$ -direction, with acceleration also in the $-x$ -direction ($a < 0$). On the other hand, in (b), the object is moving in the $-x$ -direction, while the acceleration points in the $+x$ -direction ($a > 0$).

55. The displacements of cars A and B are tabulated below:

time (s)	0	10	20	30	40	50	60
$x_A(\text{m})$	200	0	-200	-400	-600	-800	-1000
$x_B(\text{m})$	-200	-100	0	100	200	300	400

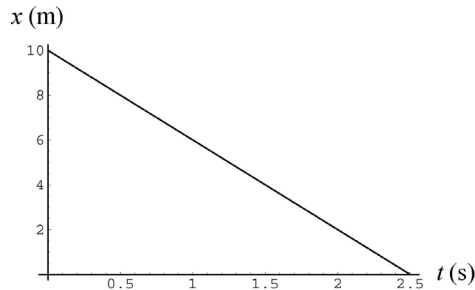
Mathematically, the positions can be written as

$$\begin{aligned} x_A(t) &= x_{A0} + v_{A0}t = (200 \text{ m}) - (20 \text{ m/s})t \\ x_B(t) &= x_{B0} + v_{B0}t = (-200 \text{ m}) + (10 \text{ m/s})t \end{aligned}$$

56. (a) From $x(t) = 10 \text{ m} - (4.0 \text{ m/s})t$, we know that the initial position of the object is $x_0 = 10 \text{ m}$ and it moves with a constant velocity $v_0 = -4.0 \text{ m/s}$ (zero acceleration).

(b) A student is 10 m east of the physics building (taken to be the origin) when he realizes that there is a special seminar on the origin of the Universe. Not wanting to miss any part of it, he starts running at 4.0 m/s toward the building.

(c) The position-versus-time graph is shown below.



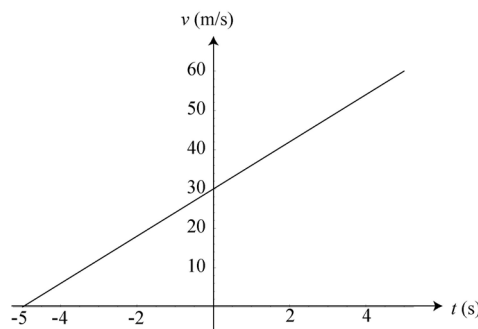
The student reaches the physics building after $\Delta t = (10 \text{ m}) / (4.0 \text{ m/s}) = 2.5 \text{ s}$.

57. (a) The function $x(t) = -100 \text{ m} + (30 \text{ m/s})t + (3.0 \text{ m/s}^2)t^2$ describes a motion with constant acceleration.

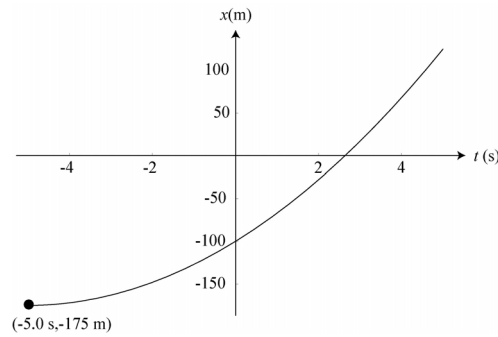
(b) The object is initially at $x_0 = -100 \text{ m}$ (with respect to some reference frame) with an initial velocity of $v_0 = +30 \text{ m/s}$, and a constant acceleration $a = 6.0 \text{ m/s}^2$.

(c) A car is initially 100 m west of an intersection (origin, with east taken to be positive) and heading east at a speed of 30 m/s when the driver notices that the traffic light has turned yellow. He steps on the gas pedal and the car starts to accelerate at 3.0 m/s^2 .

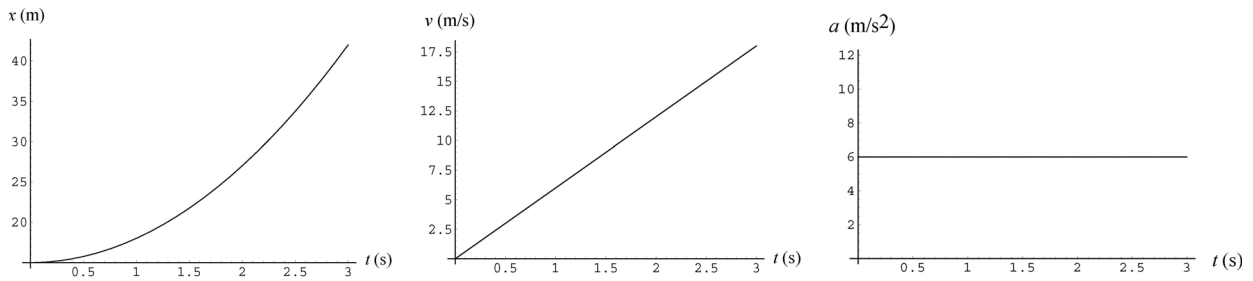
(d) The velocity function of the car is $v(t) = 30 \text{ m/s} + (6.0 \text{ m/s}^2)t$.



(e) The time $t = -5.0 \text{ s}$ is the only time where the speed of the car is zero. It is at $x = -175 \text{ m}$. One can think of this instant as 5.0 s before some reference time.

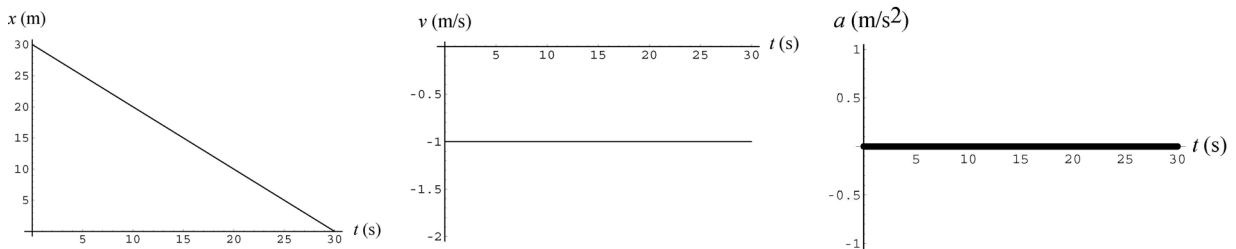


58. (a) From $x(t) = 15 \text{ m} - (-3.0 \text{ m/s}^2)t^2 = 15 \text{ m} + (3.0 \text{ m/s}^2)t^2$, we find the initial position of the object to be at $x_0 = 15 \text{ m}$. Its initial velocity is $v_0 = 0$, and the object is undergoing a constant acceleration with $a = 6.0 \text{ m/s}^2$. Its velocity can be written as $v(t) = (6.0 \text{ m/s}^2)t$. The graphs of position, velocity, and acceleration as function of time are plotted below.



Story: A car is initially parked illegally 15.0 m due east of an intersection (taken to be the origin). Seeing a police car coming, the driver starts his car and moves away (eastbound) with acceleration 3.0 m/s^2 .

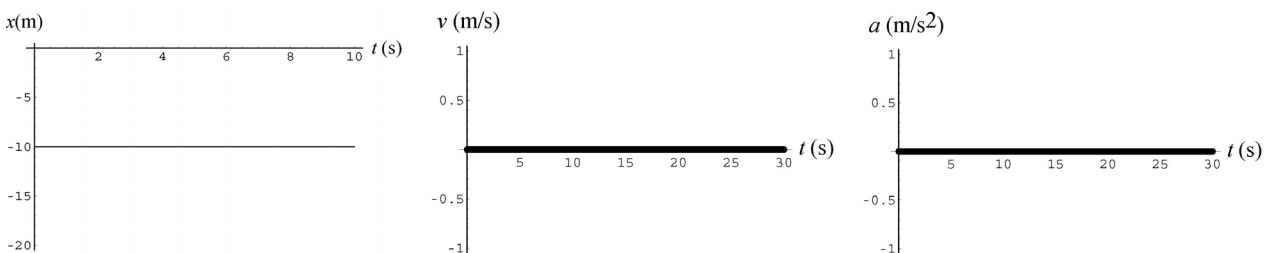
(b) From $x(t) = 30.0 \text{ m} - (1.0 \text{ m/s}^2)t$, we find the initial position of the object to be at $x_0 = 30 \text{ m}$. Its initial velocity is $v_0 = -10 \text{ m/s}$, with zero acceleration. The graphs of position, velocity, and acceleration as function of time are plotted below.



Story: A student is 30 m due east of his classroom (taken to be the origin, with east positive) when he realizes that he forgot to turn in his homework. He starts walking at a speed of 1.0 m/s toward the classroom, and gets there after 30 s.

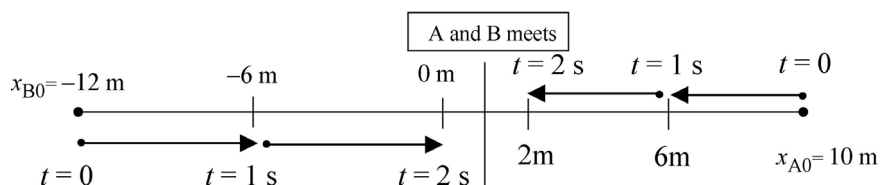
(c) For $x = -10$ m, the initial position of the object is at $x_0 = -10$ m (with respect to some reference point). The object is at rest there with zero velocity and zero acceleration.

Story: A car has been parked 10 m due west of a stop sign (the origin, with east being the positive x -direction). The graphs of position, velocity, and acceleration as function of time are plotted below.



59. Sketch and translate We choose Earth as the object of reference with the observer's position as the reference point. The positive direction points to the east. We have two objects of interest here – A and B, which we treat as point-like.

Simplify and diagram The initial position of object A is $x_{A0} = 10$ m and its constant velocity is $v_{A0} = -4.0$ m/s (the velocity is negative since it's moving westward). Object B's initial position is $x_{B0} = -12$ m and its constant velocity is $v_{B0} = +6.0$ m/s. The motion diagram of the objects is shown below.



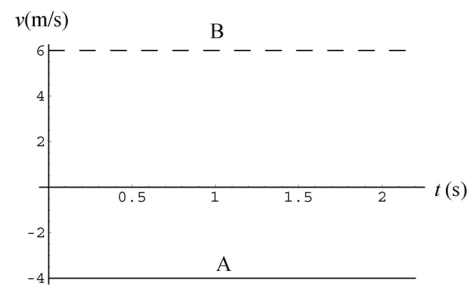
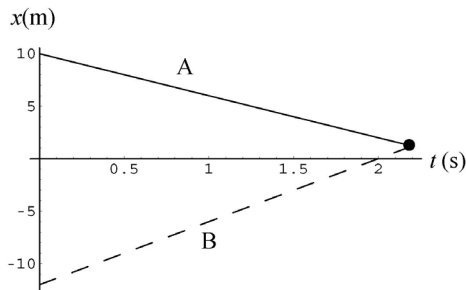
Represent mathematically The positions of objects A and B are given by

$$x_A(t) = x_{A0} + v_{A0}t = (10 \text{ m}) - (4.0 \text{ m/s})t$$

$$x_B(t) = x_{B0} + v_{B0}t = (-12 \text{ m}) + (6.0 \text{ m/s})t$$

Solve and evaluate From the graph shown above, we see that the two objects would meet between 2.0 and 3.0 s later at a position between $x = 0$ and $x = 2.0$ m.

We also plot the graphs of position and velocity as function of time.



Solving the equation mathematically, we find

$$(10 \text{ m}) - (4.0 \text{ m/s})t = (-12 \text{ m}) + (6.0 \text{ m/s})t$$

which gives $t = 2.2 \text{ s}$. So the two object meet at

$$x_A = x_B = 10 \text{ m} - (4.0 \text{ m/s})(2.2 \text{ s}) = 1.2 \text{ m}.$$

60. The velocities and positions of the two cars can be written as

$$v_1(t) = 30 \text{ m/s} - (6.0 \text{ m/s}^2)t$$

$$v_2(t) = 24 \text{ m/s} - (6.0 \text{ m/s}^2)t$$

and

$$x_1(t) = (30 \text{ m/s})t - \frac{1}{2}(6.0 \text{ m/s}^2)t^2$$

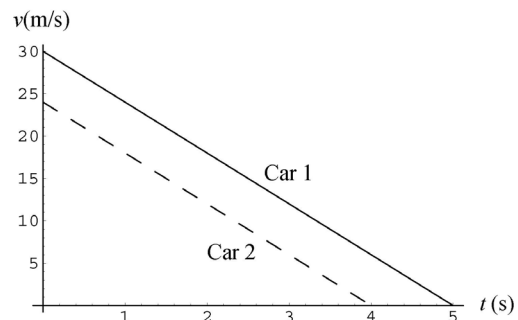
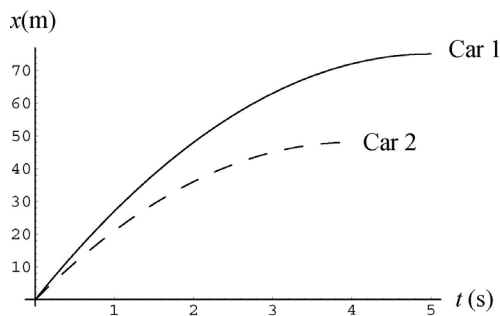
$$x_2(t) = (24 \text{ m/s})t - \frac{1}{2}(6.0 \text{ m/s}^2)t^2$$

Using $v = v_0 + at$, we find that Car 1 stops after 5.0 s, and Car 2 stops after 4.0 s. The distances traveled while stopping are

$$x_1(t = 5.0 \text{ s}) = (30 \text{ m/s})(5.0 \text{ s}) - \frac{1}{2}(6.0 \text{ m/s}^2)(5.0 \text{ s})^2 = 75 \text{ m}$$

$$x_2(t = 4.0 \text{ s}) = (24 \text{ m/s})(4.0 \text{ s}) - \frac{1}{2}(6.0 \text{ m/s}^2)(4.0 \text{ s})^2 = 48 \text{ m}$$

The graphs below show how their velocities and positions change with time.



61. (a) At $t = 0$, a car is moving at a constant velocity of 20 m/s. At $t = 8.0$ s, the driver begins to slow down by applying a negative constant acceleration, and the car comes to a complete stop at $t = 20$ s.

(b) The acceleration of the car is $a = \Delta v / \Delta t = (-20 \text{ m/s}) / (12 \text{ s}) = -1.67 \text{ m/s}^2$. So the velocity of the car at $t = 10$ s is $v_{10} = 20 \text{ m/s} - (1.67 \text{ m/s}^2)(10 \text{ s} - 8.0 \text{ s}) = 16.67 \text{ m/s}$. The displacement of the car during a time interval is given by the area under the velocity-versus-time curve. Thus, between 10 s and 20 s, we have

$$x_{10 \rightarrow 20} = \frac{1}{2}(16.67 \text{ m/s})(20 \text{ s} - 10 \text{ s}) = 83.3 \text{ m}$$

(c) The displacement for the first 8 seconds is

$$x_{0 \rightarrow 8} = (20 \text{ m/s})(8.0 \text{ s}) = 160 \text{ m}$$

Similarly, from 8.0 s to 20 s, we have

$$x_{8 \rightarrow 20} = \frac{1}{2}(20 \text{ m/s})(20 \text{ s} - 8.0 \text{ s}) = 120 \text{ m}$$

Thus, the total distance traveled is $x_{0 \rightarrow 20} = x_{0 \rightarrow 8} + x_{8 \rightarrow 20} = 160 \text{ m} + 120 \text{ m} = 280 \text{ m}$, and the corresponding average speed is

$$v_{\text{avg}} = \frac{x_{0 \rightarrow 20}}{\Delta t} = \frac{280 \text{ m}}{20 \text{ s}} = 14 \text{ m/s}$$

62. (a) The following story could be used to describe the graph: At $t = 0$, the driver of a car that is moving at a constant velocity of -20 m/s due west (with east taken to be the positive x -direction) has just realized that he was going the wrong way. He steps on the brake (which applies a constant deceleration) and the car comes to a stop 30 s later. He quickly turns around and heads east with a constant acceleration, reaching a speed of 10 m/s at $t = 45$ s. Between 45 s and 60 s, the car maintains this speed. At $t = 60$ s, the driver thinks that he might be late for his appointment and begins to accelerate; the car reaches 20 m/s at $t = 70$ s.

(b) The acceleration between 0 and 45 s is

$$a = \frac{\Delta v}{\Delta t} = \frac{10 \text{ m/s} - (-20 \text{ m/s})}{45 \text{ s}} = 0.67 \text{ m/s}^2$$

During this interval, the displacement of the car is the area under the curve:

$$\begin{aligned}
 x_{0 \rightarrow 45} &= x_{0 \rightarrow 30} + x_{30 \rightarrow 45} = \frac{1}{2}(-20 \text{ m/s})(30 \text{ s}) + \frac{1}{2}(10 \text{ m/s})(45 \text{ s} - 30 \text{ s}) \\
 &= -300 \text{ m} + 75 \text{ m} = -225 \text{ m}
 \end{aligned}$$

The same result can be obtained using

$$x_{0 \rightarrow 45} = (-20 \text{ m/s})(45 \text{ s}) + \frac{1}{2}(0.67 \text{ m/s}^2)(45 \text{ s})^2 = -225 \text{ m}$$

The path traveled is $l = |-300 \text{ m}| + |75 \text{ m}| = 375 \text{ m}$.

(c) For the next two time intervals, we have

$$\begin{aligned}
 x_{45 \rightarrow 60} &= (10 \text{ m/s})(60 \text{ s} - 45 \text{ s}) = 150 \text{ m} \\
 x_{60 \rightarrow 70} &= \frac{1}{2}(20 \text{ m/s} + 10 \text{ m/s})(10 \text{ s}) = 150 \text{ m}
 \end{aligned}$$

Thus, the total displacement for the first 70 s is

$$x_{0 \rightarrow 70} = x_{0 \rightarrow 30} + x_{30 \rightarrow 45} + x_{45 \rightarrow 60} + x_{60 \rightarrow 70} = -300 \text{ m} + 75 \text{ m} + 150 \text{ m} + 150 \text{ m} = 75 \text{ m}$$

and the average velocity is $v_{\text{avg}} = \frac{x_{0 \rightarrow 70}}{\Delta t} = \frac{75 \text{ m}}{70 \text{ s}} = 1.07 \text{ m/s}$. On the other hand, the path length the car has traveled is

$$l_{0 \rightarrow 70} = |x_{0 \rightarrow 30}| + |x_{30 \rightarrow 45}| + |x_{45 \rightarrow 60}| + |x_{60 \rightarrow 70}| = 300 \text{ m} + 75 \text{ m} + 150 \text{ m} + 150 \text{ m} = 675 \text{ m}$$

giving an average speed of $s_{\text{avg}} = \frac{l_{0 \rightarrow 70}}{\Delta t} = \frac{675 \text{ m}}{70 \text{ s}} = 9.64 \text{ m/s}$.

63. From the figure, we see that Car 2 is moving at a constant velocity since the spacing between dots is uniform. On the other hand, the non-uniformity in the spacing between dots for Car 1 implies that it has non-zero acceleration.

Qualitatively, we see that both cars are at the same position at $t = 1.0 \text{ s}$ and have the same displacement between $t = 3$ and $t = 4 \text{ s}$. Thus, the cars should have roughly the same speed at $t = 3.5$ seconds.

We can also analyze the problem more quantitatively. Let's assume that the velocity of Car 1 is initially zero, so its position is represented as $x_1(t) = x_{10} + \frac{1}{2}a_1t^2$, where a_1 is the

constant acceleration. Similarly, for Car 2, we have $x_2(t) = v_{20}t$. Now at $t = 1$ s, both cars are at the same position:

$$x_{10} + \frac{1}{2}a_1 = v_{20}.$$

In addition, both cars have the same displacements between 3.0 s and 4.0 s. This gives:

$$v_{20} = x_{10} + \frac{1}{2}a_1(4^2 - 3^2) = \frac{7}{2}a_1.$$

(a) The speeds of the two cars are

$$v_1(t) = a_1 t, \quad v_2(t) = v_{20}$$

We readily see that the two speeds are the same at $t = 3.5$ s.

(b) The positions of the cars as function of time can be written as (in SI units)

$$x_1(t) = 3a_1 + \frac{1}{2}a_1 t^2$$

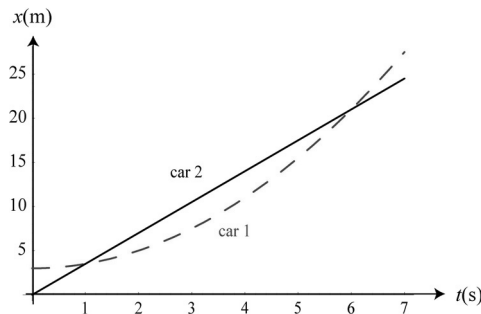
$$x_2(t) = 3.5a_1 t$$

For simplicity, we set $a_1 = 1.0$ m/s², so that $v_{20} = 3.5$ m/s, and $x_{10} = 3.0$ m. Thus, according to this model,

$$x_1(t) = (3.0 \text{ m}) + \frac{1}{2}(1.0 \text{ m/s}^2)t^2$$

$$x_2(t) = (3.5 \text{ m/s})t$$

we find that the cars are at the same positions at two different times: $t = 1.0$ s and $t = 6.0$ s. However, exactly when both cars would meet again depends on the details of the initial conditions.



64. A possible story for the processes is as follows: A truck with failed brake begins to slide down an inclined ramp at constant acceleration. At $t = t_1$, it runs into another parked truck, and finally comes to a complete stop 0.40 s later.

From the two equations given, the speed of the truck before hitting the second truck is $v_1 = 20 \text{ m/s}$. With $a = 5.0 \text{ m/s}^2$, the time it takes for the truck to slide down is $t_1 = v_1 / a_1 = (20 \text{ m/s}) / (5.0 \text{ m/s}^2) = 4.0 \text{ s}$, and the distance traveled is (Part I)

$$x_1 = \frac{1}{2} a_1 t_1^2 = \frac{1}{2} (5.0 \text{ m/s}^2) (4.0 \text{ s})^2 = 40 \text{ m}$$

For Part II, we find the acceleration to stop the truck in 0.40 s to be

$$a_2 = \frac{-v_1}{t_2} = \frac{-20 \text{ m/s}}{0.40 \text{ s}} = -50 \text{ m/s}^2$$

The total distance the truck has traveled (relative to its initial position at the top of the ramp) is

$$x_2 = x_1 + (20 \text{ m/s})(0.40 \text{ s}) + \frac{1}{2} (-50 \text{ m/s}^2) (0.40 \text{ s})^2 = 44 \text{ m}$$

65. (a) Using $y = y_0 - \frac{1}{2} g t^2$ with $g = 9.8 \text{ m/s}^2$, the time it takes for the eraser to reach the ground ($y = 0$) is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(15 \text{ m})}{9.8 \text{ m/s}^2}} = 1.75 \text{ s}$$

(b) The speed just before striking the ground is $v = g t = (9.8 \text{ m/s}^2)(1.75 \text{ s}) = 17.1 \text{ m/s}$.

(c) The result would be $v t = (17.1 \text{ m/s})(1.75 \text{ s}) = 30 \text{ m}$, which is greater than 15 m . The result is expected since the speed in (b) represents the maximum speed of the eraser.

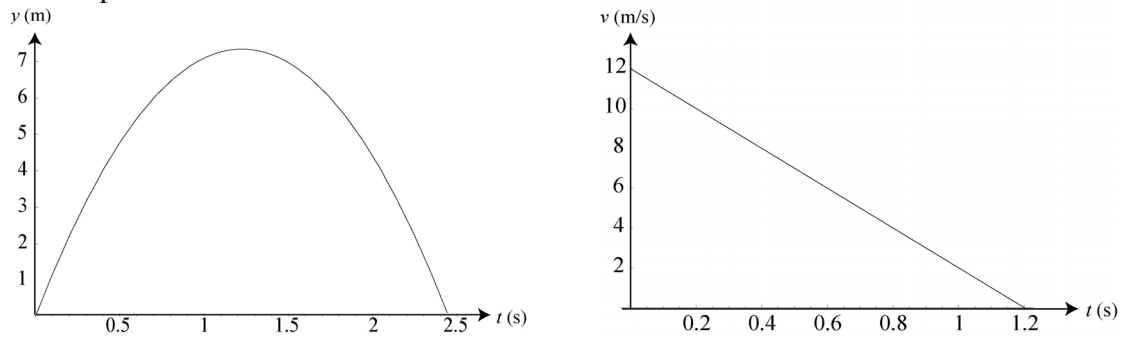
66. The average speed of the eraser is

$$v_{\text{avg}} = \frac{y_0}{t} = \frac{15 \text{ m}}{1.75 \text{ s}} = 8.57 \text{ m/s}$$

which is half of the speed just before impact. Using the average value we obtain $v_{\text{avg}} t = (8.57 \text{ m/s})(1.75 \text{ s}) = 15 \text{ m}$, which is the height from which the eraser was dropped.

67. **Sketch and translate** We choose Earth as the object of reference with the ground observer's position as the reference point ($y = 0$). The positive direction points up. The tennis ball is our object of interest, which we treat as point-like.

Simplify and diagram The object's acceleration is -9.8 m/s^2 on the way up; -9.8 m/s^2 on the way down; and even -9.8 m/s^2 at the instant when the object is momentarily at rest at the highest point of its motion. At all times during the object's flight, its velocity is changing at a rate of -9.8 m/s each second. The position and velocity as a function of time are plotted below.



Represent mathematically The vertical motion of the tennis ball can be represented by

$$y(t) = v_0 t - \frac{1}{2} g t^2$$

$$v(t) = v_0 - g t$$

Note that $y(t)$ is the position of the ball relative to where it was first released.

Solve and evaluate With an initial speed of $v_0 = 12 \text{ m/s}$, the ball reaches its maximum height y_{max} in

$$t = v_0 / g = (12 \text{ m/s}) / (9.8 \text{ m/s}^2) = 1.22 \text{ s}$$

with

$$y_{\text{max}} = (12 \text{ m/s})(1.22 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.22 \text{ s})^2 = 7.35 \text{ m}$$

The total flight time of the tennis ball is $T = 2t = 2(1.22 \text{ s}) = 2.44 \text{ s}$. Note that a speed of 12 m/s , or about 27 mi/h , is a reasonable value with a human throw.

68. The acceleration of the parachute is

$$a = \frac{\Delta v}{\Delta t} = \frac{8.0 \text{ m/s} - 50 \text{ m/s}}{0.80 \text{ s}} = -52.5 \text{ m/s}^2$$

Using $v^2 = v_0^2 + 2a\Delta y$, we find the distance fallen to be

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{(8.0 \text{ m/s})^2 - (50 \text{ m/s})^2}{2(-52.5 \text{ m/s}^2)} = 23.2 \text{ m}$$

The magnitude of the acceleration is 52.5 m/s^2 , or about $5.36g$. Since it exceeds the $5g$ limit, there's good probability that the skydiver will faint. In our calculation, we have modeled the skydiver as a point-like object and ignored the air resistance, which will greatly reduce the acceleration of the skydiver.

69. The vertical motion of the helmet can be represented by

$$y(t) = v_0 t - \frac{1}{2} g t^2, \quad v(t) = v_0 - g t$$

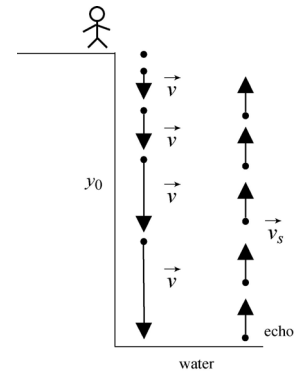
where upward is the $+y$ -direction. One can show that at $t = v_0 / g$ the helmet reaches its maximum height $y_{\text{max}} = v_0^2 / 2g$. With $y_{\text{max}} = 5.0 \text{ m}$, we find the initial speed to be

$$v_0 = \sqrt{2g y_{\text{max}}} = \sqrt{2(9.8 \text{ m/s}^2)(5.0 \text{ m})} = 9.9 \text{ m/s}$$

The total flight time is simply

$$T = 2t = 2v_0 / g = 2(9.9 \text{ m/s}) / (9.8 \text{ m/s}^2) = 2.02 \text{ s}.$$

70. Sketch and translate We choose Earth as the object of reference with your position as the reference point ($y = 0$). The positive direction points downward. The rock is our object of interest, which we treat as point-like. The situation is depicted in the figure to the right.



Simplify and diagram With our convention, the rock's acceleration is $+9.8 \text{ m/s}^2$ on the way down. After hitting the water, the echo propagates up at a speed of $v_s = 340 \text{ m/s}$.

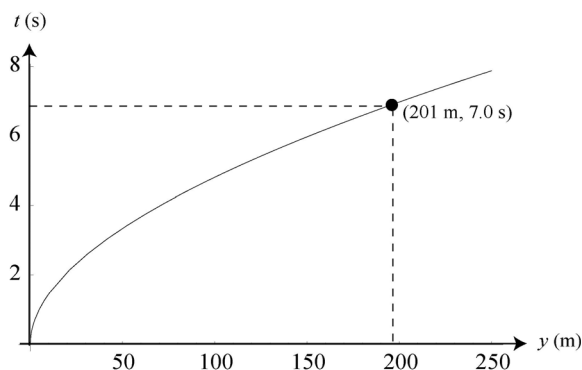
Represent mathematically Let $y(t) = v_0 t + \frac{1}{2} g t^2$ (with downward as $+y$), the time taken for the rock to drop a distance y_0 is $t_1 = \sqrt{2y_0 / g}$. It then takes $t_2 = y_0 / v_s$ for the sound to travel back. So the time elapsed before you hear the echo is

$$t = \sqrt{\frac{2y_0}{g}} + \frac{y_0}{v_s}$$

Solve and evaluate To solve for y_0 , we first bring the second term to the left-hand-side and then square both sides to obtain

$$\frac{2y_0}{g} = \left(t - \frac{y_0}{v_s} \right)^2$$

With $t = 7.0$ s and $v_s = 340$ m/s, after some algebra, we obtain $y_0 = 201$ m. In our calculation, we have ignored air resistance and assumed that the rock and the echo travel in a straight path. Taking these factors into consideration will result in a lower acceleration, and hence a smaller depth. The time it takes for you to hear the echo as a function of canyon depth is plotted below.



71. The vertical distance fallen t seconds after an object undergoes free fall is $y = \frac{1}{2}gt^2$. Thus, we find the reaction time to be

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(0.12 \text{ m})}{9.8 \text{ m/s}^2}} = 0.156 \text{ s}$$

or about 0.16 s.

72. Using $y(t) = \frac{1}{2}gt^2$ (with downward taken to be $+y$), the time taken for the diver to fall at a distance y is $t = \sqrt{2y/g}$. With $y = 36$ m, we obtain

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(36 \text{ m})}{9.8 \text{ m/s}^2}} = 2.7 \text{ s}$$

The speed of the diver before entering the water is $v = gt = (9.8 \text{ m/s}^2)(2.7 \text{ s}) = 26.6 \text{ m/s}$. In this calculation, we have modeled the diver as a point-like object and ignored air resistance. Taking air resistance into consideration would result in a lower speed for the diver.

73. The time required for the first rock to strike the ground is

$$t_1 = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(55 \text{ m})}{9.8 \text{ m/s}^2}} = 3.35 \text{ s}$$

For the second rock to reach the ground the same time as the first one, its travel time would be $t_2 = t_1 - \Delta t = 3.35 \text{ s} - 0.50 \text{ s} = 2.85 \text{ s}$. Let its initial speed be v_{20} . We then have

$$y = v_{20}t + \frac{1}{2}gt^2 \Rightarrow 55 \text{ m} = (v_{20})(2.85 \text{ s}) + \frac{1}{2}(9.8 \text{ m/s}^2)(2.85 \text{ s})^2$$

which can be readily solved to give $v_{20} = 5.33 \text{ m/s}$.

74. Sketch and translate Upon release, the lunch bag will have an initial upward speed of 7.0 m/s , as the hot air balloon. An observer on the ground will first see the bag go up and then come down. The situation is depicted to the right.

Represent mathematically Using $y(t) = y_0 + v_0t - \frac{1}{2}gt^2$,

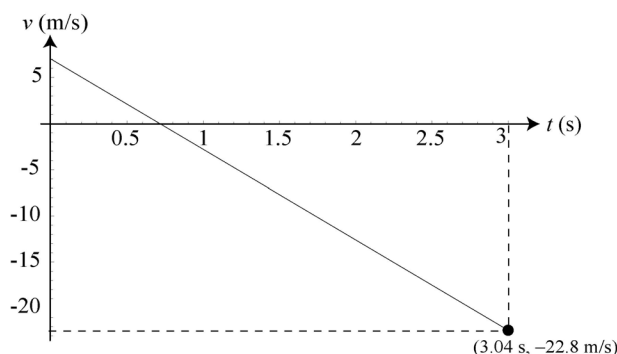
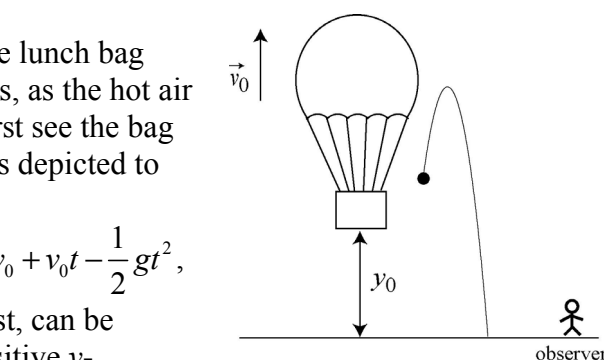
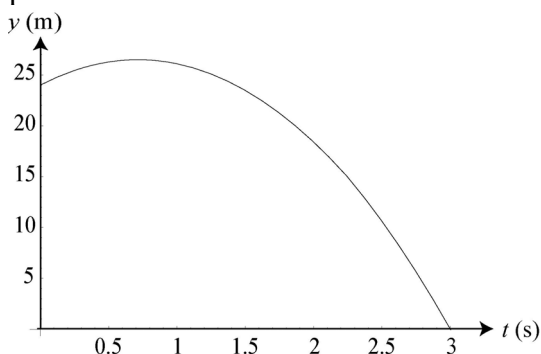
the position of the bag, the object of interest, can be written as (with upward taken to be the positive y -direction)

$$y(t) = (24 \text{ m}) + (7.0 \text{ m/s})t - \frac{1}{2}(9.8 \text{ m/s}^2)t^2$$

Solve and evaluate Solving the quadratic equation for $y = 0$, the condition that the bag has reached the ground, we obtain $t = 3.04 \text{ s}$ (the other solution $t = -1.61 \text{ s}$ can be rejected). With $v(t) = v_0 - gt$, the velocity of the bag right before hitting the ground is

$$v(t) = v_0 - gt = 7 \text{ m/s} - (9.8 \text{ m/s}^2)(3.04 \text{ s}) = -22.8 \text{ m/s}$$

The negative sign indicates that the bag is traveling in the $-y$ -direction. The speed of the bag is simply $+22.8 \text{ m/s}$. The position and velocity of the bag as a function of time are plotted below.



75. The position of the knife can be written as, with upward taken to be positive y -direction, $y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$, where $y_0 = 20$ m and $v_0 = -10$ m/s. Solving the quadratic equation for $y = 0$, the condition that the knife has reached the ground, we obtain $t = 1.24$ s (the other solution $t = -3.3$ s can be rejected). The velocity of the knife right before reaching the ground is

$$v(t) = v_0 - g t = -10 \text{ m/s} - (9.8 \text{ m/s}^2)(1.24 \text{ s}) = -22.2 \text{ m/s}$$

The negative sign indicates that the knife is traveling in the $-y$ -direction. The speed of the knife is simply $+22.2$ m/s.

76. Assuming that the driver of the car before you applied an acceleration of -5.0 m/s^2 to stop his car, it would take $t_1 = \Delta v / a = (-20 \text{ m/s}) / (-5.0 \text{ m/s}^2) = 4.0$ s for the car to stop, and the distance traveled during this time interval is

$$x_1 = v_0 t_1 + \frac{1}{2} a t_1^2 = (20 \text{ m/s})(4.0 \text{ s}) + \frac{1}{2}(-5.0 \text{ m/s}^2)(4.0 \text{ s})^2 = 40 \text{ m}$$

Driving at a distance 20 m behind the first car, you have a total of 4.0 s to respond to avoid a collision. With a reaction time of $\Delta t = 0.60$ s, your car has advanced a “reaction distance” of $\Delta x = (20 \text{ m/s})(0.60 \text{ s}) = 12$ m. This means that you must be able to stop your car within $t_2 = t_1 - \Delta t = 4.0 \text{ s} - 0.60 \text{ s} = 3.4$ s, and the “braking distance” be less than $x_c = 40 \text{ m} + 20 \text{ m} - 12 \text{ m} = 48$ m. The acceleration to stop in 3.4 s is $a_2 = \Delta v / t_2 = (-20 \text{ m/s}) / (3.4 \text{ s}) = -5.88 \text{ m/s}^2$, and the distance traveled is

$$x_2 = v_0 t_2 + \frac{1}{2} a_2 t_2^2 = (20 \text{ m/s})(3.4 \text{ s}) + \frac{1}{2}(-5.88 \text{ m/s}^2)(3.4 \text{ s})^2 = 34 \text{ m}$$

This is less than 40 m, so a collision can be avoided.

77. Suppose Car 1 traveling with a speed of v_{10} applies a constant acceleration a_1 . It would take $t_1 = -v_{10} / a_1$ ($a_1 < 0$) for the car to stop, and the distance traveled during this time interval is

$$x_1 = v_{10} t_1 + \frac{1}{2} a_1 t_1^2 = -\frac{v_{10}^2}{2a_1}$$

Similarly, if Car 2 is traveling with a speed of v_{20} , with a constant acceleration a_2 , it would take $t_2 = -v_{20} / a_2$ ($a_2 < 0$) for Car 2 to stop, and the distance traveled during this time interval is $x_2 = -v_{20}^2 / 2a_2$. For the driver in Car 2, his reaction time is t_R , so his

reaction distance is $x_R = v_{20}t_R$. If the initial separation between the two cars is d , then the condition for avoiding a rear-end collision is $x_2 + x_R < d + x_1$, with $t_2 + t_R = t_1$. The above expressions can be combined to give

$$d > \frac{v_{10}^2}{2a_1} - \frac{v_{20}^2}{2a_2} + v_{20}t_R$$

If both cars have the same initial speed, $v_{10} = v_{20} = v_0$, then the condition for stopping $0 = v_0 + a_2t_2 = v_0 + a_1t_1$ would imply

$$a_2 = a_1 \frac{t_1}{t_2} = a_1 \frac{t_1}{t_1 - t_R} = -\frac{v_0}{t_1 - t_R}$$

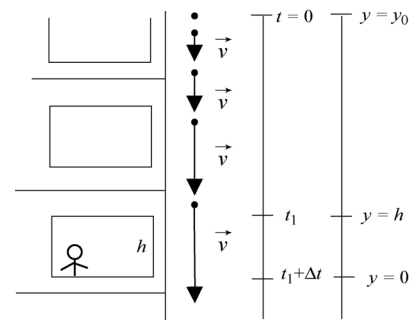
and the above expression can be simplified to $d > v_0t_R / 2$. Note that this represents the minimum distance required with $a_2 = -v_0 / (t_1 - t_R)$ given above to avoid collision. For example, if the reaction time is $t_R = 0.60$ s, with $v_0 = 80$ km/h = 22.2 m/s, the minimum distance of separation would be $d = v_0t_R / 2 = (22.2 \text{ m/s})(0.60 \text{ s})/2 = 6.7$ m, provided that the condition $a_2t_2 = a_1t_1$ is met (car 2 brakes harder than car 1).

78. The distance traveled during reaction time is $x_R = v_0t_R = (21 \text{ m/s})(0.80 \text{ s}) = 16.8$ m. After applying the brake, the distance traveled before coming to a stop is

$$x_b = -\frac{v_0^2}{2a} = -\frac{(21 \text{ m/s})^2}{2(-7.0 \text{ m/s}^2)} = 31.5 \text{ m}$$

79. Sketch and translate We choose Earth as the object of reference with your position as the reference point ($y = 0$). The positive direction points upward. The water balloon is our object of interest, which we treat as point-like. The situation is depicted in the figure to the right.

Simplify and diagram The balloon's acceleration is -9.8 m/s^2 as it is dropped. The position of the water balloon can be written as, with upward taken to be positive y -direction, $y(t) = y_0 - \frac{1}{2}gt^2$.



Represent mathematically The vertical distance traveled between t_1 and $t_1 + \Delta t$ is

$$h = y(t_1) - y(t_1 + \Delta t) = \frac{1}{2}g(t_1 + \Delta t)^2 - \frac{1}{2}gt_1^2 = gt_1\Delta t + \frac{1}{2}g(\Delta t)^2$$

Solve and evaluate With $h = 1.6$ m and $\Delta t = 0.15$ s, we solve for t_1 and obtain $t_1 = 1.01$ s. Therefore, we conclude that the water balloon must have been dropped from a height

$$y_0 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(1.01 \text{ s})^2 = 5.0 \text{ m}$$

above you. So, look for guest that live 5.0 m, or two floors, above you!

80. According to the problem, to incur zygomatic bone injury, the acceleration of the puck must exceed 900 g , with a contact time of 6.0 ms or longer. Suppose the puck hits the cheek and the contact time interval lasts 0.0060 s (6.0 ms), during which its speed changes by Δv . The change of speed needed to have an acceleration of 900 g would then be

$$a_c = -900g = -8.8 \times 10^3 \text{ m/s}^2 = -\frac{\Delta v}{6.0 \times 10^{-3} \text{ s}} \Rightarrow \Delta v = -53 \text{ m/s}$$

or a change of about 118 mi/h. If the puck hits the face and stops, the stopping distance is given by $v_f^2 = v_i^2 + 2a_c d$, or

$$d = \frac{v_f^2 - v_i^2}{2a_c} = \frac{0 - (53 \text{ m/s})^2}{2(-8.8 \times 10^3 \text{ m/s}^2)} = 0.16 \text{ m}$$

This stopping distance is unphysical, as it would mean that the puck has gone inside the skull. Thus, the puck is not likely to be in contact with the bone long enough to break it.

81. Let the speed of the rocket at the end of 1.6 s be v_1 . The maximum height it reaches is $y_{\max} = v_1^2 / 2g$. With $y_{\max} = 80$ m, we find the speed to be

$$v_1 = \sqrt{2gy_{\max}} = \sqrt{2(9.8 \text{ m/s}^2)(80 \text{ m})} = 39.6 \text{ m/s}$$

Thus, the average acceleration during the launch is $a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{39.6 \text{ m/s}}{1.6 \text{ s}} = 24.7 \text{ m/s}^2$.

Note that we have assumed the rocket to be a point-like object and neglected air resistance. Taking air resistance into consideration would have raised the value of a_{avg} during launch.

82. The results are tabulated below:

Speed (mi/h)	Speed v_0 (m/s)	Reaction distance d_R (m)	Reaction time t_R (s)	Braking distance d_B (m)	Total stopping distance (m)	Acceleration a (m/s ²)
20	8.94	7	0.783	7	14	-5.7
40	17.88	13	0.727	32	45	-4.99
60	26.82	20	0.75	91	111	-3.95

In the above, the reaction time is d_R/v_0 , while the acceleration is $a = -v_0^2 / 2d_B$.

83. Suppose the truck is going at $v_0 = 55$ mi/h (25 m/s), and you're 20 m behind the truck. You decide to accelerate at $a = 1.0$ m/s² to pass the truck in Δt seconds. To pass safely, you also want to pass the truck by 20 m before moving back to the lane. Since the truck has moved a distance $v_0\Delta t$ in time Δt , the distance you need to travel is

$$d = 20 \text{ m} + 20 \text{ m} + v_0\Delta t = 40 \text{ m} + (25 \text{ m/s})\Delta t$$

Equating with $\frac{1}{2}a(\Delta t)^2 = \frac{1}{2}(1.0 \text{ m/s}^2)(\Delta t)^2$ and solving for Δt , we have $\Delta t = 9$ s. This is a reasonable time for passing a truck.

84. We first ignore reaction time. Suppose Car A traveling with a speed of v_{A0} applies a constant acceleration a_A . It would take $t_A = -v_{A0} / a_A$ ($a_A < 0$) for the car to stop, and the distance traveled during this time interval is

$$x_A = v_{A0}t_A + \frac{1}{2}a_At_A^2 = -\frac{v_{A0}^2}{2a_A}$$

Similarly, if Car B is traveling with a speed of v_{B0} , with a constant acceleration a_B , it would take $t_B = -v_{B0}/a_B$ ($a_B < 0$) for Car B to stop, and the distance traveled during this time interval is $x_B = -v_{B0}^2/2a_B$. Substituting the values given, we find the total distance traveled by the two cars to be

$$\begin{aligned} x_{\text{total}} = x_A + x_B &= -\frac{v_{A0}^2}{2a_A} - \frac{v_{B0}^2}{2a_B} = \frac{-(30 \text{ m/s})^2}{2(-7.0 \text{ m/s}^2)} + \frac{-(20 \text{ m/s})^2}{2(-9.0 \text{ m/s}^2)} \\ &= 64.3 \text{ m} + 22.2 \text{ m} = 86.5 \text{ m} \end{aligned}$$

which is less than 100 m. Thus, the two cars will not collide.

Realistically, one should take reaction time t_R into consideration. Suppose $t_R = 0.60$ s for both drivers. Then the total distance traveled during reaction time would be

$x_R = x_{R,A} + x_{R,B} = (v_{A0} + v_{B0})t_R = (30 \text{ m/s} + 20 \text{ m/s})(0.60 \text{ s}) = 30 \text{ m}$. In this case, we would have

$$x_{\text{total}} = x_A + x_B + x_{R,A} + x_{R,B} = 86.5 \text{ m} + 30 = 116.5 \text{ m}$$

which exceeds their initial separation of 100m, resulting in collision.

85. (c) The time interval is $\Delta t = \frac{\Delta v}{a} = \frac{10 \text{ m/s}}{1000 \text{ m/s}^2} = 10^{-2} \text{ s}$.

86. (c) The average speed is $v_{\text{avg}} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(10 \text{ m/s} + 0) = 5.0 \text{ m/s}$.

87. (a) The stopping distance would be $\Delta x = v_{\text{avg}} \Delta t = (4.0 \text{ m/s})(0.01) = 0.04 \text{ m}$.

88. (e) The stopping distance is $\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (10 \text{ m/s})^2}{2(1000 \text{ m/s}^2)} = 0.05 \text{ m}$.

89. (d) Acceleration can be calculated using $a = \frac{v - v_0}{\Delta t}$ or $a = \frac{v^2 - v_0^2}{2\Delta x}$. Thus, we see that a longer impact time interval (Δt), a longer stopping distance (Δx), and a smaller initial speed v_0 , all reduce the magnitude of the acceleration.

90. (b) From the reading passage, air in the atmosphere absorbs the X-rays before they reach Earth-based detectors.

91. (a) The speed of the rocket at the end of the burn is

$$v = at = (300 \text{ m/s}^2)(8.0 \text{ s}) = 2400 \text{ m/s}$$

92. (b) The height reached at the end of the fuel burn is

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}(300 \text{ m/s}^2)(8.0 \text{ s})^2 = 9600 \text{ m}$$

or about 10,000 m.

93. (c) The speed of the rocket at the end of the burn is $v_1 = 2400 \text{ m/s}$. Its subsequent speed is given by $v(t) = v_1 - gt$, and its instantaneous speed at the maximum height is zero. The time it takes to get there is

$$\Delta t = \frac{\Delta v}{g} = \frac{2400 \text{ m/s}}{9.8 \text{ m/s}^2} = 245 \text{ s}$$

or about 250 s.

94. (a) The maximum height the rocket reaches is

$$y_{\text{max}} = \frac{v_1^2}{2g} = \frac{(2400 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 2.94 \times 10^5 \text{ m}$$

or about 300,000 m.