Units Convetsion

(a) $\dfrac{1}{640} \times 5280^2 = \underline{\underline{4.36 \times 10^4 \ ft^3}}$

acte ft $\quad \dfrac{\text{mile}^2}{\text{acte}} \quad \dfrac{ft^2}{\text{miles}^2}$

(b) $4.36 \times 10^4 \times 7.48 = \underline{\underline{3.26 \times 10^5 \ \cancel{gal}}}$

$ft^3 \quad \dfrac{\cancel{gal}}{ft^3}$

(c) $4.36 \times 10^4 \times \dfrac{1}{3.281^3} = \underline{\underline{1,233 \quad m^3}}$

$ft^3 \quad \dfrac{m^3}{ft^3}$

(d) $M = \rho V$

$= \underset{m^3}{\overset{V}{1,233}} \times \underset{\frac{\cancel{kg}}{m^3}}{\overset{\rho}{1,000}} = 1.233 \times 10^6 \ \underline{kg}$

$= \underline{\underline{1,233 \quad \text{tonnes}}} \quad (t)$

Units Convetsion

Viscosity

$$\mu = 10 \text{ centipoise} = \underbrace{10 \times 0.01 \times 10^{-1} \frac{kg}{m\,s}}_{1 \text{ poise}} = \underline{\underline{0.01 \frac{kg}{m\,s}}}$$

$$= \underset{\frac{kg}{m\,s}}{0.01} \times \frac{0.3048}{\underset{\frac{lbm}{kg}}{0.4536}} \;\; \left(\frac{m}{ft}\right) = \underline{\underline{6.72 \times 10^{-3} \frac{lbm}{ft\,s}}}$$

(Useful conversion factors: 1 cp = 0.000672 lbm/ft s
= 2.42 lbm/(ft hr))

Density

$$\rho = \underset{\frac{g}{cm^3}}{0.8 \times 1} \times \frac{(100)^3}{\underset{\frac{kg}{g}}{1000}} \;\; \left(\frac{cm}{m}\right)^3 = \underline{\underline{800 \frac{kg}{m^3}}}$$

$$= 0.8 \times 62.4 = \underline{\underline{49.9 \frac{lbm}{ft^3}}}$$

1.3 Units Conversion

Gravitational Acceleration

$$g = \frac{981}{100} \frac{cm}{s^2} \frac{m}{cm} = 9.81 \frac{m}{s^2}$$

Pressure

$$p = \frac{14.7 \times 32.2 \times 144 \times 3.281}{2.205}$$

$$\frac{lbf}{in^2} \frac{lbm\,ft}{lbf\,s^2} \frac{in^2}{ft^2} \frac{kg}{lbm} \frac{ft}{m}$$

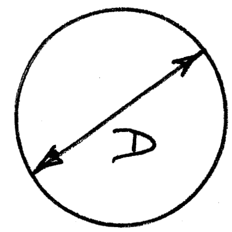
$$= 1.01 \times 10^5 \frac{kg}{m\,s^2} = \frac{N}{m^2} = Pa$$

Since $1\ bar = 10^5\ Pa$

$$p = 1.01\ bar$$

1.4

Meteorite Density



Mass $\quad M = \dfrac{\pi D^3}{6}\rho$

$$\rho = \frac{6M}{\pi D^3} = \frac{6 \times 10^6 \times 1000}{\pi \times 60^3}$$

$$\rho = 8{,}842 \ \frac{kg}{m^3}$$

$$s = \frac{8{,}842}{1{,}000} = 8.84$$

Specific gravities of some elements are

Fe	7.86	Co	8.9	Ag	10.5
Ni	8.9	Pb	11.3	Au	19.3
				U	18.5

Most likely candidate is iron, the deviation being due to the "ball park" figures in the article.

Kinetic Energy

$$\frac{1}{2} M u^2 = \frac{1}{2} 10^9 \times (15000)^2 = 1.125 \times 10^{17} \ J$$

$$\text{TNT equivalent} = \frac{1.125 \times 10^{17}}{5 \times 10^9} = 2.25 \times 10^7 \text{ tonnes}$$

1.5

Reynolds Number

Cross-sectional area

$$A = \frac{\pi D^2}{4} = \frac{\pi \left(\frac{1.05}{12}\right)^2}{4} = 0.00601 \ ft^2$$

Volumetric flow rate

$$Q = \frac{35}{7.48 \times 60} = 0.0780 \ \frac{ft^3}{s}$$

Mean velocity

$$u_m = \frac{Q}{A} = \frac{0.0780}{0.00601} = 12.98 \ \frac{ft}{s}$$

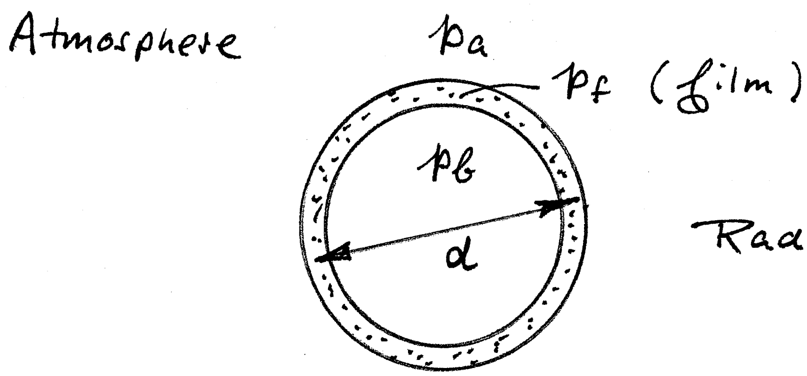
Reynolds number

$$Re = \frac{\rho u_m D}{\mu} = \frac{62.3 \times 12.98 \times 1.05/12}{1.2 \times 0.000672}$$

$$\frac{lbm}{ft^3} \quad \frac{ft}{s} \quad ft \qquad cP \ \frac{cP}{lbm/ft\,s} \quad \Big\} \text{ All units cancel}$$

$$\doteq \underline{\underline{87{,}740}} \quad \text{(dimensionless)}$$

Pressure in Bubble



Radius $a = \frac{d}{2}$

From notes, the increase in pressure as we go inwards across a convex surface is

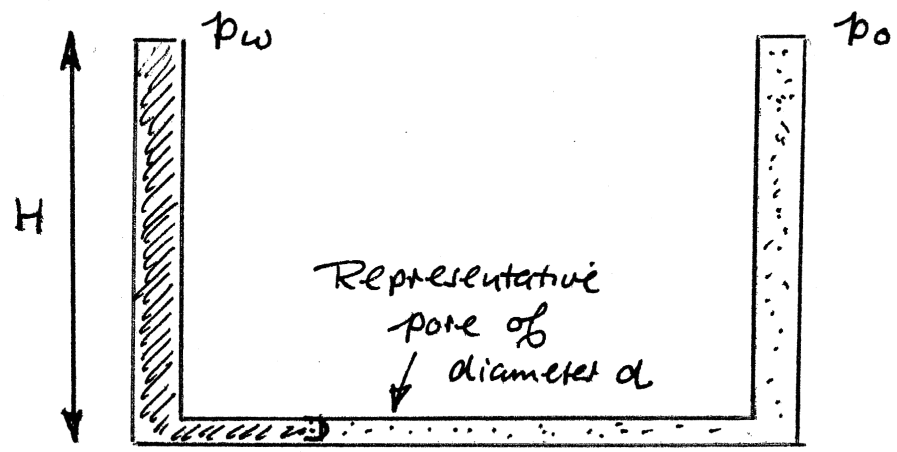
$$\left.\begin{aligned} p_f - p_a &= \frac{2\sigma}{a} \\ p_b - p_f &= \frac{2\sigma}{a} \end{aligned}\right\} \text{Two surfaces are involved}$$

Hence, by addition,

$$p_b - p_a = \frac{4\sigma}{a} = \frac{8\sigma}{d}$$

$$p_b = p_a + \frac{8\sigma}{d}$$

1·7
Reservoir Water-Flooding



$$p_o + \rho_o g H + \frac{4\sigma}{d} = p_w + \rho_w g H$$

Increase in pressure
from oil into water

Hence required water inlet pressure is

$$p_w = p_o - \underbrace{(\rho_w - \rho_o)}_{\text{Positive}} g H + \frac{4\sigma}{d}$$

1.8-1

Barometer Reading

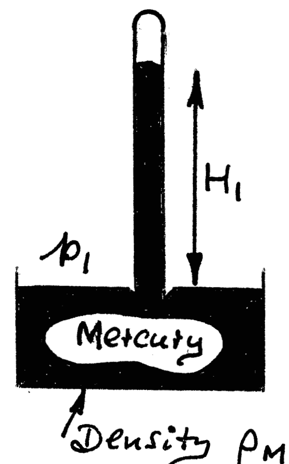
② • House $\quad z_2 = 950 \text{ ft} \quad p_2 = ?$

$H_2 = ?$

① • Weather Station $\quad z_1 = 700 \text{ ft} \quad p_1 = 0.966 \text{ bar}$

$H_1 = ?$

At the weather station



The atmospheric pressure p_1 is balanced by a column of mercury of height H_1:

$$p_1 = \rho_M g H_1$$

Hence

$$H_1 = \frac{p_1}{\rho_M g} = \frac{0.966 \times 10^5 \times 3.281 \times 12}{13.57 \times 1000 \times 9.81}$$

$$\frac{kg}{m^2}\frac{m}{s^2} \quad \frac{m^3}{kg} \quad \frac{s^2}{m} \quad \frac{in}{m} = in$$

$$= \underline{\underline{28.57 \text{ in mercury}}}$$

(Note: $g = 9.81 \text{ m/s}^2$, $3.281 \text{ ft} = 1 \text{ m}$,

$\rho_W = 1000 \frac{kg}{m^3}$, $1 \text{ bar} = 10^5 \text{ pascal} = 10^5 \frac{kg \frac{m}{s^2}}{m^2}$)

<u>Correction for elevation increase</u>. Since $z_2 - z_1$ is "small" for air, we can take ρ_A as essentially constant between ① and ②. Now pressure at weather station is

$$0.966 \text{ bar} \times \frac{14.7 \text{ psia}}{1.01 \text{ bar}} \left(\begin{matrix}\text{see}\\ \text{Problem}\\ 3\end{matrix}\right) = 14.06 \text{ psia}$$

Hence the appropriate mean pressure between ① and ② for purposes of estimating the density can be taken as 14.06 or (as done here, with a trifling change in the answer) slightly less — say 14.0 psia.

$$\rho_A = \frac{M_A p}{RT} = \frac{28.8 \times 14.0}{10.73 \times (460 + \underset{\uparrow}{25})} = 0.0775 \ \frac{\text{lbm}}{\text{ft}^3}$$

Average for January

<u>Change in Pressure</u> . $p_2 - p_1 = -\rho_A g (z_2 - z_1)$

<u>Change in Barometer Reading</u> $H_2 - H_1 = \dfrac{p_2 - p_1}{\rho_M g} = -\dfrac{\rho_A (z_2 - z_1)}{\rho_M}$

$$H_2 - H_1 = -\frac{0.0775\,(950 - 700) \times 12}{13.57 \times 62.4} = -0.27 \text{ in}$$

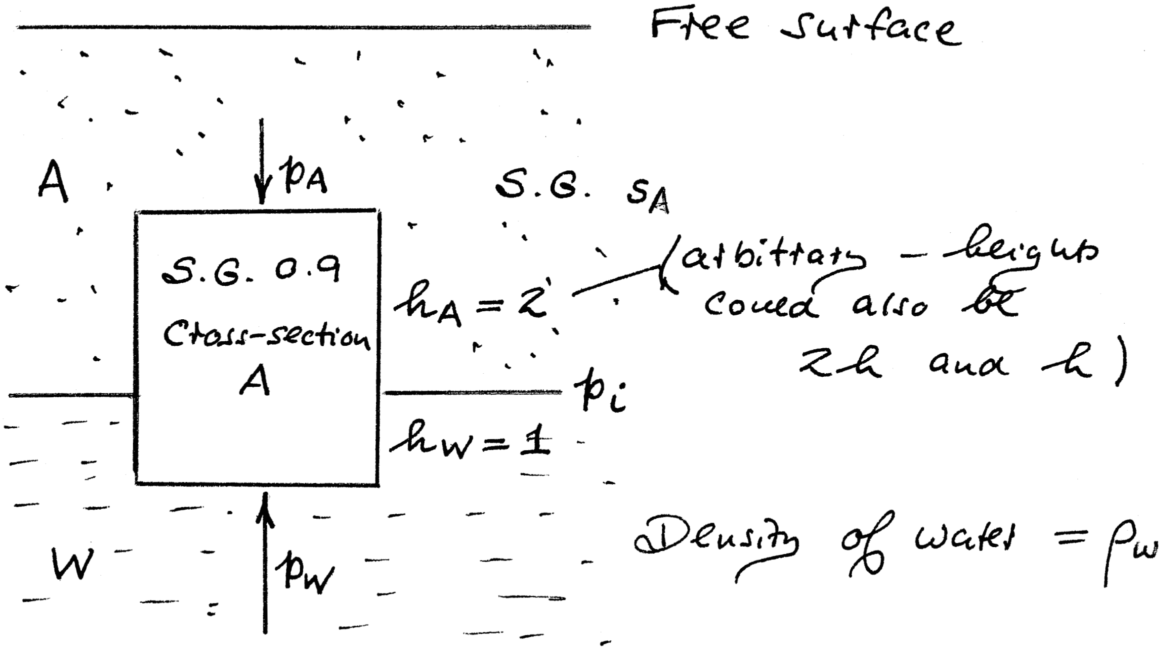
Hence $H_2 = H_1 - 0.27 = 28.57 - 0.27 = \underline{\underline{28.3 \text{ in Hg}}}$

<u>Pressure</u> $p_2 = \rho_M g H_2 = \dfrac{13.57 \times 62.4 \times 32.2 \times 28.3}{32.2 \times 144 \times 12}$

$\underline{\underline{p_2 = 13.87 \text{ psia}}}$ $\qquad \dfrac{\text{lbm}}{\text{ft}^3} \ \dfrac{\text{ft}}{\text{sec}^2} \ \text{in} \ \dfrac{\text{lbf sec}^2}{\text{lbm ft}} \ \dfrac{\text{ft}^2}{\text{in}^2} \ \dfrac{\text{ft}}{\text{in}}$

1.9

Two-Layer Buoyancy



Method 1

Weight displaced (upwards buoyant force) = Weight of cylinder downwards

$$\rho_w A g\,(1 + 2 s_A) = 0.9\,\rho_w A\, 3 g$$

$$\underline{\underline{s_A = 0.85}}$$

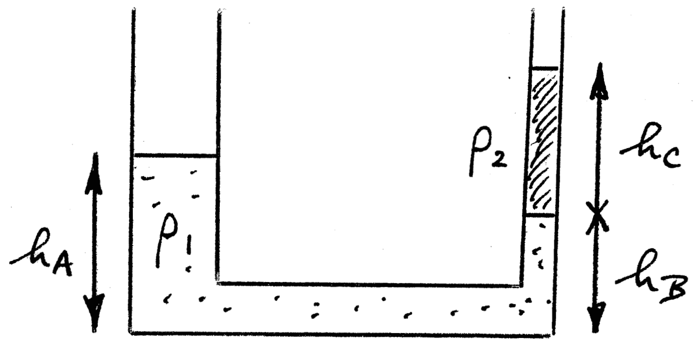
Method 2 Force balance on cylinder ↓

$$\underbrace{p_A A + 0.9 \rho_w A\, 3 g}_{\text{weight}} - p_W A = 0$$

$$p_i - 2 s_A \rho_w g + 2.7 \rho_w g - (p_i + \rho_w g) = 0$$

$$\underline{\underline{s_A = 0.85}}$$

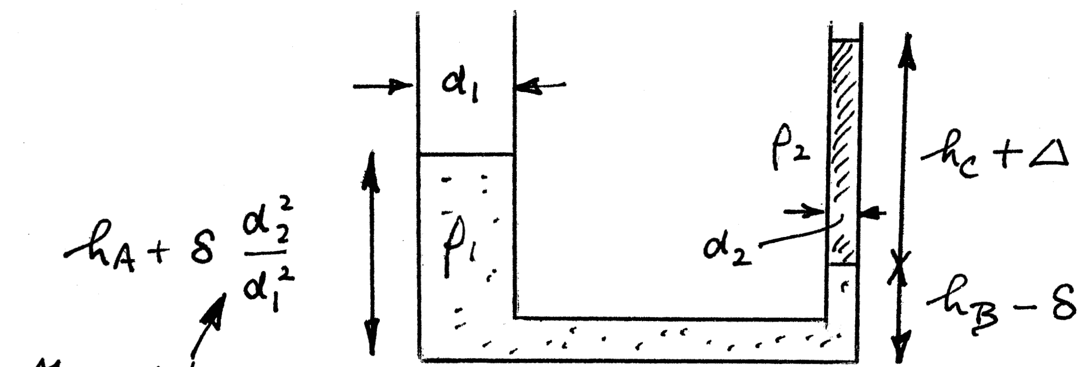
Differential Manometer



$$\rho_1 (h_A - h_B) = \rho_2 h_C \qquad (1)$$

Before

Now add Δ to h_C and let h_B decline by δ



After

$$\rho_1 \left(h_A + \delta \frac{d_2^2}{d_1^2} - (h_B - \delta) \right) = \rho_2 (h_C + \Delta) \qquad (2)$$

Subtract (1) from (2)

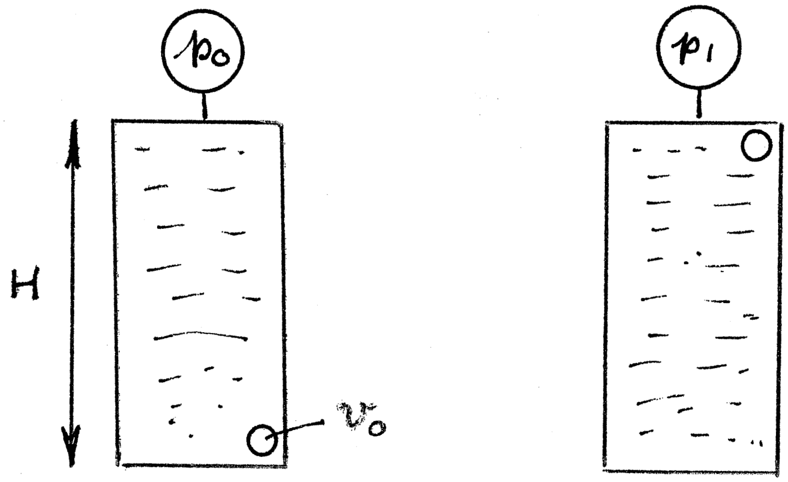
$$\rho_1 \, \delta \left(\frac{d_2^2}{d_1^2} + 1 \right) = \rho_2 \Delta = \rho_2 \frac{v_2}{\frac{\pi}{4} d_2^2} \qquad (3)$$

Equation (3) gives δ as a function of v_2, and also enables ρ_2 to be found by observing the value of δ.

Solution for δ gives:

$$\delta = \frac{\rho_2}{\rho_1} \frac{4 v_2}{\pi d_2^2} \left(\frac{d_1^2}{d_1^2 + d_2^2} \right)$$

Ascending Bubble



Since the cylinder and oil volumes don't change, the bubble volume must remain constant at v_0.

But

$$p V_0 = nRT$$

Therefore, since T is constant, p within the bubble does not change. Hence

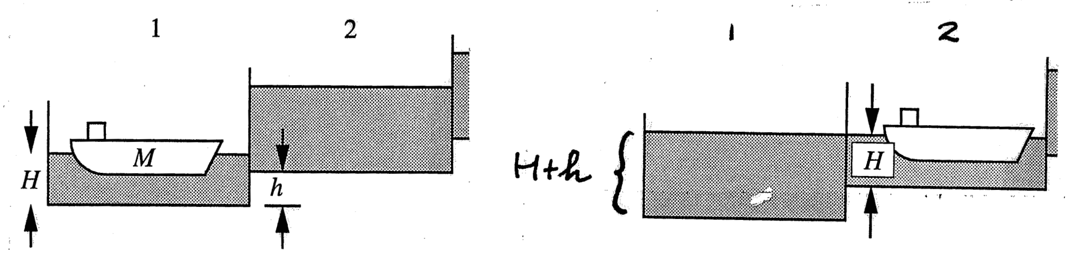
$$p = \underbrace{p_0 + \rho g H}_{\text{Before}} = \underset{\text{After}}{p_1}$$

Thus $\quad p_1 = p_0 + \rho g H$

Ship Passing Through Locks

Uphill The ship must increase its elevation by an amount h as it passes from lock 1 to lock 2. Consider the water in lock 1 before and after;

Before **After**



Mass of water in lock $= \rho A H - M$

Mass of water in lock $= \rho A(H+h)$

From Archimedes law, the ship displaces a mass M of water

Hence mass of water to be supplied to lock 1 is

$$\rho A(H+h) - (\rho A H - M) = \rho A h + M$$

Downhill A similar analysis gives the water loss from a lock as

$$\underbrace{\rho A H - M}_{\text{mass at start}} - \underbrace{\rho A(H-h)}_{\text{mass at end (note that final depth of water in lock is } H-h\text{)}} = \rho A h - M$$

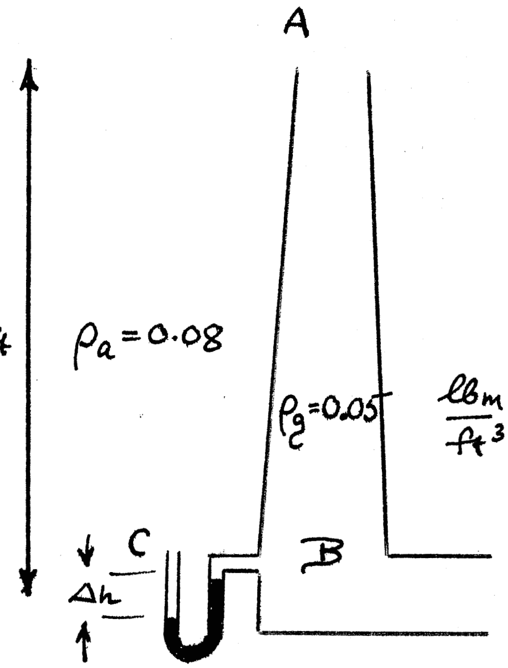
Total water supply

(i) Uphill only: $\rho A h + M$ (depends on M)

(ii) Up and down: $\rho A h + M + \rho A h - M = 2\rho A h$ (independent of M)

Furnace Stack

Start from point A and consider hydro-static increase of pressure in both cases:



$$p_B = p_A + \rho_g g H$$

$$p_C = p_A + \rho_a g H$$

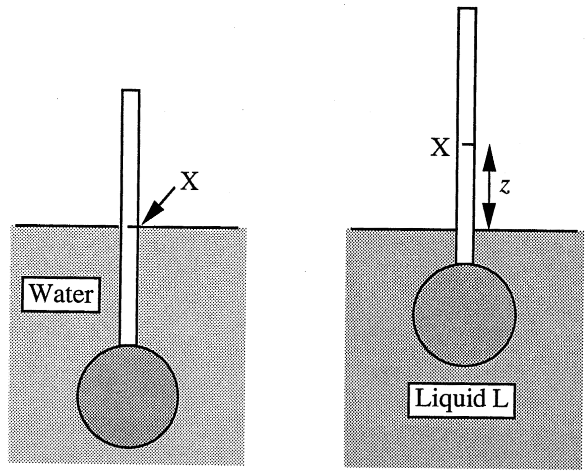
Hence $p_C = p_B + \underbrace{(\rho_a - \rho_g)}_{\text{positive}} g H$

Hence the water moves <u>up</u> in the right-hand leg by Δh given by

$$\rho_w g \Delta h = (\rho_a - \rho_g) g H$$

$$\Delta h = \frac{\rho_a - \rho_g}{\rho_w} H = \frac{(0.08 - 0.05) \times 100 \times 12}{62.4} = \underline{\underline{0.58 \text{ in.}}}$$

Hydrometer



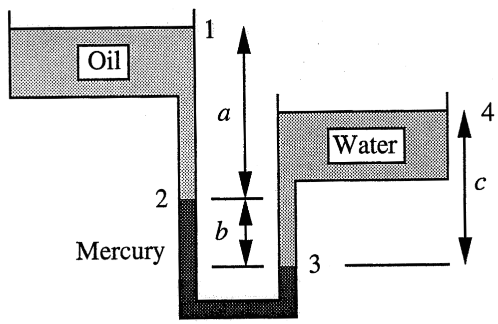
Since the same weight (that of the hydrometer) is supported by the displaced liquid in both cases:

$$\underset{\text{mass of hydrometer}}{Mg} = V \underset{\substack{\uparrow \\ \text{Density of water}}}{\rho_w} g = (V - Az)\,\rho_w s g$$

Cancellation of $\rho_w g$ and solution for s gives

$$s = \frac{1}{1 - \dfrac{Az}{V}}$$

Three-Liquid Manometer



From hydrostatics:

$$p_4 = p_1 + \rho_O a g + \rho_M b g - \rho_W c g = p_1$$

Cancel p_1 and divide by $\rho_W g$, with $s = \frac{\rho}{\rho_W}$

$$s_O a + s_M b = c$$

$$b = \frac{c - s_O a}{s_M} = \frac{48 - 0.8 \times 72}{13.6} = -0.706$$

Thus the diagram is incorrect as drawn, and the mercury rises on the right by 0.706 in.

1.16

Pressure on Mt. Erebus

From lecture notes, $\dfrac{dp}{dz} = -\rho g = -\dfrac{Mp}{RT} g$

Separating variables $\displaystyle\int_{p_0}^{p_H} \frac{dp}{p} = -\frac{Mg}{RT}\int_0^H dz$

$$\ln \frac{p_H}{p_0} = -\frac{MgH}{RT} = \frac{28.8 \times 32.2 \times 13500}{10.73 \times (460-5) \times 32.2 \times 144}$$

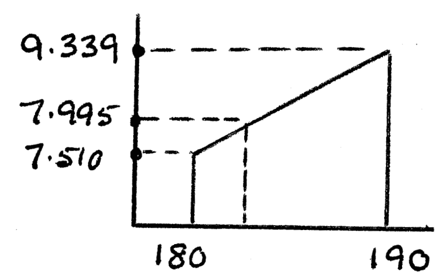
$$\frac{lb_m}{mole}\ \frac{ft}{sec^2}\ ft\ \frac{mole\ {}^\circ R}{psia\ ft^3}\ {}^\circ R\ \frac{lb_f\ sec^2}{lb_m ft}\ \frac{ft^2}{in^2}$$

$$= -0.553 \text{ (dimensionless)}$$

$$p_H = p_0\, e^{-0.553} = 13.9 \times 0.575 = \underline{\underline{7.995 \text{ psia}}}$$

From Perry's Chemical Engineers Handbook

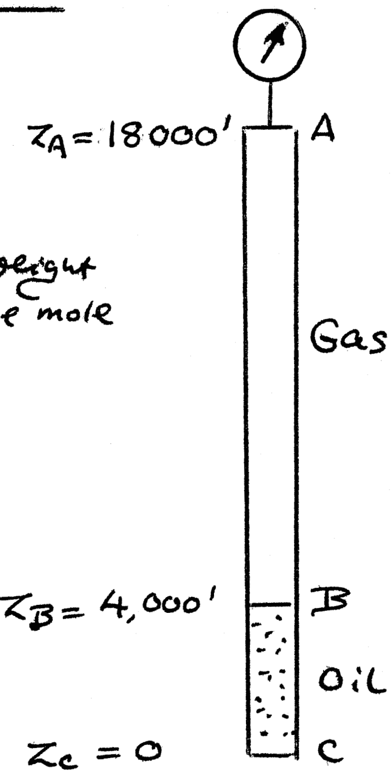
T (°F)	Vapor Pressure of Water (psia)
180	7.510
190	9.339



Using linear interpolation

$$T \doteq 180 + \frac{7.995 - 7.510}{9.339 - 7.510} \times 10 \doteq \underline{\underline{182.7\ {}^\circ F}}$$

Oil and Gas Well Pressures



Hydrostatic variation of pressure

$$\frac{dp}{dz} = -\rho g$$

Imperfect Gas Equation

$$pV = zRT \quad \text{or} \quad \rho = \frac{M}{V} = \frac{Mp}{zRT}$$

per mole

Molecular weight = mass of one mole

For Gas Column

$$\frac{dp}{dz} = -\frac{Mp}{zRT}g \quad \text{or} \quad \int_{p_B}^{p_A} \frac{dp}{p} = -\frac{Mg}{zRT}\int_{z_B}^{z_c} dz$$

$$\ln\frac{p_A}{p_B} = \frac{-0.65 \times 28.8 \times 32.2 \times (18{,}000 - 4{,}000)}{0.8 \times 10.73 \times 520 \times 144 \times 32.2} = -0.408$$

$$p_B = p_A e^{0.408} = 1.503\, p_A$$

$$= 1.503\,(2000 + 14.7) = 3028.9 \text{ psia}$$

$$p_B = 3014.2 \text{ psig}$$

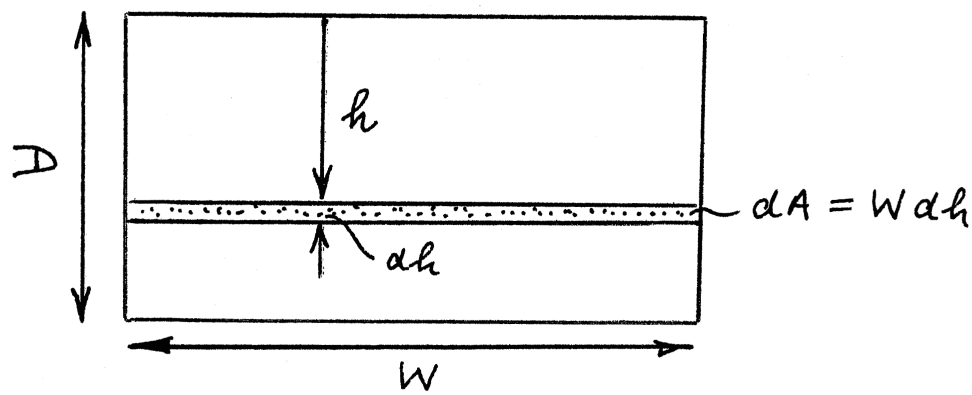
For Oil Column

$$p_c = p_B + \rho g(z_B - z_c)$$

$$= 3014.2 + \frac{0.70 \times 62.4 \times 32.2 \times 4{,}000}{32.2 \times 144}$$

$$= 3014.2 + 1{,}213.3 = 4{,}227.5 \text{ psig}$$

Thrust on Dam



Pressure at depth h is $p = \rho g h$

Force on differential strip

$$dF = p\,dA = \rho g h\, W\, dh$$

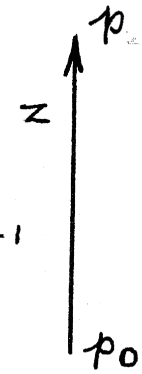
Total thrust is

$$F = \int_0^F dF = \int_0^D \rho g h\, W\, dh$$

$$= \frac{1}{2} \rho g W D^2$$

1.19

Pressure Variations in Air



Accurately, $p = p_o e^{-(M_w g z/RT)}$

For air at $40°F = 500°R$

$$\frac{M_w g}{RT} = \frac{28.8 \times 32.2 \times 5{,}280}{10.73 \times 500 \times 32.2 \times 144} = 0.197 \text{ mile}^{-1}$$

$$\frac{lbm}{lb\ mole}\ \frac{ft}{s^2}\ \frac{ft}{mile}\ \frac{in^2}{lbf}\ \frac{lb\ mole\ °R}{ft^3}\ \frac{}{°R}\ \frac{lbf\ s^2}{lbm\ ft}\ \frac{ft^2}{in^2} = \frac{1}{mile}$$

Approximately, $p^a = p_o \left(1 - \frac{M_w g}{RT} z\right)$

For a 1% error, assuming $p > p^a$ (to be checked)

$$\frac{p - p^a}{p} = 1 - \frac{p^a}{p} = 1 - \frac{1 - 0.197z}{e^{-0.197z}} = 0.01$$

That is, $0.99 e^{-0.197z} + 0.197z - 1 = 0$

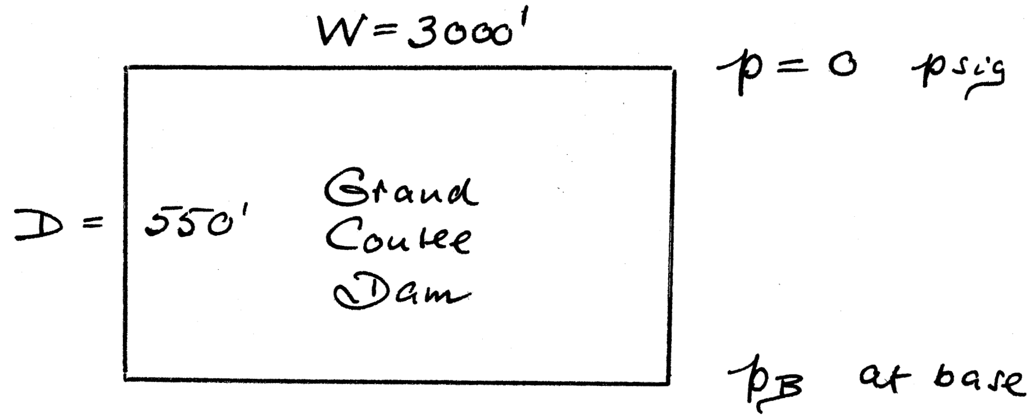
From Excel Solver, $z = 0.686$

That is, the approximation is good within 1% for elevations up to **0.686 miles**

Note, for $f(z) = 0$, Solver was asked to find z that minimized $|f(z)|$.

1.20

Grand Coulee Dam



(a) $p_B = \rho g D = \dfrac{62.4 \times 32.2 \times 550}{32.2 \times 144}$

$= \underline{\underline{238.3 \text{ psig}}}$

(b) From lecture notes, horizontal force is

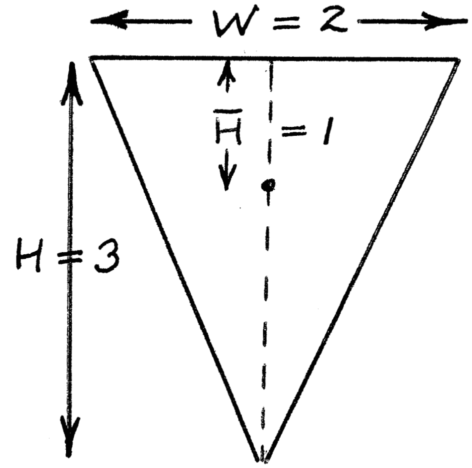
$$F = \underbrace{\frac{1}{2} p_B}_{\bar{p}} W D$$

$$= \frac{238.3 \times 3000 \times 550 \times 144}{2}$$

$$= \underline{\underline{2.83 \times 10^{10} \text{ lbf}}}$$

1.21

Force on V-Shaped Dam



$$A = \frac{2 \times 3}{2} = 3 \text{ m}^2$$

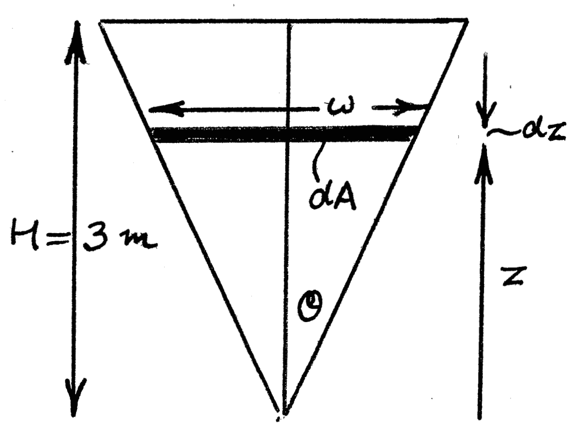
Pressure at centroid

$$\bar{p} = \rho g \bar{H}$$

$$= 10^3 \times 9.81 \times 1$$

$$= 9.81 \times 10^3 \text{ N/m}^2 \text{ or Pa}$$

$$F = \bar{p} A = \underline{\underline{2.943 \times 10^4 \text{ N}}}$$



$$w = 2 z \tan\theta$$

$$F = \int_{z=0}^{H} \underbrace{\rho g (H-z)}_{p} \underbrace{w\, dz}_{dA}$$

where $w = 2 z \tan\theta$

$$F = 2 \rho g \tan\theta \int_0^H (H-z)\, z\, dz$$

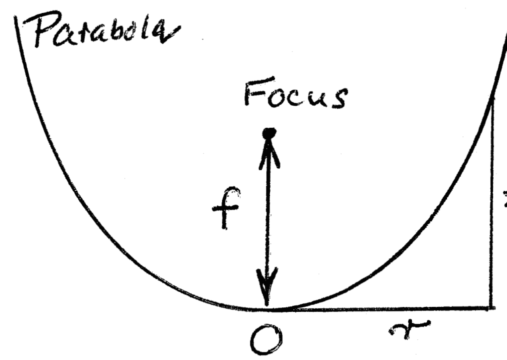
$$= 2 \rho g \tan\theta \left[\frac{Hz^2}{2} - \frac{z^3}{3} \right]_0^H = \frac{1}{3} \rho g H^3 \tan\theta$$

$$= \frac{1}{3} \times 10^3 \times 9.81 \times 3^3 \times \frac{1}{3} = \underline{\underline{29{,}430 \text{ N}}}$$

Rotating Mercury Mirror

Equation of parabola with focal length f is

$$r^2 = 4fz \qquad (1)$$



But from fluid mechanics, we know that

$$z = \frac{\omega^2 r^2}{2g} \qquad (2)$$

Angular velocity is $\omega = 2\pi N = 2\pi \times \frac{1}{6} = 1.05 \ \frac{rad}{s}$

Focal length, from (1) and (2), is

$$f = \frac{g}{2\omega^2} = \frac{9.81}{2 \times 1.05^2} = \underline{\underline{4.45 \ m}}$$

Depression at center is also elevation z when $r = \frac{D}{2}$

$$\Delta z = \frac{\omega^2 \left(\frac{D}{2}\right)^2}{2g} = \frac{1.05^2 \left(\frac{40}{12 \times 3.281 \times 2}\right)^2}{2 \times 9.81} \quad \text{(since } z = 0 \text{ at } r = 0\text{)}$$

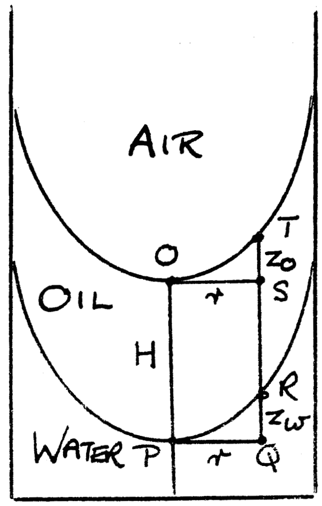
$$= \underline{\underline{0.0145 \ m = 1.45 \ cm}}$$

For new $D = 30$ m mirror (with f unchanged)

$$\Delta z = \frac{1.05^2 \times \left(\frac{30}{2}\right)^2}{2 \times 9.81} = \underline{\underline{12.64 \ m}} \quad \text{(a very large increase)}$$

1.23

Oil & Water in Rotating Container



Pressure in water at Q

$$\rho_o H g + \rho_w \frac{\omega^2 r^2}{2} = \rho_o z_o g + \rho_o (H - z_w) g + \rho_w z_w g$$

OP   PQ   TS   SR   RQ

$$\rho_w \frac{\omega^2 r^2}{2} = \rho_o z_o g + (\rho_w - \rho_o) z_w g \qquad (1)$$

Pressure in Oil at S

$$\rho_o \frac{\omega^2 r^2}{2} = \rho_o z_o g \qquad (2)$$

Subtract (2) from (1)

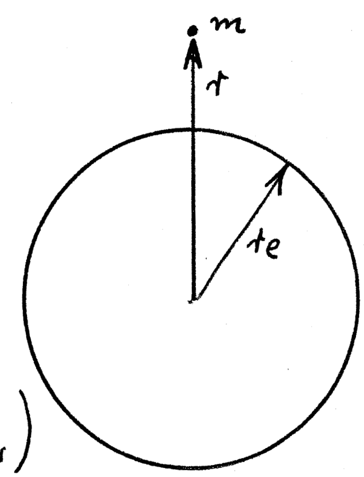
$$(\rho_w - \rho_o) \frac{\omega^2 r^2}{2} = (\rho_w - \rho_o) z_w g$$

Hence

$$\left.\begin{aligned} z_w &= \frac{\omega^2 r^2}{2g} \\ \text{and} \quad z_o &= \frac{\omega^2 r^2}{2g} \end{aligned}\right\} \text{Hence shapes of both interfaces are identical.}$$

1.24

Energy to Place Satellite in Orbit



Work done in pushing the satellite through a distance dr is

$$dw = F\,dr = G\,m\,M_e\,\frac{dr}{r^2}$$

Total energy expended is

$$w = \int_0^w dw = \int_{r_e}^{r_s} G\,m\,M_e\,\frac{dr}{r^2} = G\,m\,M_e\left(\frac{1}{r_e} - \frac{1}{r_s}\right)$$

For a mass m at the earth's surface,

$$F = m g_e = \frac{G\,m\,M_e}{r_e^2} \quad \text{or} \quad G\,M_e = g_e r_e^2$$

Thus, $$w = m\,g_e\,r_e\left(1 - \frac{r_e}{r_s}\right)$$

Numerical values $\quad m = 6{,}700 \times 0.4536 = 3{,}039\ \text{kg}$

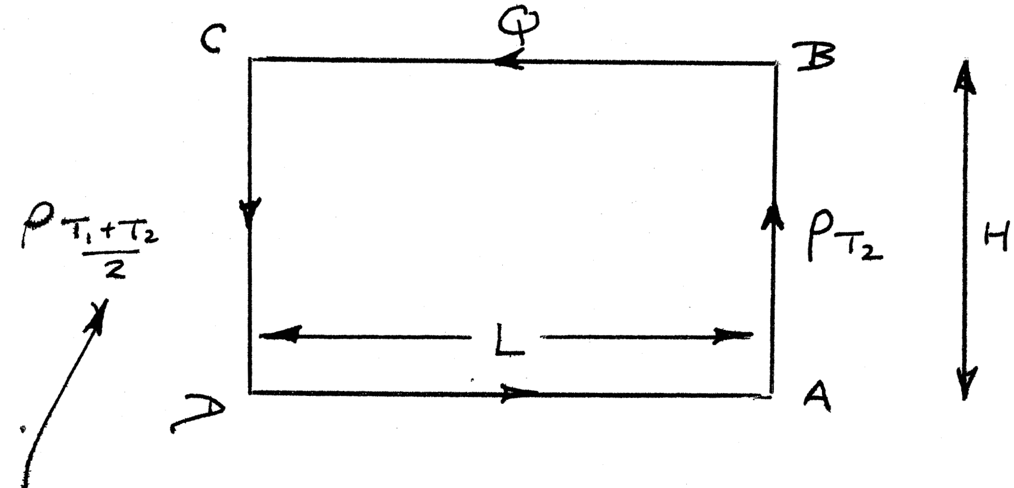
$$r_s - r_e = 360 \times 5{,}280 \times 0.3048 = 5.79 \times 10^5\ \text{m}$$

$$r_s = 5.79 \times 10^5 + 6.37 \times 10^6 = 6.95 \times 10^6\ \text{m}$$

$$w = 3{,}039 \times 9.81 \times 6.37 \times 10^6\left(1 - \frac{6.37 \times 10^6}{6.95 \times 10^6}\right) = \underline{\underline{1.585 \times 10^{10}\ \text{J}}}$$

$$\text{Average power} = \frac{1.585 \times 10^{10}}{78 \times 60} = 3.39 \times 10^6\ \text{W} = \underline{\underline{3.39\ \text{MW}}}$$

Central-Heating Loop



Note that the midpoint temperature along CD is $\frac{1}{2}(T_1+T_2)$

Going round the loop ABCD, there is no net change in the pressure:

$$\underbrace{\underset{\text{Loss in AB}}{-\rho_{T_2} g H} \quad + \underset{\text{Gain in CD}}{\rho_{\frac{T_1+T_2}{2}} g H}}_{\text{Hydrostatic changes}} \quad - \underbrace{\frac{cu^2}{D}(2L+2H)}_{\text{Total frictional loss}} = 0$$

But $\rho = \bar{\rho}\left[1 - \alpha(T-\bar{T})\right]$

$$\rho_{\frac{T_1+T_2}{2}} - \rho_{T_2} = \alpha\bar{\rho}\left(T_2 - \frac{T_1+T_2}{2}\right) = \frac{1}{2}\alpha\bar{\rho}(T_2-T_1)$$

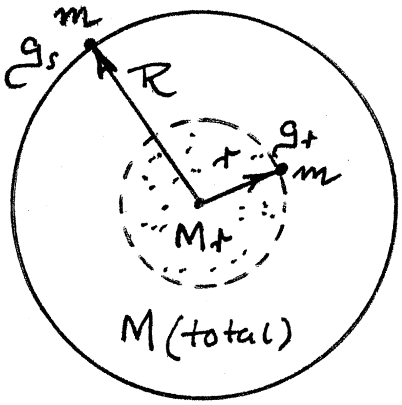
Hence

$$\frac{1}{2} g H \alpha \bar{\rho}(T_2-T_1) = \frac{2cu^2}{D}(L+H)$$

But $\quad Q = \frac{\pi D^2}{4} u \quad$ or $\quad u^2 = \frac{16 Q^2}{\pi^2 D^4}$

Hence $\quad Q = \sqrt{\dfrac{\pi^2 g H \alpha \bar{\rho} D^5 (T_2-T_1)}{64 c (L+H)}}$

Pressure at Center of Earth



Assume earth is liquid with uniform density ρ.

For mass m at radii r and R, the gravitational attractions are

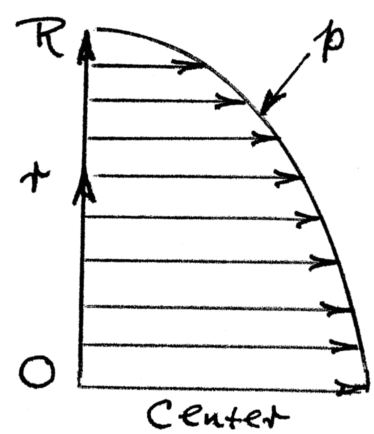
$$\left.\begin{aligned} m g_r &= \frac{G m M_r}{r^2} \\ m g_s &= \frac{G m M}{R^2} \end{aligned}\right\} \quad \frac{g_r}{g_s} = \frac{M_r R^2}{M r^2}$$

Masses:
$$\left.\begin{aligned} M_r &= \frac{4}{3}\pi \rho r^3 \\ M &= \frac{4}{3}\pi \rho R^3 \end{aligned}\right\} \quad \frac{M_r}{M} = \frac{r^3}{R^3} \qquad \boxed{\frac{g_r}{g_s} = \frac{r}{R}}$$

Hydrostatic Variations

$$\frac{dp}{dr} = -\rho g_r = -\frac{\rho g_s}{R} r \qquad \int_{p_r}^{0} dp = -\frac{\rho g_s}{R}\int_r^R r\,dr$$

Hence $p_r = \dfrac{\rho g_s R}{2}\left(1 - \dfrac{r^2}{R^2}\right)$



Thus, there is a parabolic variation of pressure.

$$p_c = \frac{\rho g_s R}{2}$$

$$p_c = \frac{g_s R}{2}\left(\frac{3M}{4\pi R^3}\right) = \frac{3 M g_s}{8\pi R^2}$$

Numerical Values

$$M = \frac{4}{3}\pi R^3 \rho$$

Hence

$$p_c = \frac{3 M g_s}{8\pi R^2} = \frac{3\left(\frac{4}{3}\pi R^3\right)\rho g_s}{8\pi R^2}$$

$$= \frac{R \rho g_s}{2}$$

$$= \frac{6.37 \times 10^6 \times 5{,}500 \times 9.81}{2} \quad \underbrace{m \frac{kg}{m^3} \frac{m}{s^2}}_{\frac{kg}{m s^2} = Pa}$$

$$\underline{\underline{p_c = 1.72 \times 10^{11} \ Pa}}$$

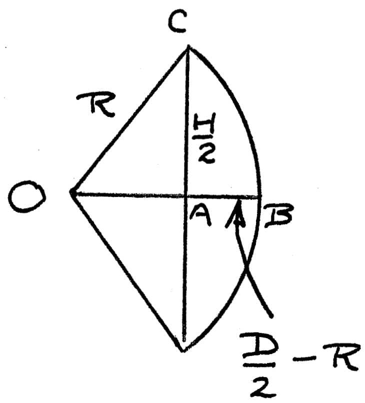
But $1 \text{ atm} = 14.7 \text{ psi} = 1.0133 \times 10^5 \text{ Pa}$

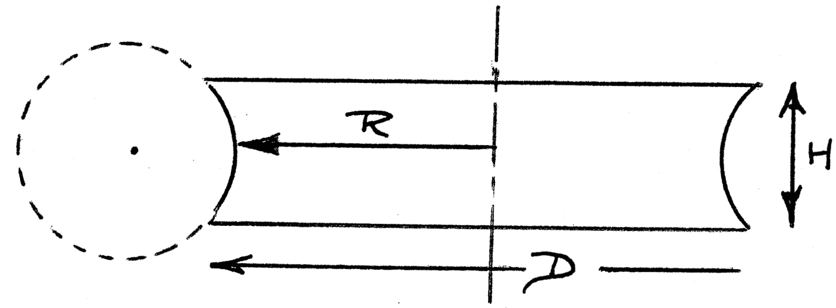
Hence

$$p_a = \frac{1.72 \times 10^{11} \times 14.7}{1.0133 \times 10^5}$$

$$= \underline{\underline{2.49 \times 10^7 \text{ psi}}}$$

1.27
Soap Film on Wire Rings





For curved surface, $\Delta p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Since there is zero pressure change across the film, the two radii of curvature must be equal, the one positive and the other negative.

Geometry gives $OA = R - \left(\frac{D}{2} - R\right) = 2R - \frac{D}{2}$

Pythagoras gives $R^2 = \left(2R - \frac{D}{2}\right)^2 + \left(\frac{H}{2}\right)^2$

$$= 4R^2 - 2RD + \frac{D^2}{4} + \frac{H^2}{4}$$

$$12R^2 - 8RD + (D^2 + H^2) = 0$$

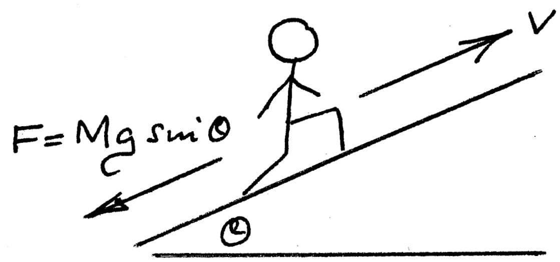
$$R = \frac{8D + \sqrt{64D^2 - 48(D^2+H^2)}}{24} = \frac{1}{6}\left(2D + \sqrt{D^2 - 3H^2}\right)$$

For $D < \sqrt{3}H$ this solution is clearly invalid.

In practice, as the rings are pulled further apart, the film would minimize its total area by forming two separate circles on the rings, with no film in between them.

1.28

Treadmill Stress Test



grade = tan θ

10%: θ = 5.71°, sin θ = 0.100

18%: θ = 10.20°, sin θ = 0.177

Power $\quad P = F v = M g \sin\theta \, V$

$$= \frac{163 \times 32.2 \times 0.177 \times 5 \times 5280}{550 \times 32.2 \times 3600} \quad \frac{lb_m\, ft}{s^2} \; \frac{mile}{hr} \; \frac{ft}{mile} \; \frac{s\, HP}{ft\, lb_f} \; \frac{lb_f\, s^2}{lb_m\, ft} \; \frac{hr}{s}$$

$$P = 0.385 \text{ HP} \quad \text{(at end of test)}$$

Power at start of test

$$= \frac{163 \times 32.2 \times 0.100 \times 1.7 \times 5280}{550 \times 32.2 \times 3600} = 0.0739 \text{ HP}$$

Mean power $\quad \bar{P} \doteq \frac{1}{2}(0.0739 + 0.385) = 0.229 \text{ HP}$

Energy expended $\quad E = \bar{P} t$

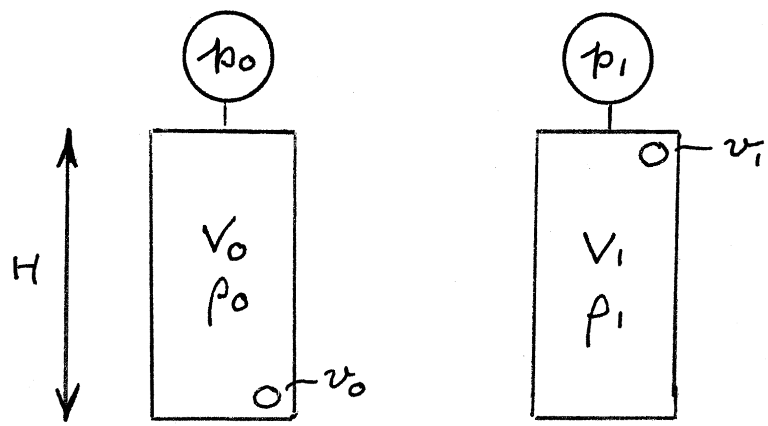
$$= \frac{0.229 \times 13.3 \times 550 \times 60 \times 0.3048}{0.2248} = 1.36 \times 10^5 \text{ J}$$

$$HP \; min \; \frac{ft\, lb_f}{s\, HP} \; \frac{s}{min} \; \frac{m}{ft} \; \frac{N}{lb_f} = N\, m = J$$

Bubble Rising in Compressible Liquid

Isothermal compressibility $\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \doteq -\frac{1}{V}\frac{\Delta V}{\Delta p}$

Hence $\Delta V \doteq -\beta V \Delta p$ (1)



Rigid container $V_0 + v_0 = V_1 + v_1$ (2)

Ideal gas $(p_0 + \rho_0 g H) v_0 = p_1 v_1$ (3)

Constant oil mass $m = \rho_0 V_0 = \rho_1 V_1$ or $\rho_1 = \rho_0 \dfrac{V_0}{V_1}$ (4)

From isothermal compressibility (1)

$$\frac{V_1 - V_0}{V_0} = -\beta\Big[\underbrace{p_1 + \tfrac{1}{2}\rho_1 g H - (p_0 + \tfrac{1}{2}\rho_0 g H)}_{\text{Take mean pressures at mid-height}}\Big]$$

$$= -\beta\left[(p_1 - p_0) + \tfrac{1}{2} g H (\rho_1 - \rho_0)\right]$$

From (2) and (3) and (4):

$$\frac{v_0 - v_1}{V_0} = -\beta\left[(p_0 + \rho_0 g H)\frac{v_0}{v_1} - p_0 + \frac{1}{2} g H \rho_0 \left(\frac{V_0}{V_1} - 1\right)\right]$$

$$= -\beta\left[p_0\left(\frac{v_0 - v_1}{v_1}\right) + \rho_0 g H\left(\underset{A}{\frac{v_0}{v_1}} + \frac{1}{2}\underset{B}{\frac{v_1 - v_0}{V_1}}\right)\right]$$

Since $V_1 >> v_1$ or v_0, $B << A$, so that:

$$(v_0 - v_1)\left[\underset{C}{\frac{1}{V_0}} + \underset{D}{\frac{\beta p_0}{v_1}}\right] = -\beta \rho_0 g H \frac{v_0}{v_1}$$

Since $\beta p_0 V_0 >> v_1$, $D >> C$, so β cancels and:

$$(v_0 - v_1)\frac{p_0}{v_1} = -\rho_0 g H \frac{v_0}{v_1}$$

or

$$\frac{v_1}{v_0} = 1 + \frac{\rho_0 g H}{p_0} = 1 + \frac{\rho g H}{p_0}$$

Thus, the bubble expands upon rising.

Numerical values

$$\frac{\rho g H}{p_0} = \frac{800 \times 9.81 \times 1}{10^5} = 0.0785$$

(1 bar = 10^5 Pa) $\quad \frac{kg}{m^3} \quad \frac{m}{s^2} \quad m \quad \frac{m\,s^2}{kg}$

Hence $\quad \dfrac{v_1}{v_0} = 1.0785$

Check on assumption that $\beta p_0 V_0 >> v_1$

$$5.5 \times 10^{-10} \times 10^5 \times 0.1 \overset{?}{>>} 10^{-8}$$

$$\frac{m^2}{N} \quad \frac{N}{m^2} \quad m^3 \quad\quad m^3$$

$$5.5 \times 10^{-6} \overset{?}{>>} 10^{-8} \quad \text{OK}$$

1.30

Pressures in Oil and Gas Well

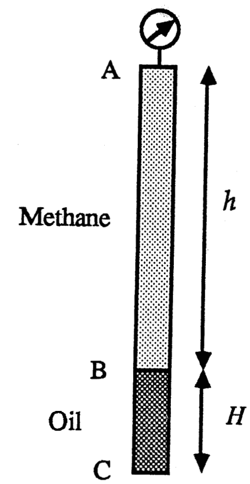


Fig. 1 Well containing oil and methane.

Variation of pressure with elevation for an ideal gas:

$$\frac{dp}{dz} = -\rho g = -\frac{Mp}{RT} g$$

Separate variables and integrate:

$$\int_{p_A}^{p_B} \frac{dp}{p} = -\frac{Mg}{RT} \int_h^0 dz$$

$$\ln \frac{p_B}{p_A} = +\frac{Mgh}{RT} = \frac{16 \times 32.2 \times 10{,}000}{10.73 \times 560 \times 32.2 \times 144}$$

$$= 0.185$$

$$[=] \quad \frac{\text{lbm}}{\text{mole}} \, \frac{\text{ft}}{\text{s}^2} \, \text{ft} \, \frac{\text{in}^2 \text{ mole } {}^\circ\text{R}}{\text{lb}_f \text{ ft}^3} \quad \frac{\text{lb}_f \text{ s}^2}{{}^\circ\text{R} \text{ lb}_m \text{ ft}} \, \frac{\text{ft}^2}{\text{in}^2} \quad \text{(all units cancel)}$$

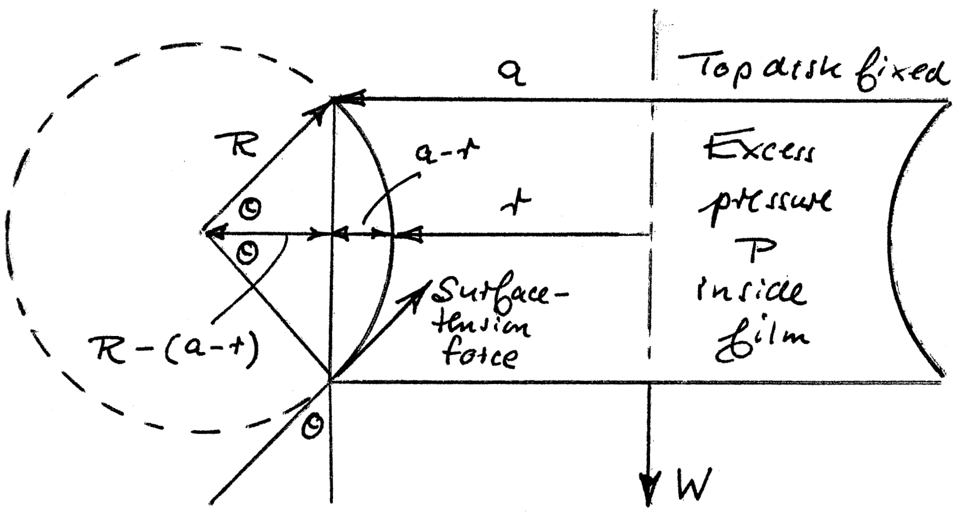
$$p_B = p_A e^{0.185} = 1014.7 \times 1.203 = \underline{\underline{1221 \text{ psia}}}$$

Pressure increase in the oil section

$$p_C - p_B = \rho_o g H = \frac{0.75 \times 62.4 \times 32.2 \times 2000}{32.2 \times 144}$$

$$= \underline{\underline{650 \text{ psi}}}$$

Soap Film Between Two Disks



Force balance ↓ on bottom disk

$$W + P\pi a^2 = 2\pi a(2\sigma)\cos\theta$$

Film has two sides

Hence $\theta = \cos^{-1}\left(\dfrac{W + P\pi a^2}{4\pi a\sigma}\right)$

Excess pressure $P = 2\sigma\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

or $\dfrac{1}{R} = \dfrac{1}{r} - \dfrac{P}{2\sigma}$

From geometry, $\cos\theta = \dfrac{R - (a - r)}{R} = 1 + \dfrac{r - a}{R}$

Hence $\dfrac{W + P\pi a^2}{4\pi a\sigma} = 1 + (r - a)\left(\dfrac{1}{r} - \dfrac{P}{2\sigma}\right)$

Newspaper Statements About the Erg

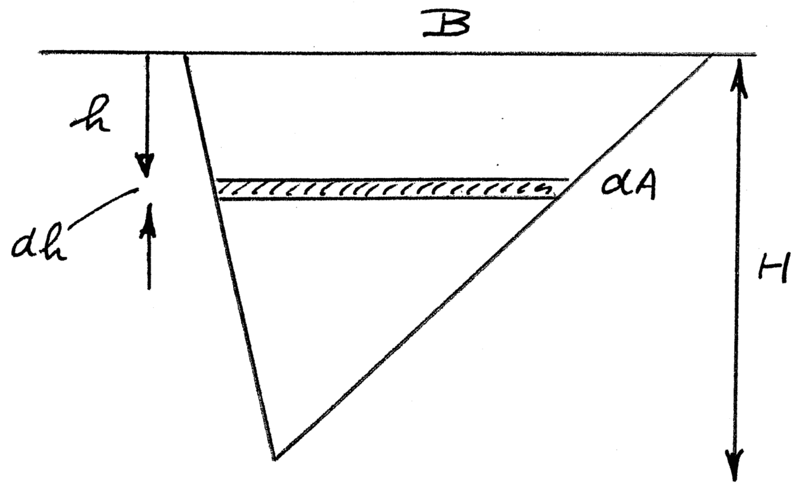
The February 10 letter probably said:

"Concerning the definition of the erg, the January 18 statement is incorrect and the January 30 letter is misleading at best. For example, if the mass were on a virtually frictionless horizontal table, it would require virtually no energy to move it through a (horizontal) distance of one centimeter.

The erg is more correctly defined as the amount of work done (equivalent to energy expended) in pushing back a resisting force of one dyne through a distance of one centimeter. One dyne is the force needed, when applied to a mass of one gram, to accelerate it at one centimeter per second per second."

1.33

Centroid of Triangle



From the text $\quad h_c \equiv \dfrac{\int_A h\,dA}{A} = \dfrac{\int_A h\,dA}{\int_A dA}$

Now the width of the triangle is proportional to $(H-h)$ at any depth h, so that

$$dA = c\,(H-h)\,dh, \text{ where } c \text{ is a constant.}$$

Thus $\quad h_c = \dfrac{\int_0^H h\,c\,(H-h)\,dh}{\int_0^H c\,(H-h)\,dh} = \dfrac{\frac{H^3}{6}}{\frac{H^2}{2}} = \dfrac{H}{3}$

Horizontal force = pressure at centroid depth × area

$$F = \left(\rho g \frac{H}{3}\right)\left(\frac{1}{2} B H\right) = \underline{\underline{\frac{1}{6}\rho g B H^2}}$$

Blake–Kozeny Equation

$$\frac{p_1 - p_2}{L} = 150 \frac{\mu v_0}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Check dimensions

$$\underbrace{\frac{M}{LT^2}\,\frac{1}{L} \qquad \frac{M}{LT}\,\frac{L}{T}\,\frac{1}{L^2} = \frac{M}{L^2T^2}}$$

Same, hence ε is dimensionless. Also, examination of $(1-\varepsilon)$ gives same conclusion assuming "1" is dimensionless.

Numerical values

$$\frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{150\,\mu v_0 L}{(p_1-p_2)D_p^2} = \frac{150 \times \overbrace{0.22}^{\frac{g}{cm\,s}} \times \overset{\frac{ft}{s}}{0.1} \times 6 \times \overset{\frac{cm}{ft^2}}{30.48}}{\underset{\frac{g}{lbm}}{453.6} \times \underset{\frac{lbf}{in^2}}{75} \times \underset{in^2}{0.1^2} \times \underset{\frac{lbm\,ft}{lbf\,s^2}}{32.2}}$$

$$= 0.05509$$

Rearrange to $\varepsilon = f(\varepsilon) = [0.05509 \times (1-\varepsilon)^2]^{1/3}$

Successive substitution

i	ε_i	$f(\varepsilon_i)$
1	0.5000	0.2397
2	0.2397	0.3170
3	0.3170	0.3014
4	0.3014	0.2996
5	0.2996	0.3001
6	0.3001	0.3000

Porosity (void fraction)

$$\varepsilon = 0.300$$

Shear Stresses for Air and Water

Viscosity of air

$$\mu_a = \mu_0 \left(\frac{T}{T_0}\right)^n = 0.0171 \left(\frac{293}{273}\right)^{0.768} = 0.0181 \text{ cP}$$

Viscosity of water

$$\mu_w = e^{a + b \ln T} = e^{29.76 - 5.24 \ln 293} = 1.00 \text{ cP}$$

Stresses

Air $$\tau_a = \mu_a \frac{V}{h} = 0.01 \times 0.0181 \times \frac{1}{0.1} = 0.00181 \ \frac{\text{dynes}}{\text{cm}^2}$$

Water $$\tau_w = \mu_w \frac{V}{h} = 0.01 \times 1.0 \times \frac{1}{0.1} = 0.1 \ \frac{\text{dynes}}{\text{cm}^2}$$

Ratio of stresses

$$\frac{\tau_w}{\tau_a} = \frac{0.1}{0.00181} = 55.2 \quad \left(= \frac{\mu_w}{\mu_a}\right)$$

At higher temperature

Air: μ_a increases, and so does τ_a

Water: μ_w and hence τ_w decrease

The shear stress τ is uniform across the gap because the velocity profile is linear (a situation itself due to zero applied pressure gradient).