Solutions Manual for Elasticity in Engineering Mechanics, Third Edition

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Solution Manual Foreword

This solution manual is intended to serve as an aid to the busy instructor. Because of the nature of the theory and the applications of elasticity, some problem solutions are lengthy and detailed. The instructor should examine the solutions before assigning problems, so that excessibely long assignments may be avoided. Some of the problems may serve as part of the take home examinations or supervised final and miterm examinations. The solutions to certain problems that involve derivations and/or verification of material presented in the text are not given, either because of their length or because the problem statement outlines the method of solutions.

The authors would appreciatate receiving notification of any errors that the reader may discover in the text and the solution manual. Corrections will be incorporated in future printings and editions.

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2-4.1

$$J = \begin{vmatrix} 1-k & k & 0 \\ k & 1-k & 0 \\ kx_3 & 0 & 1+kx_1 \end{vmatrix} = (1-2k)(1+kx_1) > 0$$

For admissible deformation, we take

$1 > 2k \quad \therefore \quad k < \frac{1}{2} \quad$ and $\quad kx_1 > -1$

2-4.2

$$J = \begin{vmatrix} 1 & ax_3 & ax_2 \\ bx_3 & 1 & bx_1 \\ cx_2 & cx_1 & 1 \end{vmatrix} > 0$$

$$\therefore \quad 1 > bcx_1^2 + acx_2^2 + abx_3^2 - 2abcx_1x_2x_3$$

For finite x_1, x_2, x_3 this is possible for sufficienty small a, b, c.

2-4.3

$$J = \begin{vmatrix} -\frac{(c-y)}{c} \sin\frac{x}{c} & -\cos\frac{x}{c} & 0 \\ \frac{c-y}{c} \cos\frac{x}{c} & -\sin\frac{x}{c} & 0 \\ 0 & 0 & 1 \end{vmatrix} > 0$$

$c \neq 0$

$\therefore \quad 1 - \frac{y}{c} > 0. \qquad$ For $\quad c > 0, \quad c > y.$

$\therefore \quad c > h. \qquad$ For $\quad c < 0, \quad c < -h.$

2-4.4

$$J = \begin{vmatrix} 1-k_1x_2 & -k_1x_1 & 0 \\ 2\nu k_2 x_1 & 1+2k_2\nu x_2 & -2\nu k_2 x_3 \\ 0 & k_3\nu x_3 & 1+k_3\nu x_2 \end{vmatrix} > 0$$

Or

$$J = 1 + x_2(\quad) + x_1^2(\quad) + x_1^2(\quad) + x_1^2 x_2(\quad) + x_2^3(\quad) > 0$$

The terms ( ) all contain combinations of k_1, k_2, k_3 and $\nu\,(<\frac{1}{2})$. Hence, if k_1, k_2, k_3 are sufficiently small, $J>0$ for finite x_1, x_2.

2-4.5

$$J = \begin{vmatrix} 1 & -\theta x_3 & -\theta x_2 \\ \theta x_3 & 1 & \theta x_1 \\ \theta x_2\left(\frac{b^2-a^2}{b^2+a^2}\right) & \theta x_1\left(\frac{b^2-a^2}{b^2+a^2}\right) & 1 \end{vmatrix} > 0$$

Or

$$J = 1 - \left(\frac{b^2-a^2}{b^2+a^2}\right)\left[2\theta^3 x_1x_2x_3 + \theta^2 x_1^2 - \theta^2 x_2^2\right] + \theta^2 x_3^2 > 0$$

$$\therefore\ (a^2+b^2)(1+\theta^2x_3^2) > (a^2-b^2)\left[\theta^2(x_2^2-x_1^2) - 2\theta^3 x_1x_2x_3\right]$$

since $a^2-b^2>0$, $J>0$ provided

$$\frac{(a^2+b^2)(1+\theta^2x_3^2)}{a^2-b^2} > \theta^2\left[x_2^2 - x_1^2 - 2\theta x_1x_2x_3\right]$$

This condition is satisfied for the usual torsion problem $\theta \ll 1$ and x_1, x_2, x_3 of typical range.

2-4.6

$$
\begin{aligned}
I_2(\bar{\boldsymbol{C}}) &= \bar{C}_{ij}\bar{C}_{ji} \\
&= a_{i\alpha}a_{j\beta}a_{j\gamma}a_{i\kappa}C_{\alpha\beta}C_{\gamma\kappa} \\
&= a_{\kappa\alpha}a_{\gamma\beta}C_{\alpha\beta}C_{\gamma\kappa} \\
&= C_{\alpha\beta}C_{\alpha\beta} = I_2(\boldsymbol{C})
\end{aligned}
$$

$$
\begin{aligned}
I_3(\bar{\boldsymbol{C}}) &= \bar{C}_{ij}\bar{C}_{jk}\bar{C}_{ki} \\
&= a_{i\alpha}a_{j\beta}a_{j\gamma}a_{k\kappa}a_{k\chi}a_{i\eta}C_{\alpha\beta}C_{\gamma\kappa}C_{\chi\eta} \\
&= a_{\alpha\eta}a_{\gamma\beta}a_{\chi\kappa}C_{\alpha\beta}C_{\gamma\kappa}C_{\chi\eta} \\
&= C_{\alpha\beta}C_{\beta\chi}C_{\chi\alpha} = I_3(\boldsymbol{C})
\end{aligned}
$$

2-4.7

By expanding the matrix and finding the determinant, it is straightforward to prove that

$$
I_A = A_{11} + A_{22} + A_{33} = A_{kk}
$$

$$
\begin{aligned}
II_A &= \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} + \begin{vmatrix} A_{11} & A_{13} \\ A_{31} & A_{33} \end{vmatrix} + \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \\
&= \frac{1}{2}\left\{ \left(A_{kk}\right)^2 - \left(A_{ij}A_{ji}\right) \right\}
\end{aligned}
$$

$$
\begin{aligned}
III_A &= \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \\
&= \frac{1}{6}\left\{ 2\left(A_{ij}A_{jk}A_{ki}\right) - 3\left(A_{kk}A_{ij}A_{ji}\right) + \left(A_{kk}\right)^3 \right\}
\end{aligned}
$$

2-6.1

By Eq. 2-6.14, given data yields

$$0.004 = 0.002\left(\tfrac{4}{5}\right) + 0.002(0) + (-.002)\left(\tfrac{1}{5}\right) + 2\epsilon_{12}(0) + 2\epsilon_{13}\left(\tfrac{2}{5}\right) + 2\epsilon_{23}(0)$$

$$\therefore \boxed{\epsilon_{13} = 0.0035}$$

$$0.003 = 0.002\left(\tfrac{9}{10}\right) + 0.002\left(\tfrac{1}{10}\right) + 2\epsilon_{12}\left(-\tfrac{3}{10}\right)$$

$$\therefore \boxed{\epsilon_{12} = -0.00166\overline{6}}$$

$$0.001 = .002\left(\tfrac{1}{3}\right) + .002\left(\tfrac{1}{3}\right) + (-.002)\left(\tfrac{1}{3}\right) + 2\epsilon_{12}\left(\tfrac{1}{3}\right) + 2\epsilon_{13}\left(\tfrac{1}{3}\right) + 2\epsilon_{23}\left(\tfrac{1}{3}\right)$$

$$\therefore \boxed{\epsilon_{23} = -0.00133\overline{3}}$$

2-6.2

$$L^2 = x^2 + y^2 \; ; \; L^{*2} = (x + u_1 - u_0)^2 + (y + v_1 - v_0)^2$$

Then

$$\frac{L^* - L}{L} = \frac{\sqrt{(x + u_1 - u_0)^2 + (y + v_1 - v_0)^2} - L}{L}$$

Let $y = 0$, $x = L$. Then

$$\frac{L^* - L}{L} = \frac{\sqrt{(L + u_1 - u_0)^2 + (v_1 - v_0)^2} - L}{L}$$

$$= \sqrt{1 + \frac{2(u_1 - u_0)}{L} + \left(\frac{u_1 - u_0}{L}\right)^2 + \left(\frac{v_1 - v_0}{L}\right)^2} - 1$$

If $u_1 - u_0 << L$, $v_1 - v_0 << L$, binomial expansion yields to first degree terms in u, v

$$\frac{L^* - L}{L} \approx \frac{u_1 - u_0}{L}$$