

Chapter 1 Introduction

1.1: PROBLEM DEFINITION

Find: List three common units for each variable:

- Volume flow rate (Q), mass flow rate (\dot{m}), and pressure (p).
- Force, energy, power.
- Viscosity, surface tension.

PLAN

Use Table F.1 to find common units

SOLUTION

- Volume flow rate, mass flow rate, and pressure.
 - Volume flow rate, m^3/s , ft^3/s or cfs, cfm or ft^3/min .
 - Mass flow rate. kg/s , lbm/s , slug/s .
 - Pressure. Pa, bar, psi or lbf/in^2 .
- Force, energy, power.
 - Force, lbf, N, dyne.
 - Energy, J, $\text{ft}\cdot\text{lbf}$, Btu.
 - Power. W, Btu/s , $\text{ft}\cdot\text{lbf}/\text{s}$.
- Viscosity.
 - Viscosity, $\text{Pa}\cdot\text{s}$, $\text{kg}/(\text{m}\cdot\text{s})$, poise.

1.2: PROBLEM DEFINITION

Situation: The hydrostatic equation has three common forms:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 = \text{constant}$$

$$p_z = p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{constant}$$

$$\Delta p = -\gamma \Delta z$$

Find: For each variable in these equations, list the name, symbol, and primary dimensions of each variable.

PLAN

Look up variables in Table A.6. Organize results using a table.

SOLUTION

Name	Symbol	Primary dimensions
pressure	p	M/LT^2
specific weight	γ	M/L^2T^2
elevation	z	L
piezometric pressure	p_z	M/LT^2
change in pressure	Δp	M/LT^2
change in elevation	Δz	L

1.3: PROBLEM DEFINITIONSituation:

Five units are specified.

Find:

Primary dimensions for each given unit: kWh, poise, slug, cfm, cSt.

PLAN

1. Find each primary dimension by using Table F.1.
2. Organize results using a table.

SOLUTION

Unit	Associated Dimension	Associated Primary Dimensions
kWh	Energy	ML^2/T^2
poise	Viscosity	$M/(L \cdot T)$
slug	Mass	M
cfm	Volume Flow Rate	L^3/T
cSt	Kinematic viscosity	L/T^2

1.4: PROBLEM DEFINITIONSituation:

The hydrostatic equation is

$$\frac{p}{\gamma} + z = C$$

p is pressure, γ is specific weight, z is elevation and C is a constant.

Find:

Prove that the hydrostatic equation is dimensionally homogeneous.

PLAN

Show that each term has the same primary dimensions. Thus, show that the primary dimensions of p/γ equal the primary dimensions of z . Find primary dimensions using Table F.1.

SOLUTION

1. Primary dimensions of p/γ :

$$\left[\frac{p}{\gamma} \right] = \frac{[p]}{[\gamma]} = \left(\frac{M}{LT^2} \right) \left(\frac{L^2 T^2}{M} \right) = L$$

2. Primary dimensions of z :

$$[z] = L$$

3. Dimensional homogeneity. Since the primary dimensions of each term is length, the equation is dimensionally homogeneous. Note that the constant C in the equation will also have the same primary dimension.

1.5: PROBLEM DEFINITIONSituation:

Four terms are given in the problem statement.

Find: Primary dimensions of each term.

- a) $\rho V^2/\sigma$ (kinetic pressure).
- b) T (torque).
- c) P (power).
- d) $\rho V^2 L/\sigma$ (Weber number).

SOLUTION

a. Kinetic pressure:

$$\left[\frac{\rho V^2}{2} \right] = [\rho] [V]^2 = \left(\frac{M}{L^3} \right) \left(\frac{L}{T} \right)^2 = \frac{M}{L \cdot T^2}$$

b. Torque.

$$[\text{Torque}] = [\text{Force}] [\text{Distance}] = \left(\frac{ML}{T^2} \right) (L) = \frac{M \cdot L^2}{T^2}$$

c. Power (from Table F.1).

$$[P] = \frac{M \cdot L^2}{T^3}$$

d. Weber Number:

$$\left[\frac{\rho V^2 L}{\sigma} \right] = \frac{[\rho] [V]^2 [L]}{[\sigma]} = \frac{(M/L^3) (L/T)^2 (L)}{(M/T^2)} = \square$$

Thus, this is a dimensionless group

1.6: PROBLEM DEFINITIONSituation:

The power provided by a centrifugal pump is given by:

$$P = \dot{m}gh$$

Find:

Prove that the above equation is dimensionally homogenous.

PLAN

1. Look up primary dimensions of P and \dot{m} using Table F.1.
2. Show that the primary dimensions of P are the same as the primary dimensions of $\dot{m}gh$.

SOLUTION

1. Primary dimensions:

$$\begin{aligned}[P] &= \frac{M \cdot L^2}{T^3} \\ [\dot{m}] &= \frac{M}{T} \\ [g] &= \frac{L}{T^2} \\ [h] &= L\end{aligned}$$

2. Primary dimensions of $\dot{m}gh$:

$$[\dot{m}gh] = [\dot{m}] [g] [h] = \left(\frac{M}{T}\right) \left(\frac{L}{T^2}\right) (L) = \frac{M \cdot L^2}{T^3}$$

Since $[\dot{m}gh] = [P]$, The power equation is dimensionally homogenous.

1.7: PROBLEM DEFINITIONSituation:

Two terms are specified.

- a. $\int \rho V^2 dA$.
- b. $\frac{d}{dt} \int_V \rho V dV$.

Find:

Primary dimensions for each term.

PLAN

1. To find primary dimensions for term a, use the idea that an integral is defined using a sum.
2. To find primary dimensions for term b, use the idea that a derivative is defined using a ratio.

SOLUTION

Term a:

$$\left[\int \rho V^2 dA \right] = [\rho] [V^2] [A] = \left(\frac{M}{L^3} \right) \left(\frac{L}{T} \right)^2 (L^2) = \boxed{\frac{ML}{T^2}}$$

Term b:

$$\left[\frac{d}{dt} \int_V \rho V dV \right] = \frac{\left[\int \rho V dV \right]}{[t]} = \frac{[\rho] [V] [V]}{[t]} = \frac{\left(\frac{M}{L^3} \right) \left(\frac{L}{T} \right) (L^3)}{T} = \boxed{\frac{ML}{T^2}}$$

Problem 1.8

No solution provided.

1.9: PROBLEM DEFINITION

Apply the grid method.

Situation:

Density of ideal gas is given by:

$$\rho = \frac{p}{RT}$$

$$p = 0.25 \text{ MPa}, R = 287 \text{ J/kg} \cdot \text{K}.$$

$$T = 310 \text{ K}.$$

Find:

Calculate density (in kg/m³).

PLAN

Follow the process given in the text. Look up conversion ratios in Table F.1.

SOLUTION

(note: unit cancellations not shown).

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \left(\frac{0.25 \text{ N}}{\text{mm}^2} \right) \left(\frac{1000 \text{ mm}}{1 \text{ m}} \right)^2 \left(\frac{\text{kg} \cdot \text{K}}{287 \text{ J}} \right) \left(\frac{1}{310 \text{ K}} \right)\end{aligned}$$

$$\boxed{\rho = 2.80 \text{ kg/m}^3}$$

1.10: PROBLEM DEFINITION

Apply the grid method.

Situation:

Wind is hitting a window of building.

$$\Delta p = \frac{\rho V^2}{2}.$$

$$\rho = 1.2 \text{ kg/m}^3, \quad V = 100 \text{ km/h}.$$

Find:

- Express the answer in pascals.
- Express the answer in meter of water column (m-H₂O).

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a) _____
Pascals.

$$\begin{aligned} \Delta p &= \frac{\rho V^2}{2} \\ &= \frac{1}{2} \left(\frac{1.2 \text{ kg}}{\text{m}^3} \right) \left(\frac{100 \text{ km/h}}{1.0} \right)^2 \left(\frac{1.0 \text{ m/s}}{3.6 \text{ km/h}} \right)^2 \left(\frac{\text{Pa} \cdot \text{m} \cdot \text{s}^2}{\text{kg}} \right) \end{aligned}$$

$$\boxed{\Delta p = 463 \text{ Pa}}$$

b) _____
Meter of water column

$$\Delta p = 463 \text{ Pa} \left(\frac{1.02 \times 10^{-4} \text{ m-H}_2\text{O}}{\text{Pa}} \right)$$

$$\boxed{\Delta p = 0.047 \text{ m-H}_2\text{O}}$$

1.11: PROBLEM DEFINITION

Apply the grid method.

Situation:

Force is given by $F = ma$.

a) $m = 10 \text{ kg}$, $a = 10 \text{ m/s}^2$.

b) $m = 10 \text{ lb}$, $a = 10 \text{ ft/s}^2$.

c) $m = 10 \text{ slug}$, $a = 10 \text{ ft/s}^2$.

Find:

Calculate force.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a) _____
Force in newtons for $m = 10 \text{ kg}$ and $a = 10 \text{ m/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10 \text{ kg}) \left(10 \frac{\text{m}}{\text{s}^2} \right) \left(\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right) \end{aligned}$$

$$\boxed{F = 100 \text{ N}}$$

b) _____
Force in lbf for $m = 10 \text{ lbm}$ and $a = 10 \text{ ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10 \text{ lbm}) \left(10 \frac{\text{ft}}{\text{s}^2} \right) \left(\frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right) \end{aligned}$$

$$\boxed{F = 3.11 \text{ lbf}}$$

c) _____
Force in newtons for $m = 10 \text{ slug}$ and acceleration is $a = 10 \text{ ft/s}^2$.

$$\begin{aligned} F &= ma \\ &= (10 \text{ slug}) \left(10 \frac{\text{ft}}{\text{s}^2} \right) \left(\frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \right) \left(\frac{4.448 \text{ N}}{\text{lbf}} \right) \end{aligned}$$

$$\boxed{F = 445 \text{ N}}$$

1.12: PROBLEM DEFINITION

Apply the grid method.

Situation:

A cyclist is travelling along a road.

$$P = FV.$$

$$V = 40 \text{ km/h}, F = 3 \text{ kgf}.$$

Find:

- Find power in watts.
- Find the energy in food calories to ride for 1 hour.

PLAN

Follow the process for the grid method given in the text. Look up conversion ratios in Table F.1.

SOLUTION

a) _____

Power

$$\begin{aligned} P &= FV \\ &= (3 \text{ kgf}) \left(\frac{9.81 \text{ N}}{\text{kgf}} \right) (40 \text{ km/h}) \left(\frac{1.0 \text{ m/s}}{3.6 \text{ km/h}} \right) \left(\frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} \right) \end{aligned}$$

$$P = 327 \text{ W}$$

b) _____

Energy

$$\begin{aligned} \Delta E &= P \Delta t \\ &= \left(\frac{327 \text{ J}}{\text{s}} \right) (1 \text{ h}) \left(\frac{3600 \text{ s}}{\text{h}} \right) \end{aligned}$$

$$\Delta E = 1177200 \text{ J}$$

1.13: PROBLEM DEFINITION

Apply the grid method.

Situation:

A pump operates for one year.

$P = 15 \text{ kW}$.

The pump operates for 20 hours/day.

Electricity costs \$0.10/kWh.

Find:

The cost (U.S. dollars) of operating the pump for one year.

PLAN

1. Find energy consumed using $E = Pt$, where P is power and t is time.
2. Find cost using $C = E \times (\$0.1/\text{kWh})$.

SOLUTION

1. Energy Consumed

$$\begin{aligned} E &= Pt \\ &= (15 \text{ kW}) \times \left(\frac{20 \text{ h}}{\text{d}} \right) \left(\frac{365 \text{ d}}{\text{y}} \right) \\ &= 109500 \text{ kWh} \end{aligned}$$

$$\boxed{E = 109500 \text{ kWh}}$$

2. Cost

$$\begin{aligned} C &= E(\$0.1/\text{kWh}) \\ &= (109500 \text{ kWh}) \left(\frac{\$0.10}{\text{kWh}} \right) \end{aligned}$$

$$\boxed{C = \$10,950}$$