CHAPTER 1 (odd-numbered problems)

Instructor's Solutions Manual

to accompany

Fundamentals of Structural Dynamics

Second Edition

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1.1 Solution

(a) Determine the natural frequency if $k = 40\,\text{N/m}$ and the mass is $m = 2.0\,\text{kg}$.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40\ \text{N/m}}{2.0\ \text{kg}}} = 4.47\ \text{rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 0.71\ \text{Hz} \qquad \textbf{Ans. (a)}$$

(b) Determine the natural frequency if $k = 100\,\text{lb/in.}$ and the mass weighs $W = 50\,\text{lb}$.

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{(100\ \text{lb/in.})(386\ \text{in./sec}^2)}{50\ \text{lb}}} = 27.78\ \text{rad/sec}$$

$$f_n = \frac{\omega_n}{2\pi} = 4.42\ \text{Hz} \qquad \textbf{Ans. (b)}$$

1.3 Solution

(a) Determine the equation of motion of mass m.

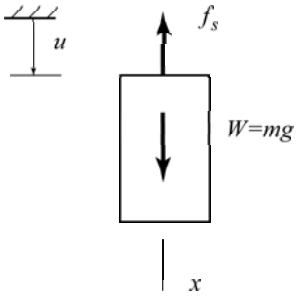

Figure 1

From Fig. 1,

$\sum F_x = m\ddot{u} = mg - f_s$

The force-elongation equation for the spring is

$f_s = ku$

Finally, the equation of motion is

$m\ddot{u} + ku = mg$ **Ans. (a)**

(b) Determine expressions for the natural frequency ω_n and the period T_n.

$$\omega_n = \sqrt{\frac{k_u}{m_u}} = \sqrt{\frac{k}{m}}\ , \quad T_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{Ans. (b)}$$

(c) If $k = 1.2\,\text{N/m}$ and $m = 0.8\,\text{kg}$, what is the resulting natural frequency f_n of this system?

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.2\ \text{N/m}}{0.8\ \text{kg}}} = 0.195\ \text{Hz} \qquad \textbf{Ans. (c)}$$

1.5 Solution

Determine an expression for the free vibration of the SDOF system in Part (c) of Prob. 1.2 if $u_0 = 2\,\text{in.}$ and $v_0 = 0$.

From Eq. 1.9,

$$u(t) = u_0 \cos\omega_n t + \frac{v_0}{\omega_n} \sin\omega_n t$$

From Part (c) of Prob. 1.2,

$$\omega_n = \sqrt{\frac{2kg}{W}} = \sqrt{\frac{2(40\ \text{lb/in.})(386\ \text{in./sec}^2)}{20\ \text{lb}}} = 39.29\ \text{rad/sec}$$

Therefore,

$$u(t) = u_0 \cos\omega_n t = 2\cos(39.29t)\ \text{in.}$$ **Ans.**