

CHAPTER 1

1.1

(a) $n \approx N_d - N_a = 10^{16} \text{ cm}^{-3}$, $q = 1.6 \times 10^{-19} \text{ coul}$, from Table 1.1,
 $\mu_n = 1194 \text{ cm}^2/\text{volt-sec}$ $\therefore \rho = (nq\mu_n)^{-1} = 0.52 \Omega\text{-cm}$

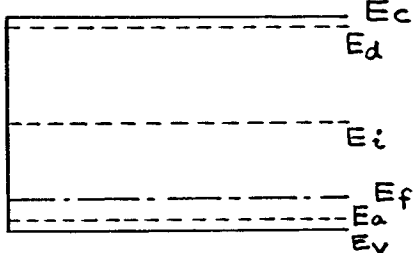
(b)

	<u>atomic weight</u>	<u>density (cm^{-3})</u>
P	31	10^{16}
Si	28	5×10^{22} (from Table 1.3)

$\therefore P/Si = \frac{31 \times 10^{16}}{28 \times 5 \times 10^{22}} = 2.21 \times 10^{-7}$

(c) P-type $p \approx N_a - N_d = 9 \times 10^{16} \text{ cm}^{-3}$, $N_a + N_d = 1.1 \times 10^{17} \text{ cm}^{-3}$, from Fig. 1.16
 $\mu_p = 310 \text{ cm}^2/\text{volt-sec}$ $\therefore \rho = (pq\mu_p)^{-1} = 0.22 \Omega\text{-cm}$

(d)



$$E_i - E_f = kT \ln \frac{N_a - N_d}{n_i}$$

$$= 0.41 \text{ eV}$$

1.2

(a) $p \approx N_a - N_d = 10^{16} \text{ cm}^{-3}$, $n = n_i^2/p = 2.1 \times 10^4 \text{ cm}^{-3}$, from Eq. (1.1.27)
 $E_i - E_f = kT \ln \frac{p}{n_i} = 0.35 \text{ eV}$, the Fermi level is 0.35 eV below E_i , or $0.56 - 0.35 = 0.21 \text{ eV}$ above E_v the edge of the valence band.

(b) $n \approx N_d - N_a = 1 \times 10^{15} \text{ cm}^{-3}$, $p = n_i^2/n = 2.1 \times 10^5 \text{ cm}^{-3}$, from Eq. (1.1.26)
 $E_f - E_i = kT \ln \frac{n}{n_i} \approx 0.29 \text{ eV}$, or $E_c - E_f = 0.56 - 0.29 = 0.27 \text{ eV}$

1.3

Arsenic: 10^{16} cm^{-3} , $E_c - E_d = 0.049 \text{ eV}$

Boron: 10^{15} cm^{-3} , $E_a - E_v = 0.045 \text{ eV}$

At 300°K $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$, $N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$

(a) Eq. (1.1.21) can be written: $n = N_c' \left(\frac{T}{300}\right)^{3/2} \exp[-(E_c - E_f)/kT]$
 where N_c' is the effective density of states at 300°K .

When $n = \frac{N_d}{2}$, $E_f \approx E_d$. With these substitutions, the equation

for n can be written: $\ln \frac{2N_c'}{N_d} + \frac{3}{2} \ln \frac{T}{300} = \frac{E_c - E_d}{KT}$

Rearranging, we get: $T = \frac{E_c - E_d}{K} \frac{1}{\ln \frac{2N_c'}{N_d} + \frac{3}{2} \ln \frac{T}{300}}$

Since the right-hand side is only a weak function of temperature, the solution may be obtained iteratively, starting with an arbitrary guess of 100°K for T .

$$\begin{aligned} \therefore T &= \frac{0.049}{8.62 \times 10^{-5}} \frac{1}{\ln \frac{2 \times 2.8 \times 10^{19}}{1 \times 10^{16}} + \frac{3}{2} \ln \frac{T}{300}} \\ &= \frac{568}{8.63 + \frac{3}{2} \ln \frac{T}{300}} \end{aligned}$$

by $T_1 = 100$, $T_2 = 81.4$, $T_3 = 85.1$, $T_4 = 84.3$, $T_5 = 84.4$

$\therefore T = 84.4^\circ\text{K}$

Similarly for boron: $T = \frac{E_a - E_v}{K} \frac{1}{\ln \frac{2N_v}{N_a} + \frac{3}{2} \ln \frac{T}{300}}$

$$\begin{aligned} \therefore T &= \frac{0.045}{8.62 \times 10^{-5}} \frac{1}{\ln \frac{2 \times 1.04 \times 10^{19}}{10^{15}} + \frac{3}{2} \ln \frac{T}{300}} \\ &= \frac{522}{9.94 + \frac{3}{2} \ln \frac{T}{300}} \end{aligned}$$

by $T_1 = 100$, $T_2 = 63.0$, $T_3 = 68.7$, $T_4 = 67.5$, $T_5 = 67.8$, $T_6 = 67.7$

$\therefore T = 67.7^\circ\text{K}$

(b) From Table 1.4, $n_i = 3.87 \times 10^{16} T^{3/2} \exp(-\frac{7014}{T})$

$$T = \frac{7014}{\ln \frac{3.87 \times 10^{16}}{n_i} + \frac{3}{2} \ln T}$$

Solve iteratively as in part (a), for $n_i = 10N_d = 10^{17} \text{ cm}^{-3}$

$$T = \frac{7014}{\frac{3}{2} \ln T - 0.95}$$

by $T_1 = 100$, $T_2 = 1177$, $T_3 = 726$, $T_4 = 785$, $T_5 = 775$, $T_6 = 777$

$\therefore T = 777^\circ\text{K} = 504^\circ\text{C}$ for $n_i = 10N_d$

For $n_i = 10N_a = 10^{16}$

$$T = \frac{7014}{\frac{3}{2} \ln T + 1.35} \quad \text{by } T_1 = 800, T_2 = 617, T_3 = 638, T_4 = 635$$

$\therefore T = 635^\circ\text{K} = 362^\circ\text{C}$

(c) Arsenic: $n \approx N_d - N_a = 10^{16} \text{ cm}^{-3} \gg n_i$, $p = \frac{n_i^2}{n} = 2.1 \times 10^4 \text{ cm}^{-3}$

Boron: $p \approx N_a - N_d = 10^{15} \text{ cm}^{-3} \gg n_i$, $n = \frac{n_i^2}{p} = 2.1 \times 10^5 \text{ cm}^{-3}$

(d) Arsenic: $E_f - E_i = KT \ln \frac{n}{n_i} = 0.35 \text{ eV}$, $E_i - E_v = 0.56 \text{ eV}$

$\therefore E_f - E_v = 0.91 \text{ eV}$

Boron : $E_i - E_f = KT \ln \frac{P}{n_i} = 0.29 \text{ eV}$, $E_i - E_v = 0.56 \text{ eV}$, $E_f - E_v = 0.27 \text{ eV}$
 Arsenic + Boron : $n \approx N_d - N_a = 9 \times 10^{15} \text{ cm}^{-3}$, $E_f - E_i = KT \ln \frac{n}{n_i} = 0.347 \text{ eV}$
 $\therefore E_f - E_v = 0.907 \text{ eV}$

1.4

From Fig. 1.15, $5 \Omega\text{-cm}$ $n \Rightarrow N_d = 9 \times 10^{14} \text{ cm}^{-3}$. $pn = n_i^2$, at 27°C (300°K) $n_i = 1.45 \times 10^{10} \text{ cm}^{-3}$
 and $p = \frac{(1.45 \times 10^{10})^2}{9 \times 10^{14}} \approx 2.3 \times 10^5 \text{ cm}^{-3}$

For 100°C , from Table 1.4,

$$n_i = 3.87 \times 10^{16} (373)^{3/2} \exp(-7014/373) = 1.9 \times 10^{12} \text{ cm}^{-3}$$

Since $N_d \gg n_i$, $n = N_d \approx 9 \times 10^{14} \text{ cm}^{-3}$ and $p = n_i^2/n = 4.0 \times 10^9 \text{ cm}^{-3}$

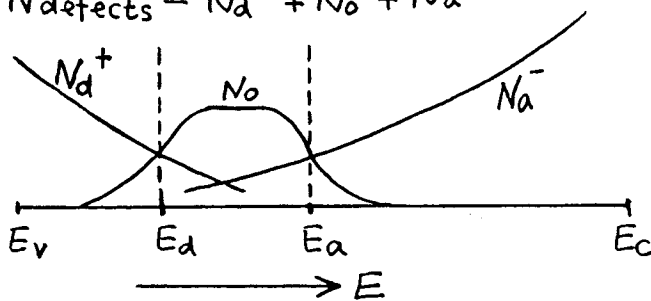
For 500°C , from Table 1.4,

$$n_i = 3.87 \times 10^{16} (773)^{3/2} \exp(-7014/773) = 9.5 \times 10^{16} \text{ cm}^{-3}$$

Since $n_i \gg N_d$, $p \approx n_i = 9.5 \times 10^{16} \text{ cm}^{-3}$

1.5

(a) $N_{\text{defects}} = N_d^+ + N_o + N_a^-$



N_d^+ dominates in heavily doped p-type material.

N_a^- dominates in n-type material.

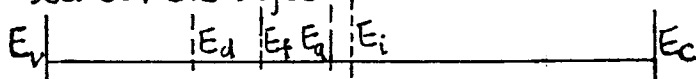
(b) $p - n + N_d^{\text{deep}+} + N_d^{\text{shallow}+} - N_a^{\text{deep}-} - N_a^{\text{shallow}-} = 0$ for charge neutrality

In n-type material $n + N_a^{\text{deep}-} \approx N_d^{\text{shallow}+}$, so increasing $N_a^{\text{deep}-}$ decreases n . Similarly, $p + N_d^{\text{deep}+} \approx N_a^{\text{shallow}-}$, increasing $N_d^{\text{deep}+}$ in heavily doped p-type material decreases p .

(c) E_f should be between E_a and E_d for charge neutrality, ($p - n = N_a^- - N_d^+$), Since E_f is below the middle of bandgap, the sample is p-type.

However p and $n \ll N_{\text{defect}}$ so $N_a^- \approx N_d^+$ and $N_{\text{defect}} e^{\frac{E_f - E_d}{KT}} \approx N_{\text{defect}} e^{\frac{E_a - E_f}{KT}}$
 so that $E_f \approx \frac{E_d + E_a}{2}$, $E_f - E_v = \frac{E_d - E_v}{2} + \frac{E_a - E_v}{2} = \frac{0.27 + 0.51}{2} = 0.39 \text{ eV}$

The sample is p-type and the Fermi level is approximately midway between the defect levels.



(d) Since $N_{\text{shallow}} > N_{\text{defects}}$, the Fermi level is above E_i for this n-type sample and $N_{\bar{a}}$ dominates the charge state of the defects. ($N_{\bar{a}} \approx N_{\text{defects}}$)
 $\therefore n - p \approx N_{\text{d phos}} - N_{\bar{a}} = 2 \times 10^{17} - 5 \times 10^{16} = 1.5 \times 10^{17} \text{ cm}^{-3}$
 Since $n - p \gg n_i \therefore n \approx 1.5 \times 10^{17} \text{ cm}^{-3}$,
 $p = \frac{n_i^2}{n} = \frac{(1.45 \times 10^{10})^2}{1.5 \times 10^{17}} \approx 1.40 \times 10^3 \text{ cm}^{-3}$
 $E_c - E_f = kT \ln \frac{N_c}{n} = 0.026 \text{ eV} \ln \frac{2.8 \times 10^{19}}{1.5 \times 10^{17}} \approx 0.136 \text{ eV}$

1.6

$$\mu_1 = 800 \text{ cm}^2/\text{V-sec}, \mu_2 = 200 \text{ cm}^2/\text{V-sec}$$

$$\frac{1}{\mu_T} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{800} + \frac{1}{200} \therefore \mu_T = 160 \text{ cm}^2/\text{V-sec}$$

1.7

The number of atoms per unit weight is N_0/A , where N_0 is Avogadro's number and A is the atomic weight. If ρ_m is the mass density, the number of atoms per unit volume is

$N = \rho_m N_0/A$. If each atom contributes Z valence electrons,

the electron concentration is $n = Z \rho_m N_0/A$

with $Z = 0.9$, $\rho_m = 2.7 \text{ g/cm}^3$, $N_0 = 6.022 \times 10^{23} (\text{g-mole})^{-1}$, $A = 27$
 then $n = 5.42 \times 10^{22} \text{ electrons/cm}^3$

$$\mu_n = 1/(p n \xi) \text{ with } p = 2.8 \times 10^{-6} \Omega\text{-cm}, \mu_n = 41.2 \text{ cm}^2/\text{V-sec}$$

$$\tau = \frac{m^* \mu_n}{q} = \frac{(9.11 \times 10^{-31} \text{ kg})(41.2 \times 10^{-4} \text{ m}^2/\text{V-sec})}{1.6 \times 10^{-19} \text{ coul}} = 2.35 \times 10^{-14} \text{ sec}$$

For silicon $m^* = 0.26 m_0$ (conductivity effective mass)

$$\tau = \frac{m^* \mu_n}{q} = \frac{0.26 \times (9.11 \times 10^{-31} \text{ kg})(1420 \times 10^{-4} \text{ m}^2/\text{V-sec})}{1.6 \times 10^{-19} \text{ coul}} = 2.00 \times 10^{-13} \text{ sec}$$

$$\therefore \tau_{\text{Si}} = 8.51 \tau_{\text{Al}}$$

1.8

$$V_{\text{th}} \approx 2.3 \times 10^7 \text{ cm/sec} \therefore V_d \approx 2.3 \times 10^6 \text{ cm/sec}$$

The electron travels a distance L in a time $T = L/V_d$

If τ is the average time between collisions, the average number of collisions in traveling the distance L is $C = \frac{T}{\tau} = \frac{L}{\tau V_d}$

With $L = 1 \mu\text{m}$ and $\tau = \frac{m^* \mu_n}{q} = 2.1 \times 10^{-13} \text{ sec}$, then $C \approx 207$ collisions

The applied voltage is $V_a = \mathcal{E}L$, with $\mu_n = 1417 \text{ cm}^2/\text{V-sec}$, then
 $\mathcal{E} = \frac{V_a}{\mu_n} = 1623 \text{ Volts/cm}$ and $V_a = 162 \text{ mV}$.

1.9

The sample is biased in the intermediate region of the velocity - field curve where the velocity is not a linear function of the field but is approaching its saturation velocity. Thus, doubling the field results in less than a doubling of velocity and, hence, of current.

1.10

$$\mu_n = 1000 \text{ cm}^2/\text{V-sec}, D_n = \left(\frac{KT}{q}\right)\mu_n = 25.8 \text{ cm}^2 \text{ s}^{-1}$$

$$\frac{dn}{dx} = - \frac{10^{17} \text{ cm}^{-3} - 6 \times 10^{16} \text{ cm}^{-3}}{2 \times 10^{-4} \text{ cm}} = -2 \times 10^{20} \text{ cm}^{-4}$$

The electron diffusion current density is

$$J_n = q D_n \frac{dn}{dx} = -825.6 \text{ amps/cm}^2$$

1.11

(a) Si replacing Ga \Rightarrow extra electron \Rightarrow donor $N_d = 0.05 \times 10^{10} = 5 \times 10^8/\text{cm}^3$
 Si replacing As \Rightarrow one less electron \Rightarrow acceptor $N_a = 0.95 \times 10^{10} = 9.5 \times 10^9/\text{cm}^3$

(b) $p \approx N_a - N_d = (9.5 - 0.5) \times 10^9 = 9.0 \times 10^9/\text{cm}^3 \gg n_i = 9.0 \times 10^6$ for GaAs @ 300 °K.

$$n = \frac{n_i^2}{p} = \frac{81 \times 10^{12}}{9 \times 10^9} = 9 \times 10^3/\text{cm}^3$$

$$\therefore E_f - E_v = KT \ln \frac{N_v}{p} = KT \ln \frac{7 \times 10^{18}}{9 \times 10^9} = 0.026 \times 20.5 = 0.53 \text{ eV}$$

$$\therefore E_f = 0.53 \text{ eV above } E_v.$$

$$(c) \sigma = nq\mu_n + pq\mu_p = (9 \times 10^3)(1.6 \times 10^{-19})(8800) + (9 \times 10^9)(1.6 \times 10^{-19})(400) \\ \approx pq\mu_p = 5.8 \times 10^{-7} / \Omega\text{-cm}$$

1.12

$$(a) -\frac{dp}{dt} = \nabla \cdot J = \nabla \cdot \sigma \mathcal{E} = \frac{\sigma}{\mathcal{E}} \nabla \cdot D = \frac{\sigma}{\mathcal{E}} p \quad \therefore \frac{dp}{p} = -\frac{\sigma}{\mathcal{E}} dt$$

$$\ln p - \ln p_0 = -\frac{\sigma}{\mathcal{E}} t \quad \therefore p = p_0 e^{-\frac{\sigma}{\mathcal{E}} t} = p_0 e^{-\frac{t}{\tau_{rel}}}$$

(b) Intrinsic Si: $\sigma = qn_i(\mu_n + \mu_p)$, $\mu_n = 1417 \frac{\text{cm}^2}{\text{V-sec}}$, $\mu_p = 471 \frac{\text{cm}^2}{\text{V-sec}}$, $\sigma = 4.38 \times 10^{-6} (\Omega\text{-cm})^{-1}$

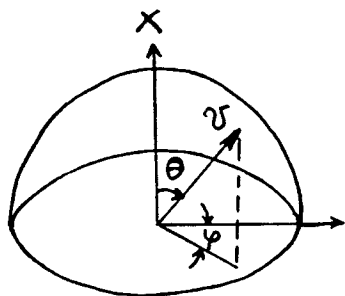
$$\therefore \tau_{rel} = \mathcal{E}/\sigma = 2.4 \times 10^{-7} \text{ sec} = 0.24 \mu\text{sec}$$

$$N_d = 10^{16} \text{ cm}^{-3}, \mu_n \approx 1190 \text{ cm}^2/\text{V-sec}, \sigma = 1.90 (\Omega\text{-cm})^{-1}$$

$$\therefore \tau_{rel} = \frac{\epsilon}{\sigma} = 5.4 \times 10^{-13} \text{ sec} = 0.54 \text{ psec}$$

$$\text{SiO}_2 \quad \tau_{rel} = \frac{3.9 \times 8.85 \times 10^{-14}}{10^{-16}} = 3450 \text{ sec} = 57.5 \text{ min}$$

1.13



Let each direction and velocity increment contain dN_v particles

$$d^3 N_{\phi\psi v} = \frac{1}{4\pi} dN_v \sin\theta d\theta d\psi$$

The average v in one direction is

$$\langle v \rangle = \frac{1}{n_0} \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{v}{4\pi} dN_v \sin\theta d\theta d\psi$$

$$= \frac{1}{n_0} \int_0^\infty v dN_v \quad \text{where } n_0 = \int d^3 N_{\phi\psi v}$$

The x-direction component of $\langle v \rangle$ is

$$\langle v_x \rangle = \langle v \cos\theta \rangle = \frac{1}{n_0} \int_0^{2\pi} \int_0^\pi \int_0^\infty \frac{v}{4\pi} dN_v \cos\theta \sin\theta d\theta d\psi$$

$$= \frac{1}{4n_0} \int_0^\infty v dN_v = \frac{1}{4} \langle v \rangle$$

$$\text{and } J_x = -\frac{q}{4} n_0 \langle v_x \rangle = -\frac{q n_0 \langle v \rangle}{4}$$

1.14

$$\lambda = \frac{hc}{E_g}$$

$$\lambda(\mu\text{m}) = \frac{1.24}{E_g(\text{eV})}$$

Material	$E_g(\text{eV})$	$\lambda(\mu\text{m})$	Range
Ge	0.67	1.85	Infrared
Si	1.124	1.10	Infrared
GaAs	1.42	0.87	(Near) infrared
SiO ₂	~ 9	~ 0.14	(Vacuum) ultraviolet

1.15

$$\frac{D}{\mu} = \frac{1}{q} \frac{dE_f}{d(\ln n)} \quad \text{For a nondegenerate semiconductor,}$$

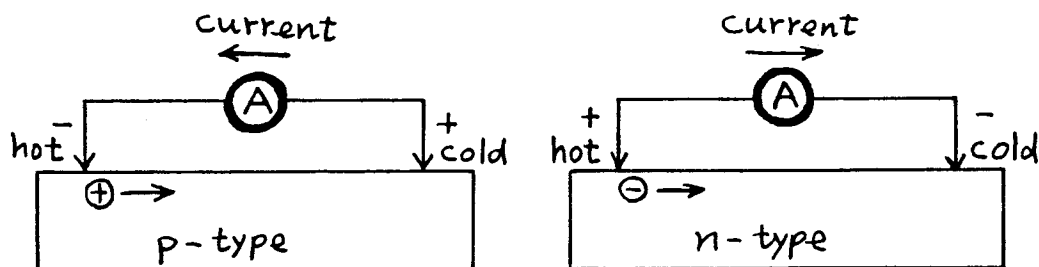
$$n \simeq N_c \exp[-(E_c - E_f)/kT] \quad \therefore E_f - E_c = kT \ln \frac{n}{N_c} = kT [\ln n - \ln N_c]$$

$$\therefore \frac{dE_f}{d(\ln n)} = kT, \text{ if } E_c \text{ and } N_c \text{ are independent of doping.}$$

$$\text{Hence: } \frac{D}{\mu} = \frac{1}{q} \frac{dE_f}{d(\ln n)} = \frac{kT}{q}$$

1.16

In a volume of an extrinsic semiconductor which is hotter than the surrounding material, the carriers have a higher thermal velocity than those in the unheated material. There is thus a net flux of carriers out of the heated volume by diffusion. The magnitude of the flux increases as the temperature difference (and thus the velocity difference) increases. For n-type material there is a net flux of electrons out of the heated region and thus a net current into the region. For p-type material there is a net flux of holes out of the heated region and thus a net current out of the region. In other words, the majority carriers at the hot end tend to move to the cold end.

1.17

$t \backslash x$	0	1	2	3	4	5	6	7	8	9	10
0	1024										
1		512									
2		512		256							
3			384		128						
4			384		256		64				
5				320		160		32			
6				320		240		96		16	
7					280		168		56		8
8						280		112		32	
9							252		168		72
10								252		210	

The distribution is symmetrical about $x = 0$

t	w	w^2	w^2/t
0	0	0	—
2	4	16	8
4	5.33	28.44	7.11
6	6.22	38.71	6.45
8	7	49	6.125
10	7.73	59	6.9

t is the time unit
 w is the half width

Example: For $t = 8$ half maximum is 140 so that $2 \leq \frac{W}{2} \leq 4$.

The straight line drawn between the known values at $x=2$ and $x=4$ is given by $y = -56x + 336$.

Letting $y=140$ we find $x=3.5 \approx \frac{W}{2}$ so that $W \approx 7$.

The other values of W shown can be found in a similar manner.

Examination of the values of W shows that the rate of diffusion slows as the distribution spreads and the gradients are reduced. The values of W^2/t indicate that for $t \gg 1$, $W^2 \propto t$, so that $W \propto (t)^{1/2}$ and $\frac{dW}{dt} \propto (t)^{-1/2}$.

1.18

(a) From Table 1.4,

$$E_g(\text{eV}) = 1.16 - \frac{7.02 \times 10^{-4} T^{3/2}}{T + 1108}$$

At $T=300\text{ K}$,

$$E_g = 1.16 - \frac{(7.02 \times 10^{-4})(300)^{3/2}}{300 + 1108} = 1.157 \text{ eV}$$

On the other hand, Table 1.3 lists 1.124 eV for E_g .

(b) From Table 1.3, at $T = 300\text{ K}$,

$$N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$$

$$N_v = 1.04 \times 10^{19} \text{ cm}^{-3}$$

From Eq. (1.1.25),

$$n_i^2 = N_c N_v \exp\left[-\frac{(E_c - E_v)}{kT}\right]$$

or

$$n_i = (N_c N_v)^{1/2} \exp\left[-\frac{E_g}{2kT}\right]$$

At 300 K, using $E_g = 1.157 \text{ eV}$ from Table 1.4,

$$\begin{aligned} n_i &= [(2.8 \times 10^{19})(1.04 \times 10^{19})]^{1/2} \exp\left[-\frac{1.157}{2(0.026)}\right] \\ &= 3.707 \times 10^9 \text{ cm}^{-3} \end{aligned}$$

At 300 K, using $E_g = 1.124 \text{ eV}$ from Table 1.3,

$$\begin{aligned} n_i &= [(2.8 \times 10^{19})(1.04 \times 10^{19})]^{1/2} \exp\left[-\frac{1.124}{2(0.026)}\right] \\ &= 6.993 \times 10^9 \text{ cm}^{-3} \end{aligned}$$

(c) From Table 1.4,

$$n_i (\text{cm}^{-3}) = 3.87 \times 10^{16} T^{3/2} \exp\left[-\frac{7014}{T}\right]$$

At $T = 300\text{K}$,

$$\begin{aligned} n_i &= (3.87 \times 10^{16})(300)^{3/2} \exp\left[-\frac{7014}{300}\right] \\ &= 1.411 \times 10^{10} \text{ cm}^{-3} \end{aligned}$$

1.19

We replace the force term \vec{F} in Eq. (1.3.1) by

$$\vec{F} = m^* \frac{d\vec{V}}{dt} + m^* \frac{\vec{V}}{\tau}$$

where the second term represents scattering. Also adding electric force term, we get

$$m \left(\frac{d}{dt} + \frac{1}{\tau} \right) \vec{V} = q (\vec{E} + \vec{V} \times \vec{B}).$$

With the standard Hall geometry of Fig. 1.20

under DC condition ($d/dt = 0$), we solve for velocity components.

For electrons,

$$V_{nx} = \mu_n \frac{-E_x + \omega_{cn} \tau_n E_y}{1 + (\omega_{cn} \tau_n)^2}$$

$$V_{ny} = \mu_n \frac{-\omega_{cn} \tau_n E_x - E_y}{1 + (\omega_{cn} \tau_n)^2}$$

$$V_{nz} = 0, \text{ where } \mu_n = \frac{q \tau_n}{m_n^*}, \omega_{cn} = \frac{q B_z}{m_n^*}$$

For holes,

$$V_{px} = \mu_p \frac{E_x + \omega_{cp} \tau_p E_y}{1 + (\omega_{cp} \tau_p)^2}$$

$$V_{py} = \mu_p \frac{-\omega_{cp} \tau_p E_x + E_y}{1 + (\omega_{cp} \tau_p)^2}$$

$$V_{pz} = 0, \text{ where } \mu_p = \frac{q \tau_p}{m_p^*}, \omega_{cp} = \frac{q B_z}{m_p^*}$$

We require the transverse current to be zero.

$$J_y = J_{ny} + J_{py} = -q n V_{ny} + q p V_{py}$$

$$\cong q (\mu_n^2 n - \mu_p^2 p) E_x B_z + q (\mu_n n + \mu_p p) E_y$$

with the approximation $\omega_{cn} \ll \frac{1}{\tau_n}$ and $\omega_{cp} \ll \frac{1}{\tau_p}$.

Now $J_y = 0$ implies

$$E_x B_z = \frac{\mu_p p + \mu_n n}{\mu_p^2 p - \mu_n^2 n} E_y \quad \text{----- (1)}$$

The longitudinal current is

$$J_x = J_{nx} + J_{px} = -q n v_{nx} + q p v_{px}$$

$$\cong q(\mu_p p + \mu_n n) E_x + q(\mu_p^2 p - \mu_n^2 n) E_y B_z$$

$$J_x B_z \cong q(\mu_p p + \mu_n n) E_x B_z$$

$$= q \frac{(\mu_p p + \mu_n n)^2}{\mu_p^2 p - \mu_n^2 n} E_y \quad (\text{replace } E_x B_z \text{ from (1)}) \text{----- (2)}$$

We neglected the term involving B_z^2 in (2).

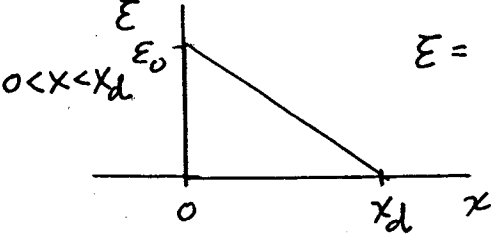
$$R_H = \frac{E_y}{J_x B_z} = \frac{\mu_p p^2 - \mu_n^2 n}{q(\mu_p p + \mu_n n)^2}$$

To see that this result is consistent with the simpler theory of sec. 1.3, we can let $p=0$ for a sample containing electrons only and find $R_H = -\frac{1}{q n}$. For a sample containing holes only, $n=0$ gives $R_H = \frac{1}{q p}$.

APPENDIX PROBLEMS

A1.1 FOR $0 < x < x_d$ $\rho = -\rho_1$ $\frac{dE}{dx} = -\frac{\rho_1}{\epsilon_s}$

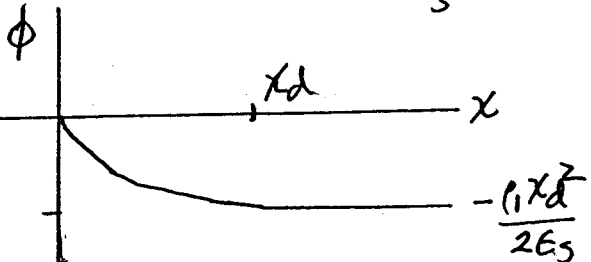
(a) $\therefore E = \epsilon_0 - \frac{\rho_1 x}{\epsilon_s}$ $0 < x < x_d$ $E = 0, x < 0$
 $0, x > x_d$
 where $\epsilon_0 = \frac{\rho_1 x_d}{\epsilon_s}$



(b) $\phi = -\int E dx$
 $\phi = -\epsilon_0 x + \frac{\rho_1 x^2}{2\epsilon_s}$ $0 < x < x_d$ or $\phi = -\frac{\rho_1}{\epsilon_s} \left(x_d x - \frac{x^2}{2} \right)$

Take $\phi(x < 0) = 0$

$\phi(x_d) = -\frac{\rho_1 x_d^2}{2\epsilon_s}$



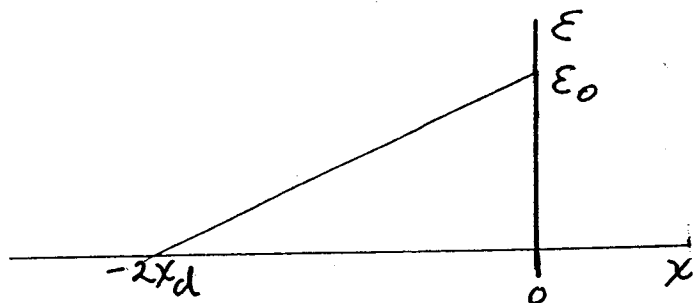
(c) $\phi(0) - \phi_{x_d} = \frac{\rho_1 x_d^2}{2\epsilon_s}$

(d) $E = 0$ $\begin{cases} x < -2x_d \\ x > 0 \end{cases}$

$E(x) = \frac{\rho_1}{\epsilon_s} \left(x_d + \frac{x}{2} \right)$

$-2x_d < x < 0$

$E(0) = \frac{\rho_1 x_d}{\epsilon_s}$

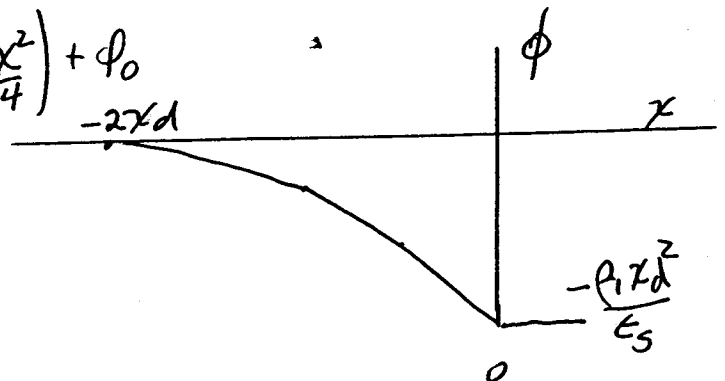


$\phi = -\int E dx = -\frac{\rho_1}{\epsilon_s} \left(x_d x + \frac{x^2}{4} \right) + \phi_0$

If $\phi_0 = 0 (x \leq -2x_d)$

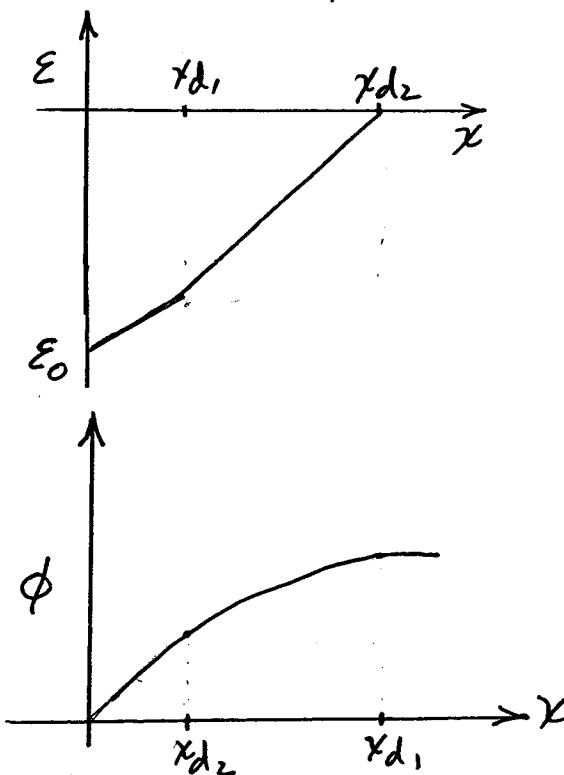
$\phi = -\frac{\rho_1}{\epsilon_s} \left(x_d + \frac{x}{2} \right)^2$

$\therefore \phi(-2x_d) - \phi(x=0) = \frac{\rho_1 x_d^2}{\epsilon_s}$



A1.2(a) For $x < 0$, $\mathcal{E} = 0$.

For $0 < x < x_{d1}$, $\frac{d\mathcal{E}}{dx} = \frac{\rho_1}{\epsilon_s}$ so $\mathcal{E} = \mathcal{E}_0 + \frac{\rho_1 x}{\epsilon_s}$



By Gauss' Law $\mathcal{E}_0 = -\frac{1}{\epsilon_s} [\rho_1 x_{d1} + 2\rho_1(x_{d2} - x_{d1})]$

$$\mathcal{E}_0 = -\frac{\rho_1}{\epsilon_s} [2x_{d2} - x_{d1}]$$

and $\mathcal{E}(x_{d1}) = \mathcal{E}_0 x_{d1} + \frac{\rho_1 x_{d1}^2}{\epsilon_s}$

$$\begin{aligned} \therefore \mathcal{E}(x_{d1}) &= -\frac{\rho_1}{\epsilon_s} [2x_{d2}x_{d1} - x_{d1}^2] \\ &= -\frac{2\rho_1}{\epsilon_s} [x_{d2}x_{d1} - \frac{x_{d1}^2}{2}] \end{aligned}$$

For $x_{d1} < x < x_{d2}$

$$\mathcal{E} = \mathcal{E}(x_{d1}) + \frac{2\rho_1}{\epsilon_s} (x - x_{d1})$$

$$= -\frac{2\rho_1}{\epsilon_s} [x_{d2}x_{d1} - \frac{x_{d1}^2}{2} - (x - x_{d1})x_{d1}]$$

$$= -\frac{2\rho_1}{\epsilon_s} [x_{d2}x_{d1} - x_{d1}^2 - x_{d1}x + \frac{x_{d1}^2}{2}]$$

(b) $\phi = 0$ $x < 0$

$\phi = -\int \mathcal{E} dx$ For $0 < x < x_{d1}$

$$\phi = \mathcal{E}_0 x - \frac{\rho_1 x^2}{2\epsilon_s}$$

(c) $\phi(0) - \phi(x_{d1})$

$$= -\frac{\rho_1}{\epsilon_s} (2x_{d2}x_{d1} - \frac{3}{2}x_{d1}^2) = \frac{\rho_1}{\epsilon_s} [2x_{d2}x_{d1} - x_{d1}^2 - \frac{x_{d1}^2}{2}]$$

$$\phi(x=x_{d1}) = \frac{\rho_1}{\epsilon_s} [2x_{d2}x_{d1} - \frac{3}{2}x_{d1}^2]$$

 $\phi(0) - \phi(x_{d2})$

$$= -\frac{\rho_1}{\epsilon_s} (x_{d2}^2 - \frac{x_{d1}^2}{2})$$

For $x_{d1} < x < x_{d2}$

$$\phi = \phi(x_{d1}) - \int_{x_{d1}}^x \mathcal{E} dx$$

$$= \frac{\rho_1}{\epsilon_s} (2x_{d2}x_{d1} - \frac{x_{d1}^2}{2} - x^2)$$

$$\phi(x_{d2}) = \frac{\rho_1}{\epsilon_s} (x_{d2}^2 - \frac{x_{d1}^2}{2})$$

A1.3

(a) $\mathcal{E} = 0 \quad x < -x_{d1}$
 $\frac{d\mathcal{E}}{dx} = -\frac{\rho_1}{\epsilon_s} \quad -x_{d1} < x < 0$

$$\therefore \mathcal{E} = -\frac{\rho_1}{\epsilon_s} (x + x_{d1})$$

$$-x_{d1} < x < 0$$

at $x = 0$; $\mathcal{E} = -\mathcal{E}_0 = -\frac{\rho_1 x_{d1}}{\epsilon_s}$

also; $\mathcal{E}_0 = -\frac{\rho_2 x_{d2}}{\epsilon_s}$

$$\mathcal{E} = 0 \text{ for } x_{d2} < x < \infty$$

(b) Take $\phi(x < -x_{d1}) = 0$

Then for $x_{d1} < x < 0$

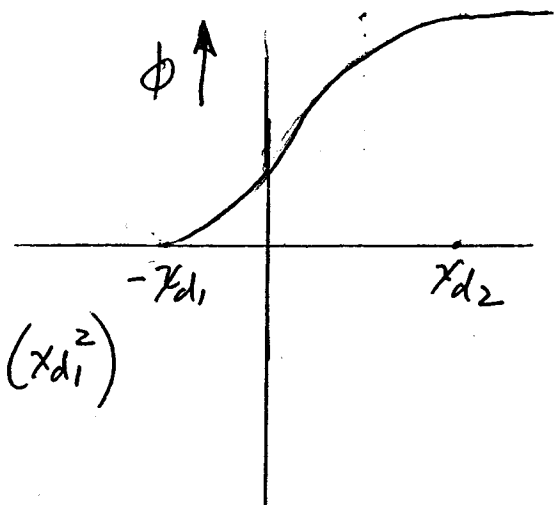
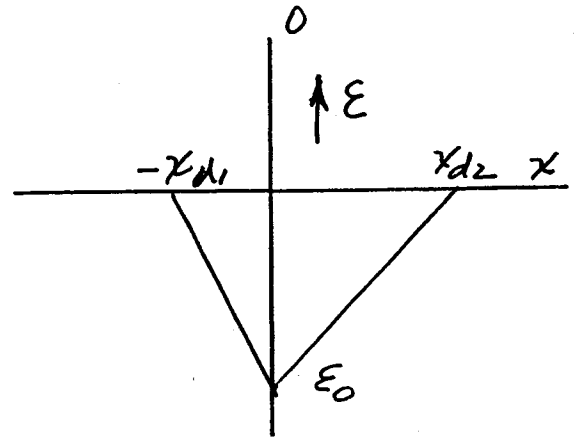
$$\phi = \frac{\rho_1}{\epsilon_s} \left(\frac{x^2}{2} + x_{d1}x \right) + \frac{\rho_1}{2\epsilon_s} (x_{d1}^2)$$

For $0 < x < x_{d2}$

$$\phi = \frac{\rho_1 x_{d1}^2}{2\epsilon_s} + \frac{\rho_2 x^2}{2\epsilon_s}$$

(c) $\phi(x_{d2}) - \phi(-x_{d1}) = \frac{1}{2\epsilon_s} (\rho_1 x_{d1}^2 + \rho_2 x_{d2}^2)$

- (d) Problem A1.1 can be constructed as a special case of the negative of the summed charges in A1.1. (a) and (d). The graphs of \mathcal{E} and ϕ superpose to the negatives of the graphs in this problem and the expressions for ϕ and \mathcal{E} compare similarly.



A1.4

For $x < 0$, $\epsilon = 0$

(a) Charge at $x=0 = -\rho x_d$

In Oxide; $\epsilon_{ox} = -\frac{\rho x_d}{\epsilon_{ox}}$ constant

At $x=x_0$, ϵ changes such that $\epsilon_{ox} \epsilon_{ox} = \epsilon_s \epsilon_{os}$

For $x_0 < x < x_d$ $\epsilon = \epsilon_{os} + \frac{\rho_1 x}{\epsilon_s}$

(b) $\phi = 0$ $x < 0$

$$\phi = \epsilon_{ox} x \quad 0 < x < x_{ox}$$

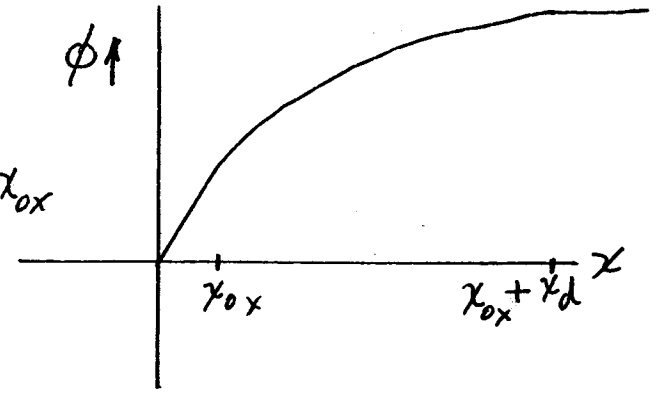
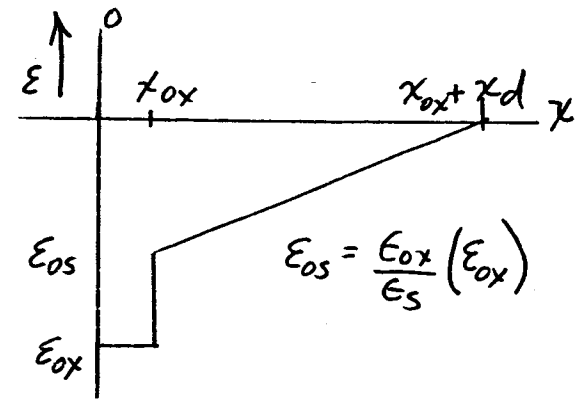
$$\phi = \epsilon_{ox} x_{ox} + \frac{\rho (x - x_{ox})^2}{2\epsilon_s} \quad x_{ox} < x < x_d$$

(c) $\Delta\phi_o = \epsilon_{ox} x_{ox}$

$$= -\frac{\rho x_d x_{ox}}{\epsilon_{ox}}$$

$$(d) \Delta\phi_d = \epsilon_{ox} x_{ox} - \frac{\rho x_d^2}{2\epsilon_s} - \epsilon_{ox} x_{ox}$$

$$= -\frac{\rho x_d^2}{2\epsilon_s}$$



(e) A convenient decomposition is into 2 charge sheets of $-\rho x_d$ and $+\rho x_d$ on either side of x_{ox} plus a sheet of $-\rho x_d$ coupled to distributed charge as shown below:

