Chapter 1: Linear Equations and Functions

Exercise 1.1

1.
$$\begin{aligned} 4x-7&=8x+2\\ 4x-7+7-8x&=8x+2+7-8x\\ -4x&=9\\ x&=-\frac{9}{4}\end{aligned}$$

2.
$$\begin{aligned} 3x+22&=7x+2\\ 22&=4x+2\\ 20&=4x\\ 5&=x\end{aligned}$$

3.
$$\begin{aligned} x+8&=8(x+1)\\ x+8&=8x+8\\ x-8x&=8-8\\ -7x&=0\\ x&=0\end{aligned}$$

4.
$$\begin{aligned} x+x+x&=x\\ 3x&=x\\ 2x&=0\\ x&=0\end{aligned}$$

5.
$$\begin{aligned} -\frac{3x}{4}&=24\\ -3x&=4(24)=96\\ x&=-32\end{aligned}$$

6.
$$\begin{aligned} \frac{-1}{6}x&=12\\ -6\left(\frac{-1}{6}x\right)&=-6(12)\\ x&=-72\end{aligned}$$

7.
$$\begin{aligned} 2(x-7)&=5(x+3)-x\\ 2x-14&=5x+15-x\\ 2x-5x+x&=15+14\\ -2x&=29\\ x&=-\frac{29}{2}\end{aligned}$$

8.
$$\begin{aligned} 3(x-4)&=4-2(x+2)\\ 3x-12&=4-2x-4\\ 3x-12&=-2x\\ 5x-12&=0\\ 5x&=12\\ x&=\frac{12}{5}\end{aligned}$$

9.
$$\begin{aligned} \frac{5x}{2}-4&=\frac{2x-7}{6}\\ 6\left(\frac{5x}{2}-4\right)&=6\left(\frac{2x-7}{6}\right)\\ 15x-24&=2x-7\\ 15x-2x&=24-7\\ 13x&=17\\ x&=\frac{17}{13}\end{aligned}$$

10.
$$\begin{aligned} \frac{2x}{3}-1&=\frac{x-2}{2}\\ 6\left(\frac{2x}{3}-1\right)&=6\left(\frac{x-2}{2}\right)\\ 4x-6&=3x-6\\ x&=0\end{aligned}$$

11.
$$\begin{aligned} x+\frac{1}{3}&=2\left(x-\frac{2}{3}\right)-6x\\ x+\frac{1}{3}&=2x-\frac{4}{3}-6x\\ 3x+1&=6x-4-18x\\ 3x+18x-6x&=-4-1\\ 15x&=-5\\ x&=\frac{-5}{15}=-\frac{1}{3}\end{aligned}$$

12. $\frac{3x}{4}-\frac{1}{3}=1-\frac{2}{3}\left(x-\frac{1}{6}\right)$

$\frac{3x}{4}-\frac{1}{3}=1-\frac{2x}{3}+\frac{2}{18}$

$36\left(\frac{3x}{4}-\frac{1}{3}\right)=36\left(1-\frac{2x}{3}+\frac{2}{18}\right)$

$27x-12=36-24x+4$

$27x-12=-24x+40$

$51x-12=40$

$51x=52$

$x=52/51$

13. $(5x)\left(\frac{33-x}{5x}\right)=5x(2)$

$33-x=10x$

$-x-10x=-33$

$-11x=-33$

$x=3$

Check: $\frac{33-3}{5(3)}\stackrel{?}{=}2$

$\frac{30}{15}\stackrel{?}{=}2$

$2=2$

$x = 3$ is the solution.

14. $\frac{3x+3}{x-3}=7$

$(x-3)\left(\frac{3x+3}{x-3}\right)=(x-3)(7)$

$3x+3=7x-21$

$-4x=-24$

$x=6$

Check: $\frac{3(6)+3}{(6)-3}\stackrel{?}{=}7$

$\frac{21}{3}=7$

15. $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{2(2x+5)}$

Multiply each term by $6(2x + 5)$.

$12x=(8x+20)-15$

$12x-8x=20-15$

$4x=5$ or $x=\frac{5}{4}$

Check: $\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}\stackrel{?}{=}\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$

$\frac{10}{10+20}\stackrel{?}{=}\frac{2}{3}-\frac{5}{15}$

$\frac{10}{30}=\frac{1}{3}$ and $\frac{2}{3}-\frac{5}{15}=\frac{1}{3}$

$x=\frac{5}{4}$ is the solution.

16. $\frac{3}{x}+\frac{1}{4}=\frac{2}{3}+\frac{1}{x}$

LCD is $12x$.

$(12x)\left(\frac{3}{x}\right)+(12x)\left(\frac{1}{4}\right)=(12x)\left(\frac{2}{3}\right)+(12x)\left(\frac{1}{x}\right)$

$36+3x=8x+12$

$-5x=-24$

$x=\frac{24}{5}$

Check: $\frac{3}{\left(\frac{24}{5}\right)}+\frac{1}{4}\stackrel{?}{=}\frac{2}{3}+\frac{1}{\left(\frac{24}{5}\right)}$

$\frac{5}{8}+\frac{1}{4}\stackrel{?}{=}\frac{2}{3}+\frac{5}{24}$

$24\left(\frac{5}{8}\right)+24\left(\frac{1}{4}\right)\stackrel{?}{=}24\left(\frac{2}{3}\right)+24\left(\frac{5}{24}\right)$

$15+6=16+5$

17. $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$

$\frac{2x-2}{x-1}+\frac{1}{3}=\frac{5}{6}$

$\frac{2(x-1)}{x-1}+\frac{1}{3}=\frac{5}{6}$

$2+\frac{1}{3}\neq\frac{5}{6}$

There is no solution.

18. $$\frac{2x}{x-3} = 4 + \frac{6}{x-3}$$
$$(x-3)\left(\frac{2x}{x-3}\right) = (x-3)(4) + (x-3)\left(\frac{6}{x-3}\right)$$
$$2x = 4x - 12 + 6$$
$$-2x = -6$$
$$x = 3$$
Not defined for $x = 3$. No solution.

19. $$3.259x - 8.638 = -3.8(8.625x + 4.917)$$
$$3.259x - 8.638 = -32.775x - 18.6846$$
$$3.259x + 32.775x = 8.638 - 18.6846$$
$$36.034x = -10.0466$$
$$x = \frac{-10.0466}{36.034} \approx -0.279$$

20. $$3.319(14.1x - 5) = 9.95 - 4.6x$$
$$46.7979x - 16.595 = 9.95 - 4.6x$$
$$51.3979x - 16.595 = 9.95$$
$$51.3979x = 26.545$$
$$x \approx 0.516$$

21. $$0.000316x + 9.18 = 2.1(3.1 - 0.0029x) - 4.68$$
$$0.000316x + 9.18 = 6.51 - 0.00609x - 4.68$$
$$0.000316x + 0.00609x = 6.51 - 4.68 - 9.18$$
$$0.006406x = -7.35$$
$$x = \frac{-7.35}{0.006406} \approx -1147.362$$

22. $$3.814x = 2.916(4.2 - 0.06x) + 5.3$$
$$3.814x = 12.2472 - 0.17496x + 5.3$$
$$3.814x = 17.5472 - 0.17496x$$
$$3.98896x = 17.5472$$
$$x \approx 4.399$$

23. $$3x - 4y = 15$$
$$-4y = -3x + 15$$
$$y = \frac{-3x}{-4} + \frac{15}{-4}$$
$$y = \frac{3}{4}x - \frac{15}{4}$$

24. $$3x - 5y = 25$$
$$-5y = -3x + 25$$
$$y = \frac{3}{5}x - 5$$

25. $$2\left(9x + \frac{3}{2}y\right) = 2(11)$$
$$18x + 3y = 22$$
$$3y = -18x + 22$$
$$y = -6x + \frac{22}{3}$$

26. $$\frac{3x}{2} + 5y = \frac{1}{3}$$
LCD is 6.
$$6\left(\frac{3x}{2}\right) + 6(5y) = 6\left(\frac{1}{3}\right)$$
$$9x + 30y = 2$$
$$30y = -9x + 2$$
$$y = -\frac{3}{10}x + \frac{1}{15}$$

27. $$S = P + Prt$$
$$Prt = S - P$$
$$t = \frac{S - P}{Pr}$$

28. $$\frac{y-b}{x-a} = \frac{m}{1}$$
$$y - b = m(x - a)$$
$$y = mx - am + b$$

29. $$3(x-1) < 2x - 1$$
$$3x - 3 < 2x - 1$$
$$x - 3 < -1$$
$$x < 2$$

30. $$2(x+1) > x - 1$$
$$2x + 2 > x - 1$$
$$x + 2 > -1$$
$$x > -3$$

31. $$1 - 2x > 9$$
$$-2x > 8$$
$$\left(-\tfrac{1}{2}\right)(-2x) > 8\left(-\tfrac{1}{2}\right)$$
$$x < -4$$

32. $$17 - x < -4$$
$$-x < -21$$
$$(-1)(-x) < (-1)(-21)$$
$$x > 21$$

33. $\dfrac{3(x-1)}{2} \le x-2$

$3(x-1) \le 2(x-2)$

$3x-3 \le 2x-4$

$x-3 \le -4$

$x \le -1$

34. $\dfrac{x-1}{2}+1 > x+1$

$\dfrac{x-1}{2} > x$

$x-1 > 2x$

$-1 > x$ or $x < -1$

35. $2(x-1)-3 > 4x+1$

$2x-2-3 > 4x+1$

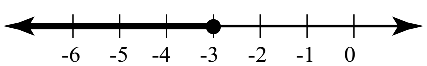
$2x-5 > 4x+1$

$-2x-5 > 1$

$-2x > 6$

$\left(-\tfrac{1}{2}\right)(-2x) > 6\left(-\tfrac{1}{2}\right)$

$x < -3$



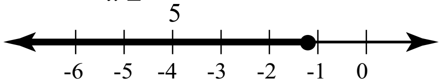
36. $7x+4 \le 2(x-1)$

$7x+4 \le 2x-2$

$5x+4 \le -2$

$5x \le -6$

$x \le -\dfrac{6}{5}$



37. $\dfrac{-3x}{2} > 3-x$

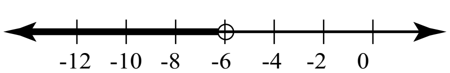
$-3x > 2(3-x)$

$-3x > 6-2x$

$-x > 6$

$(-1)(-x) > 6(-1)$

$x < -6$



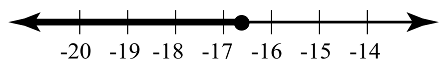
38. $\dfrac{-2x}{5} \le -10-x$

$-2x \le 5(-10-x)$

$-2x \le -50-5x$

$3x \le -50$

$x \le -\dfrac{50}{3}$



39. $12\left(\dfrac{3x}{4}-\dfrac{1}{6}\right) < 12\left(x-\dfrac{2(x-1)}{3}\right)$

$9x-2 < 12x-8(x-1)$

$9x-2 < 12x-8x+8$

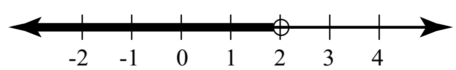
$9x-2 < 4x+8$

$5x-2 < 8$

$5x < 10$

$\left(\tfrac{1}{5}\right)(5x) < \left(\tfrac{1}{5}\right)(10)$

$x < 2$



40. $12\left(\dfrac{4x}{3}-3\right) > 12\left(\dfrac{1}{2}+\dfrac{5x}{12}\right)$

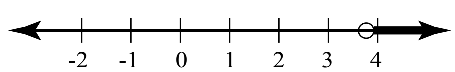
$16x-36 > 6+5x$

$11x-36 > 6$

$11x > 42$

$\left(\tfrac{1}{11}\right)(11x) > \left(\tfrac{1}{11}\right)(42)$

$x > \dfrac{42}{11}$



41. $y = 648{,}000 - 1800x$

$387{,}000 = 648{,}000 - 1800x$

$1800x = 648{,}000 - 387{,}000 = 261{,}000$

$x = \dfrac{261{,}000}{1800} = 145$ months

42. Fully depreciated means

$810{,}000 - 2250x = 0$

$2250x = 810{,}000$

$x = 360$ months

43. $\dfrac{I}{175.393}+0.663=r$

$$\frac{I}{175.393}+0.663=19.8$$
$$\frac{I}{175.393}=19.8-0.663=19.137$$
$$I=19.137(175.393)$$
$$I=\$3356.50$$

44. **a.** $33p-18d=495$

$$p=\frac{18d+495}{33}=\frac{6d+165}{11}$$

b. When d = 12,460,

$$p=\frac{6(12,460)+165}{11}=\frac{74,925}{11}$$

$p\approx 6811$ lbs/sq in.

45. $R=C$ for breakeven point

$20x=2x+7920$

$18x=7920$

$x=440$ packs or 220,000 CD's

46. $4P=81x-29970$

$P=0$ if $81x-29970=0$

$$81x=29970$$
$x=370$ systems

47. $170,500=5.76x$

$$x=\frac{170,500}{5.76}=\$29,600$$

48. Let the pre-tax price of the car be P. Then

$$P+0.06P=21,041$$
$$1.06P=21,041$$
$$P=\frac{21,041}{1.06}$$
$$P=19,850$$

Therefore, the tax on the car is 0.06.
0.06(19,850) = 1191
We could also find the tax by subtracting the pre-tax price from the total price:
21,041 – 19,850 = 1191
$1191.00

49. $959C-1000I=456.8$

$$959C-1000(75)=456.8$$
$$959C-75000=456.8$$
$$959C=75456.8$$
$$C=78.68\%$$

50. $B=1.052W-18,691.28$

$$50,000=1.052W-18,691.28$$
$$68,691.28=1.052W$$
$$W=\$65,295.89$$

51. $\dfrac{93+69+89+97+FE+FE}{6}=90$

$$2FE+348=540$$
$$2FE=192$$
$$FE=96$$

A 96 is the lowest grade that can be earned on the final.

52. Let x = the lowest score on the final.
If the 52 earned during the semester is not replaced,

$$\frac{x+83+67+52+90}{5}=80$$
$$x+292=400$$
$$x=108.$$

This indicates that a grade of 80 is not possible under these circumstances. If the grade of 52 is replaced with the final score x, then

$$\frac{x+83+67+x+90}{5}=80$$
$$2x+240=400$$
$$2x=160$$
$$x=80$$

53. x = amount in safe fund
120,000 – x = amount in risky fund
Yield: $0.09x + 0.13(120,000 - x) = 12,000$

$$0.09x+15,600-0.13x=12,000$$
$$-0.04x=-3600$$
$$x=90,000$$

x = \$90,000 in 9% fund
120,000 – 90000 = \$30,000 in 13% fund.

54. x = amount in safe fund
145,600 – x = amount in risky fund
Yield: $0.10x+0.18(145,600-x)=20,000$

$$0.10x+26,208-0.18x=20,000$$
$$-0.08x=-6208$$
$$x=77,600$$

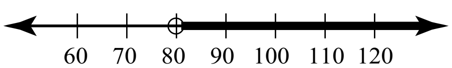
x = \$77,600 in 10% fund
145,600 – 77,600 = \$68,000 in 18% fund.

55. Reduced salary: 2000 – 0.10(2000) = \$1800
Increased salary: 1800 + 0.20(1800) = \$2160
160 = R% of 2000
$R=\frac{160}{2000}=\frac{8}{100}$
\$160 is an 8% increase.

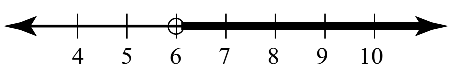
56. a. $\frac{3}{100}=\frac{100}{x}$
$3x=100(100)$
$3x=10,000$
$x=3333$ (rounded)

b. $\frac{63}{1000}=\frac{1000}{x}$
$63x=1000(1000)$
$63x=1,000,000$
$x=15,873$ (rounded)

57. $40x>20x+1600$
$20x>1600$
$x>80$



58. $20d+78<33d$
$78<13d$
$6<d$ or $d>6$



59. $695+5.75t\le 900$
$5.75t\le 205$
$t\le 35.65$
He could buy 35 tapes.

60. Let T be the tax and B be the amount of the monthly bill.
If $0\le B<60$, then $T=0.02B$.
If $60\le B<80$, then $T=0.04B$.
If $B\ge 80$, then $T=0.06B$.

61. a. $2010-1995=15$
b. $6.205+11.23t>150$
$11.23t>143.795$
$t>12.8$
c. 2008

62. $p=75.75-0.74t$
a. $t=2010-1975=35$
b. $p=75.75-0.74(35)=49.85\%$
c. $75.75-0.74t<0$
$75.75<0.74t$
$102.4<t$
In $1975+103=2078$ the model ceases to be effective.

63. $A=90.2+41.3h$
a. $90.2+41.3h\ge 110$
$41.3h\ge 19.8$
$h\ge 0.48$
b. $90.2+41.3h<100$
$41.3h<9.8$
$h<0.24$

64. $WC=1.337t-24.094$
$1.337t-24.094\le t-30$
$0.337t-24.094\le -30$
$0.337t\le -5.906$
$t\le -17.53$

Exercise 1.2

1. a. For each value of x there is only one y.
b. $D=\{-7,-1,0,3,4.2,9,11,14,18,22\}$
$R=\{0,1,5,9,11,22,35,60\}$
c. $f(0)=1$, $f(11)=35$

2. a. $f(9)$ is an output of f.
b. x is not a function of y. For $y=0$ there are two x's.

3. This is a function, since for each x there is only one y. $D=\{1,2,3,8,9\}$, $R=\{-4,5,16\}$

4. No, the relation is not a function because the x-value 1 has two y-values, 4 and 9.
D = {–1, 0, 1, 3}, R = {0, 2, 4, 6, 9}

5. The vertical line test shows that graph (a) is a function of x, and that graph (b) is not a function of x.

6. (b) is a function since for each x there is one y.

7. If $y=3x^3$, then y is a function of x.

8. If $y=6x^2$, then y is a function of x.

9. If $y^2 = 3x$, then y is not a function. If, for example, $x = 3$, there are two possible values for *y*.

10. y is not a function. For $x \neq 0$, there are two *y*'s.

11. $R(x) = 8x - 10$

a. $R(0) = 8(0) - 10 = -10$
b. $R(2) = 8(2) - 10 = 6$
c. $R(-3) = 8(-3) - 10 = -34$
d. $R(1.6) = 8(1.6) - 10 = 2.8$

12. $h(x) = 3x^2 - 2x$

a. $h(3) = 3(3)^2 - 2(3) = 27 - 6 = 21$

b. $h(-3) = 3(-3)^2 - 2(-3) = 27 + 6 = 33$

c. $h(2) = 3(2)^2 - 2(2) = 12 - 4 = 8$

d. $$h\left(\frac{1}{6}\right) = 3\left(\frac{1}{6}\right)^2 - 2\left(\frac{1}{6}\right) = \frac{3}{36} - \frac{2}{6}$$
$$= \frac{3}{36} - \frac{12}{36} = -\frac{9}{36} = -\frac{1}{4}$$

13. $C(x) = 4x^2 - 3$

a. $C(0) = 4(0)^2 - 3 = -3$

b. $C(-1) = 4(-1)^2 - 3 = 1$

c. $C(-2) = 4(-2)^2 - 3 = 13$

d. $$C\left(-\frac{3}{2}\right) = 4\left(-\frac{3}{2}\right)^2 - 3 = 6$$

14. $R(x) = 100x - x^3$

a. $R(1) = 100(1) - 1^3 = 100 - 1 = 99$

b. $R(10) = 100(10) - (10)^3 = 1000 - 1000 = 0$

c. $R(2) = 100(2) - 2^3 = 200 - 8 = 192$

d. $$R(-10) = 100(-10) - (-10)^3$$
$$= -1000 - (-1000) = 0$$

15. $f(x) = x^3 - \dfrac{4}{x}$

a. $$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - \frac{4}{-\frac{1}{2}} = -\frac{1}{8} + 8 = \frac{63}{8}$$

b. $$f(2) = 2^3 - \frac{4}{2} = 8 - 2 = 6$$

c. $$f(-2) = (-2)^3 - \frac{4}{-2} = -8 + 2 = -6$$

16. $C(x) = \dfrac{x^2 - 1}{x}$

a. $$C(1) = \frac{1^2 - 1}{1} = \frac{0}{1} = 0$$

b. $$C\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2 - 1}{\frac{1}{2}} = \frac{\frac{1}{4} - 1}{\frac{1}{2}} = \frac{-\frac{3}{4}}{\frac{1}{2}} = -\frac{3}{2}$$

c. $$C(-2) = \frac{(-2)^2 - 1}{-2} = \frac{4 - 1}{-2} = -\frac{3}{2}$$

17. $f(x) = 1 + x + x^2$

a. $f(2+1) = f(3) = 1 + 3 + 3^2 = 13$
$f(2) + f(1) = 7 + 3 = 10$
$f(2) + f(1) \neq f(2+1)$

b. $f(x+h) = 1 + (x+h) + (x+h)^2$

c. No. This is equivalent to a.

d. $f(x) + h = 1 + x + x^2 + h$
No. $f(x+h) \neq f(x) + h$

e. $$f(x+h) = 1 + (x+h) + (x+h)^2$$
$$= 1 + x + h + x^2 + 2xh + h^2$$
$$f(x) = 1 + x + x^2$$
$$f(x+h) - f(x) = h + 2xh + h^2$$
$$= h(1 + 2x + h)$$
$$\frac{f(x+h) - f(x)}{h} = 1 + 2x + h$$

18. $f(x) = 3x^2 - 6x$

a. $f(3+2) = f(5) = 3 \cdot 5^2 - 6 \cdot 5 = 75 - 30 = 45$
$f(3) + 2 = (3 \cdot 3^2 - 6 \cdot 3) + 2 = 27 - 18 + 2 = 11$
So, $f(3+2) \neq f(3) + 2$.

b. $$f(x+h) = 3(x+h)^2 - 6(x+h)$$
$$= 3(x^2 + 2xh + h^2) - 6x - 6h$$
$$= 3x^2 + 6xh + 3h^2 - 6x - 6h$$

c. $f(x) + h = 3x^2 - 6x + h$
So, $f(x+h) \neq f(x) + h$

d. $f(x) + f(h) = 3x^2 - 6x + 3h^2 - 6h$
So, $f(x+h) \neq f(x) + f(h)$

e. $$f(x+h) = 3(x+h)^2 - 6(x+h)$$
$$= 3x^2 + 6xh + 3h^2 - 6x - 6h$$
$$f(x+h) - f(x)$$
$$= 3x^2 + 6xh + 3h^2 - 6x - 6h - (3x^2 - 6x)$$
$$= 6xh + 3h^2 - 6h$$
$$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 - 6h}{h}$$
$$= 6x + 3h - 6$$

19. $f(x) = x - 2x^2$

a. $f(x+h) = (x+h) - 2(x+h)^2$
$= -2x^2 - 4xh - 2h^2 + x + h$

b. $f(x+h) - f(x)$
$= (x+h) - 2(x+h)^2 - (x - 2x^2)$
$= x + h - 2x^2 - 4xh - 2h^2 - x + 2x^2$
$= h - 4xh - 2h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{h-4xh-2h^2}{h} = 1 - 4x - 2h$$

20. $f(x) = 2x^2 - x + 3$

a. $f(x+h) = 2(x+h)^2 - (x+h) + 3$
$= 2(x^2 + 2xh + h^2) - x - h + 3$
$= 2x^2 + 4xh + 2h^2 - x - h + 3$

b. $f(x+h) - f(x)$
$= 2x^2 + 4xh + 2h^2 - x - h + 3 - (2x^2 - x + 3)$
$= 2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3$
$= 4xh + 2h^2 - h$

$$\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2-h}{h} = 4x + 2h - 1$$

21. Since (9, 10) and (5, 6) are points on the graph:

a. $f(9) = 10$

b. $f(5) = 6$

22. a. From the figure, $g(0) = 0$.

b. There are three values of x that satisfy $g(x) = 0$.

23. a. The coordinates of $Q = (1, -3)$. Since the point is on the curve, the coordinates satisfy the equation.

b. The coordinates of $R = (3, -3)$. They satisfy the equation.

c. The ordered pair (a, b) satisfies the equation. Thus $b = a^2 - 4a$.

d. The x values are 0 and 4. These values are also solutions of $x^2 - 4x = 0$.

24. a. The point (1, 1) does not lie on the graph. The coordinates do not satisfy the equation.

b. From the graph, the coordinates of point R are (1, 2). These coordinates do satisfy the equation.

c. If $P(a, b)$ is a point on the graph, then $b = 2a^2$.

d. The x-coordinate of the point whose y-coordinate is 0 is 0. This value of x does satisfy the equation $0 = 2x^2$.

25. $y = x^2 + 4$

There is no division by zero or square roots. Domain is all the reals, i.e., $\{x : x \in \text{Reals}\}$. Since $x^2 \ge 0, x^2 + 4 \ge 4$, the range is reals ≥ 4 or $\{y : y \ge 4\}$.

26. Domain: all reals
Range: reals ≥ -1

27. $y = \sqrt{x+4}$

There is no division by zero. To get a real number y, we must have $x + 4 \ge 0$ or $x \ge -4$. Domain: $x \ge -4$. The square root is always nonnegative.
Thus, the range is $\{y : y \in \text{reals}, y \ge 0\}$.

28. Domain: all reals
Range: reals ≥ 1

29. $D: \{x : x \ge 1, x \ne 2\}$

30. Domain: reals > -3

31. $D: \{x : -7 \le x \le 7\}$

32. Domain: $-3 \le x \le 3$

33. $f(x) = 3x$, $g(x) = x^3$

a. $(f+g)(x) = 3x + x^3$

b. $(f-g)(x) = 3x - x^3$

c. $(f \cdot g)(x) = 3x \cdot x^3 = 3x^4$

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x}{x^3} = \dfrac{3}{x^2}$

34. $f(x)=\sqrt{x}, \quad g(x)=\dfrac{1}{x}$

a. $(f+g)(x)=f(x)+g(x)=\sqrt{x}+\dfrac{1}{x}$

b. $(f-g)(x)=\sqrt{x}-\dfrac{1}{x}$

c. $(f\cdot g)=f(x)\cdot g(x)=\sqrt{x}\cdot\dfrac{1}{x}=\dfrac{\sqrt{x}}{x}$

d. $(f/g)(x)=\dfrac{f(x)}{g(x)}=\dfrac{\sqrt{x}}{\frac{1}{x}}=x\sqrt{x}$

35. $f(x)=\sqrt{2x}$ **,** $g(x)=x^2$

a. $(f+g)(x)=\sqrt{2x}+x^2$

b. $(f-g)(x)=\sqrt{2x}-x^2$

c. $(f\cdot g)(x)=\sqrt{2x}\cdot x^2=x^2\sqrt{2x}$

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{\sqrt{2x}}{x^2}$

36. $f(x)=(x-1)^2, \quad g(x)=1-2x$

a. $(f+g)(x)=f(x)+g(x)$
$=(x-1)^2+1-2x$
$=x^2-2x+1+1-2x$
$=x^2-4x+2$

b. $(f-g)(x)=f(x)-g(x)$
$=(x-1)^2-(1-2x)$
$=x^2-2x+1-1+2x=x^2$

c. $(f\cdot g)(x)=f(x)\cdot g(x)=(x-1)^2(1-2x)$

d. $(f/g)(x)=\dfrac{f(x)}{g(x)}=\dfrac{(x-1)^2}{1-2x}$

37. $f(x)=(x-1)^3, \quad g(x)=1-2x$

a. $(f\circ g)(x)=f(1-2x)=(1-2x-1)^3=-8x^3$

b. $(g\circ f)(x)=g((x-1)^3)=1-2(x-1)^3$

c. $f(f(x))=f((x-1)^3)=[(x-1)^3-1]^3$

d. $(f\cdot f)(x)=(x-1)^3\cdot(x-1)^3=(x-1)^6$
$\left[\left(f\cdot f\right)(x)\neq f\left(f(x)\right)\right]$

38. $f(x)=3x, \quad g(x)=x^3-1$

a. $(f\circ g)(x)=f(g(x))=f(x^3-1)$
$=3(x^3-1)=3x^3-3$

b. $(g\circ f)(x)=g(f(x))=g(3x)$
$=(3x)^3-1=27x^3-1$

c. $f(f(x))=f(3x)=3(3x)=9x$

d. $f^2(x)=(f\cdot f)(x)=f(x)\cdot f(x)$
$=3x\cdot 3x=9x^2$

39. $f(x)=2\sqrt{x}$ **,** $g(x)=x^4+5$

a. $(f\circ g)(x)=f(x^4+5)=2\sqrt{x^4+5}$

b. $(g\circ f)(x)=g(2\sqrt{x})=(2\sqrt{x})^4+5$
$=16x^2+5$

c. $f(f(x))=f(2\sqrt{x})=2\sqrt{2\sqrt{x}}$

d. $(f\cdot f)(x)=2\sqrt{x}\cdot 2\sqrt{x}=4x$
$\left[\left(f\cdot f\right)(x)\neq f\left(f(x)\right)\right]$

40. $f(x)=\dfrac{1}{x^3}, \quad g(x)=4x+1$

a. $(f\circ g)(x)=f(g(x))$
$=f(4x+1)=\dfrac{1}{(4x+1)^3}$

b. $(g\circ f)(x)=g(f(x))=g\left(\dfrac{1}{x^3}\right)$
$=4\cdot\dfrac{1}{x^3}+1=\dfrac{4}{x^3}+1$

c. $f(f(x))=f\left(\dfrac{1}{x^3}\right)=\dfrac{1}{\left(\frac{1}{x^3}\right)^3}=\dfrac{1}{\frac{1}{x^9}}=x^9$

d. $f^2(x)=(f\cdot f)(x)=f(x)\cdot f(x)$
$=\dfrac{1}{x^3}\cdot\dfrac{1}{x^3}=\dfrac{1}{x^6}$

41. a. $f(20)=103,000$ means it will take 20 years to pay off a debt of \$103,000 (at \$800 per month and 7.5% compounded monthly.)

b. $f(5+5)=f(10)=69,000$;
$f(5)+f(5)=80,000$; No.

c. It will take 15 years to pay off the debt, i.e., $89,000=f(15)$.

42. a. From the table, the monthly payment is \$775.30 if they refinance for 20 years, i.e. $775.30 = f(20)$.

b. From the table, $f(10) = 1161.09$. the value of $f(10)$ is the monthly payment to repay a \$100,000 loan in 10 years when the interest rate is 7%.

c. $f(5+5) = f(10) = 1161.09$

$f(5) + f(5) = 1980.12 + 1980.12 = 3960.24$

$f(5+5) \neq f(5) + f(5)$

43. a. $f(1950) = 16.5$ means that in 1950 there were 16.5 workers supporting each person receiving Social Security benefits.

b. $f(1990) = 3.4$

c. The points based on known data must be the same and those based on projections might be the same.

d. Domain: $1950 \leq t \leq 2050$
Range: $1.9 \leq n \leq 16.5$

44. a. $f(0) \approx 14{,}023$ and $f(6.5) \approx 13{,}968$.
These values represent the opening value and the closing value, respectively, for the Dow Jones average on October 4, 2007.

b. The domain is $0 \leq t \leq 6.5$. The range is approximately 13,950 to 14,050.

c. There are six t-values that satisfy $f(t) = 14{,}000$. Answers will vary.

45. a. $f(95) = 1{,}000{,}000, \quad g(95) = 600{,}000$

b. $f(100) = 1{,}200{,}000$. This was the number of prisoners in 2000.

c. $g(90) = 500{,}000$. This was the number of parolees in 1990.

d. $(f-g)(100) = 1{,}200{,}000 - 700{,}000$
$= 500{,}000$. There were this many more in prison than were out on parole.

e. $(f-g)(93) = 900{,}000 - 600{,}000 = 300{,}000$

$(f-g)(98) = 1{,}100{,}000 - 600{,}000 = 500{,}000$

$(f-g)(98)$ is greater. Possible reason is increased prison capacity but parolee level is constant.

46. a. $f(1970) = 30{,}000{,}000$

b. $f(1930) = 10{,}000{,}000$. There were 10,000,000 women in the labor force in 1930.

c. $f(1990) - f(1980) = 16{,}000{,}000$. The labor force increased 16,000,000.

47. a. Since the wind speed cannot be negative, $s \geq 0$.

b. $f(10) = 45.694 + 1.75(10) - 29.26\sqrt{10} = -29.33$
At a temperature of –5°F and a wind speed of 10 mph, the temperature feels like $-29.33°\text{F}$.

c. $f(0) = 45.694$, but $f(0)$ should equal the air temperature, –5°F.

48. $C = \frac{5}{9}F - \frac{160}{9}$

a. C is a function of F.

b. Mathematically, the domain is all reals.

c. Domain: $\{F : 32 \leq F \leq 212\}$

Range: $\{C : 0 \leq C \leq 100\}$

d. $C(40) = \frac{5}{9}(40) - \frac{160}{9}$

$= \frac{200-160}{9} = \frac{40}{9} = 4.44°C$

49. $C(x) = 300x + 0.1x^2 + 1200$

a. $C(10) = 300(10) + 0.1(10)^2 + 1200$
$= 3000 + 0.1(100) + 1200$
$= 3000 + 10 + 1200$
$= \$4210$

b. The value $C(100)$ is the total cost of producing 100 units.

c. $C(100) = 300(100) + 0.1(100)^2 + 1200$
$= \$32{,}200$

50. a. $P(2000) = 47(2000) - 0.01(2000)^2 - 8000$
$= 94000 - 0.01(4{,}000{,}000) - 8000$
$= \$46{,}000$

b. $P(5000)$
$= 47(5000) - 0.01(5000)^2 - 8000$
$= 235{,}000 - 0.01(25{,}000{,}000) - 8000$
$= -\$23{,}000$

c. $P(5000)$ is negative, which means it is not profitable for the company to product 5000 units.

51. $C(p) = \dfrac{7300p}{100-p}$

a. Domain: $\{p : 0 \le p < 100\}$

b. $C(45) = \dfrac{7300(45)}{100-45} = \dfrac{328{,}500}{55} = \$5972.73$

c. $C(90) = \dfrac{7300(90)}{100-90} = \dfrac{657{,}000}{10} = \$65{,}700$

d. $C(99) = \dfrac{7300(99)}{100-99} = \dfrac{722{,}700}{1} = \$722{,}700$

e. $C(99.6) = \dfrac{7300(99.6)}{100-99.6} = \dfrac{727{,}080}{0.4} = \$1{,}817{,}700$

In each case above, to remove p% of the particulate pollution would cost $C(p)$.

52. $R(n) = \dfrac{0.6n}{0.4+0.6n}$

a. $R(1) = \dfrac{0.6(1)}{0.4+0.6(1)} = \dfrac{0.6}{1.0} = 0.6$

b. $R(2) = \dfrac{0.6(2)}{0.4+0.6(2)} = \dfrac{1.2}{1.6} = 0.75$

c. Improvement is 0.75 – 0.6 = 0.15. The percentage improvement is $\dfrac{0.15}{0.6} = 25\%$.

53. a. A is a function of x.

b. $A(2) = 2(50 - 2) = 96$ sq ft
$A(30) = 30(50 - 30) = 600$ sq ft

c. For the problem to have meaning we have $0 < x < 50$.

54. a. $V(x) = x^2(108-4x)$ is a function of x.

b.
$$\begin{aligned} V(10) &= (10)^2(108-4(10)) \\ &= 100(108-40) = 100(68) \\ &= 6800 \text{ cubic inches} \end{aligned}$$
$$\begin{aligned} V(20) &= (20)^2(108-4(20)) \\ &= 400(108-80) = 400(28) \\ &= 11{,}200 \text{ cubic inches} \end{aligned}$$

c. The values for x must be such that $0 < x < 27$, otherwise the volume would be less than or equal to 0.

55. a.
$$\begin{aligned} P(q(t)) &= P(1000+10t) \\ &= 180(1000+10t) - \frac{(1000+10t)^2}{100} - 200 \\ &= 169{,}800 + 1600t - t^2 \end{aligned}$$

b. $q(15) = 1000 + 10(15) = 1150$
$P(q(15)) = \$193{,}575$

56. $W(L) = kL^3$
When $k = 0.02$, we have $W(L) = 0.02L^3$ and
$L = L(t) = 50 - \dfrac{(t-20)^2}{10}$, $0 \le t \le 20$.

So, $(W \circ L)(t) = W(L(t)) = 0.02\left[50 - \dfrac{(t-20)^2}{10}\right]^3$.

57. $R = f(C) \quad C = g(A)$

a. $(f \circ g)(x) = f(C) = R$

b. $(g \circ f)(x)$ is not defined.

c. A is the independent variable and R is the dependent variable. Revenue depends on money spent for advertising.

58. a. sanding the door
b. painting the door
c. sanding the door and then painting
d. painting the door and then sanding
e. painting the door with two coats

59. length = x width = y $L = 2x + 2y$

$1600 = xy$ or $y = \dfrac{1600}{x}$

$L = 2x + 2\left(\dfrac{1600}{x}\right) = 2x + \dfrac{3200}{x}$

60. Let x = the length of the base.

Then $\frac{1}{2}x$ = the height. Bottom: x^2 sq. ft.

Sides: $\frac{1}{2}x \cdot x = \frac{1}{2}x^2$ sq. ft. for each of 4 sides for a total of $4 \cdot \frac{1}{2}x^2 = 2x^2$ sq. ft.

Top: x^2 sq. ft

Cost:
$$C(x) = \underset{\text{bottom}}{\underset{\updownarrow}{2x^2}} + \underset{\text{sides}}{\underset{\updownarrow}{2(2x^2)}} + \underset{\text{top}}{\underset{\updownarrow}{1.5x^2}} = 7.5x^2$$

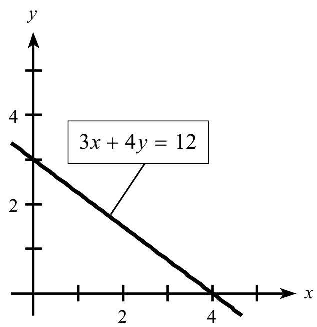
61. Revenue = (no. of people)(price per person)
Example: $R = 30 \times 10$
$R = 31 \times 9.80$
$R = 32 \times 9.60$
Solution: $R = (30 + x)(10 – 0.20x)$

62. Let x = number of \$10 increases.
Then $R(x) = \underset{\text{Rent/unit}}{\underset{\updownarrow}{(360+10x)}}\underset{\text{\#rented}}{\underset{\updownarrow}{(50-x)}}$.

Exercise 1.3

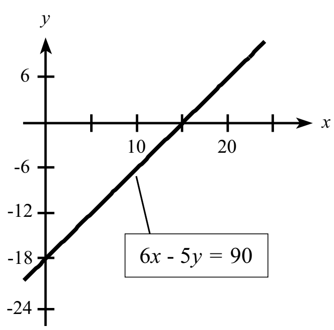
1. $3x+4y=12$

x-intercept: $y = 0$ then $x = 4$.
y-intercept: $x = 0$ then $y = 3$.



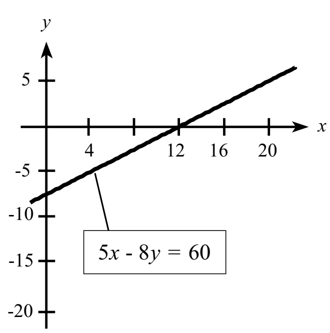
2. $6x-5y=90$

x-intercept: $y = 0$ then $x=15$.
y-intercept: $x = 0$ then $y=-18$



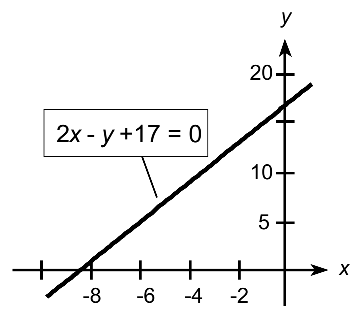
3. $5x-8y=60$

x-intercept: $y = 0$ then $x=12$.
y-intercept: $x = 0$ then $y=-7.5$.



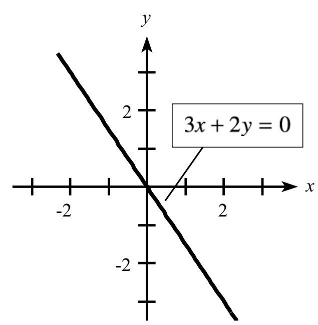
4. $2x-y+17=0$

x-intercept: $y = 0$ then $x=-8.5$.
y-intercept: $x = 0$ then $y=17$.



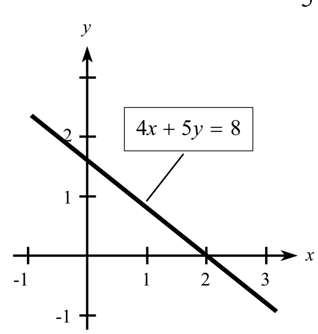
5. $3x+2y=0$

x-intercept: $y = 0$ then $x = 0$
Likewise, y-intercept is $y = 0$.



6. $4x + 5y = 8$

x-intercept: $y = 0$ then $x = 2$
y-intercept: $x = 0$ then $y=\frac{8}{5}$



7. $(22,11)$ and $(15,-17)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-17-11}{15-22}=\frac{-28}{-7}=4$$

8. $(-6,-12)$ and $(-18,-24)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-24-(-12)}{-18-(-6)}=\frac{-12}{-12}=1$$

9. $(3,-1)$ and $(-1,1)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-(-1)}{-1-3}=\frac{2}{-4}=-\frac{1}{2}$$

10. $(-5,6)$ and $(1,-3)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-6}{1-(-5)}=\frac{-9}{6}=-\frac{3}{2}$$

11. (3, 2) and (–1, 2)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-2}{3-(-1)}=\frac{0}{4}=0$$

12. $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-2-2}{-4-(-4)}=\dfrac{-4}{0}$, undefined

13. A horizontal line has a slope of 0.

14. The slope of a vertical line is undefined.

15. $(3,2)$ and $(-1,2)$

The rate of change is equivalent to the slope of the line.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-2}{-1-3}=\frac{0}{-4}=0$$

16. $(11,-5)$ and $(-9,-4)$

The rate of change is equivalent to the slope of the line.

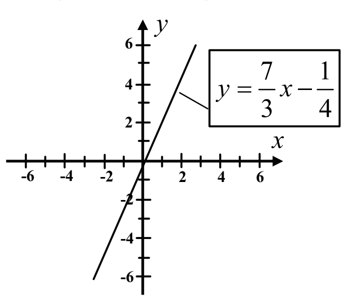
$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-(-5)}{-9-11}=\frac{-1}{20}$$

17. **a.** Slope is negative.
b. Slope is undefined.

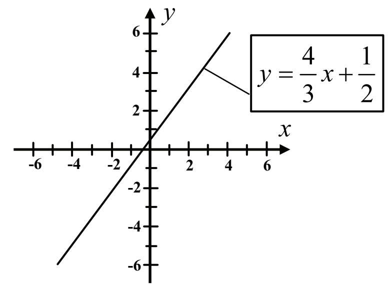
18. **a.** $m=0$

b. $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{5-(-1)}{2-0}=\dfrac{6}{2}=3$

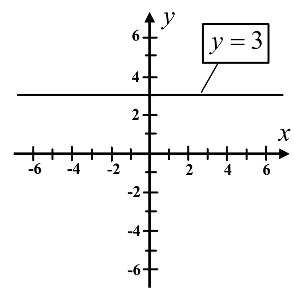
19. $y=\dfrac{7}{3}x-\dfrac{1}{4},\ m=\dfrac{7}{3},\ b=-\dfrac{1}{4}$



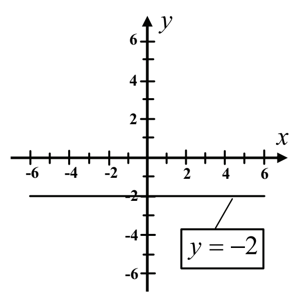
20. $y=\dfrac{4}{3}x+\dfrac{1}{2}, m=\dfrac{4}{3}, b=\dfrac{1}{2}$



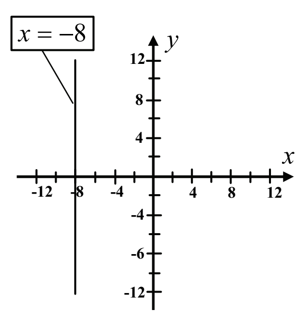
21. $y=3$ or $y=0x+3,\ m=0,\ b=3$



22. $y=-2$ horizontal line
$m=0,\ b=-2$

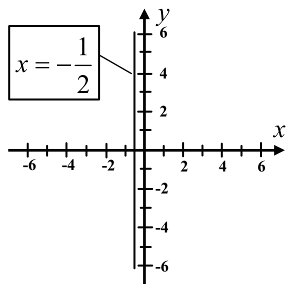


23. $x=-8$
Slope is undefined.
There is no y-intercept.

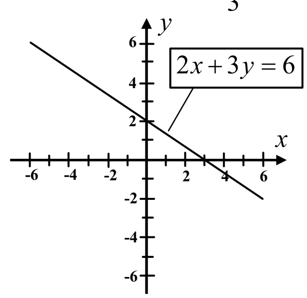


24. $x = -\frac{1}{2}$ vertical line

m is undefined; there is no y-intercept.



25. $2x + 3y = 6$ or $y = -\frac{2}{3}x + 2, m = -\frac{2}{3}, b = 2$.

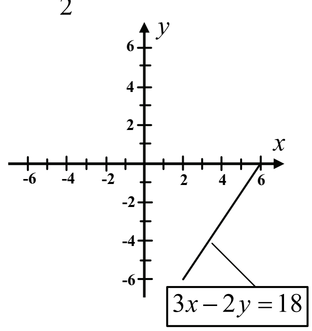


26. $3x - 2y = 18$

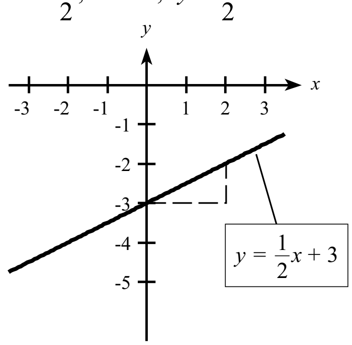
$$-2y = -3x + 18$$

$$y = \frac{3}{2}x - 9$$

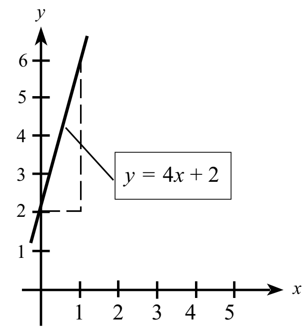
$m = \frac{3}{2}, b = -9$



27. $m = \frac{1}{2}, b = -3;\ y = \frac{1}{2}x - 3$

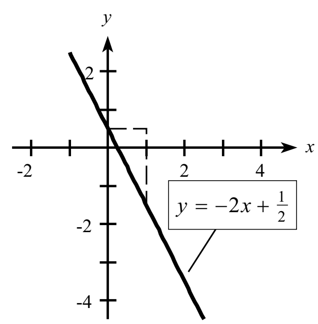


28. $m = 4$, $b = 2$, $y = 4x + 2$

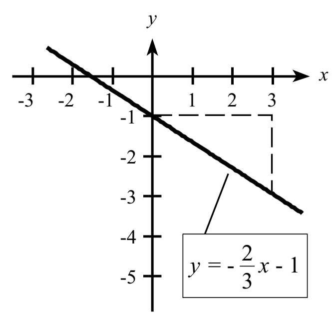


29. $m = -2,\ b = \frac{1}{2}$

$$y = -2x + \frac{1}{2}$$

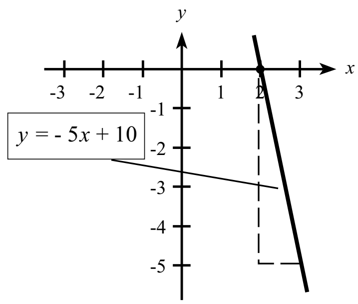


30. $m = -\frac{2}{3}, b = -1, y = -\frac{2}{3}x - 1$



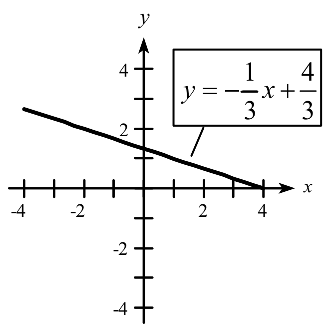
31. $P(2, 0),\ m = -5$

$$y - 0 = -5(x - 2)$$
$$y = -5x + 10$$



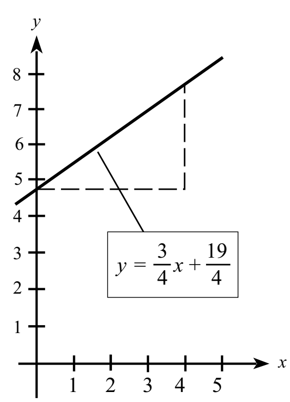
32. $(1, 1), m = -\dfrac{1}{3}$

$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



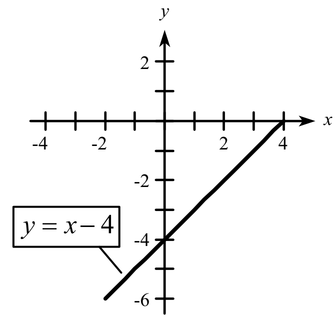
33. $P(-1, 4),\ m = \dfrac{3}{4}$

$$y - 4 = \frac{3}{4}(x - (-1))$$
$$y = \frac{3}{4}x + \frac{19}{4}$$

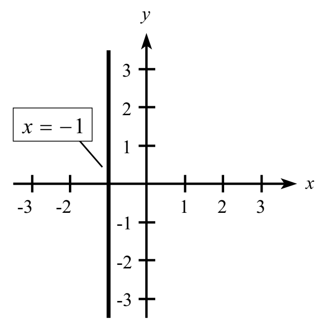


34. $(3, -1), m = 1$

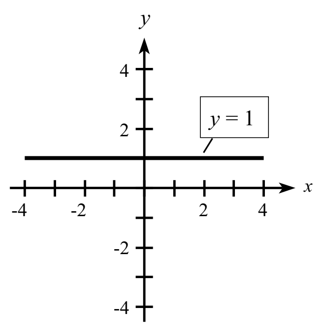
$$y - y_1 = m(x - x_1)$$
$$y + 1 = 1(x - 3)$$
$$y + 1 = x - 3$$
$$y = x - 4$$



35. $P(-1, 1)$, m is undefined
$x = -1$



36. $(1, 1)$, $m = 0$; horizontal line, $y = 1$



37. $P_1 = (3, 2), P_2 = (-1, -6)$

$$m = \frac{-6 - 2}{-1 - 3} = 2$$
$$y - 2 = 2(x - 3)$$
$$y = 2x - 4$$

38. $(-4, 2), (2, 4),\ m = \dfrac{4-2}{2-(-4)} = \dfrac{2}{6} = \dfrac{1}{3}$

$$y-4 = \frac{1}{3}(x-2)$$
$$y-4 = \frac{1}{3}x - \frac{2}{3}$$
$$y = \frac{1}{3}x + \frac{10}{3}$$

39. $P_1 = (7, 3), P_2 = (-6, 2)$

$$m = \frac{2-3}{-6-7} = \frac{-1}{-13} = \frac{1}{13}$$
$$y-3 = \frac{1}{13}(x-7)$$
$$y = \frac{1}{13}x - \frac{7}{13} + 3$$
$$y = \frac{1}{13}x + \frac{32}{13} \text{ or } -x + 13y = 32$$

40. $(10, 2), (5, 7),\ m = \dfrac{7-2}{5-10} = -1$

$$y-7 = -1(x-5)$$
$$y-7 = -x+5$$
$$y = -x+12$$

41. $3x+2y=6 \qquad 2x-3y=6$

$$y = -\frac{3}{2}x+3 \qquad y = \frac{2}{3}x-2$$

Lines are perpendicular since $\left(-\dfrac{3}{2}\right)\left(\dfrac{2}{3}\right) = -1$.

42. $5x-2y=8 \qquad 10x-4y=8$

$$y = \frac{5}{2}x-4 \qquad y = \frac{5}{2}x-2$$

Lines are parallel since the slopes are equal.

43. $6x-4y=12 \qquad 3x-2y=6$

$$y = \frac{6}{4}x - \frac{12}{4} \qquad y = \frac{3}{2}x-3$$

or $y = \dfrac{3}{2}x-3$

Lines are the same.

44. $5x+4y=7 \qquad y = \dfrac{4}{5}x+7$

$$4y = -5x+7$$
$$y = -\frac{5}{4}x + \frac{7}{4}$$

Lines are perpendicular since $\left(-\dfrac{5}{4}\right)\left(\dfrac{4}{5}\right) = -1$.

45. If $3x + 5y = 11$, then $y = -\dfrac{3}{5}x + \dfrac{11}{5}$. So, $m = -\dfrac{3}{5}$. A line parallel will have the same slope. Thus, $m = -\dfrac{3}{5}$ and $P = (-2, -7)$ gives $y-(-7) = -\dfrac{3}{5}(x-(-2))$ which simplifies to $y = -\dfrac{3}{5}x - \dfrac{41}{5}$.

46. Through $(6, -4)$, parallel to $4x - 5y = 6$
Find the slope of $4x - 5y = 6$.

$$-5y = -4x+6$$
$$y = \frac{4}{5}x - \frac{6}{5}$$
$$m = \frac{4}{5}$$

Use the same slope.

$$y-(-4) = \frac{4}{5}(x-6)$$
$$y+4 = \frac{4}{5}x - \frac{24}{5}$$
$$y = \frac{4}{5}x - \frac{44}{5}$$

47. If $5x - 6y = 4$, then $y = \dfrac{5}{6}x - \dfrac{4}{6}$. Slope of the perpendicular line is $-\dfrac{6}{5}$. Thus $m = -\dfrac{6}{5}$ and $P = (3, 1)$ gives $y-1 = -\dfrac{6}{5}(x-3)$ which simplifies to $y = -\dfrac{6}{5}x + \dfrac{23}{5}$.

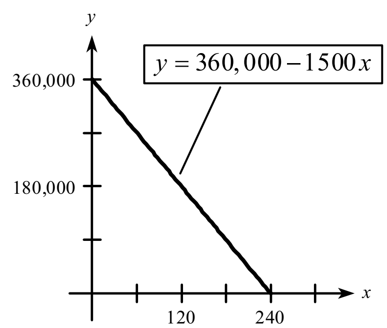
48. $(-2, -8)$, perpendicular to $x = 4y + 3$
Find the slope of $x = 4y + 3$.

$$-4y = -x+3$$
$$y = \frac{1}{4}x - \frac{3}{4}$$
$$m = \frac{1}{4}$$

Slope of new line is $-\dfrac{1}{\frac{1}{4}} = -4$.

$$y-(-8) = -4(x-(-2))$$
$$y+8 = -4x-8$$
$$y = -4x-16$$

49. a.



b. $0 = 360{,}000 - 1500x$

$x = \frac{360000}{1500} = 240$ months

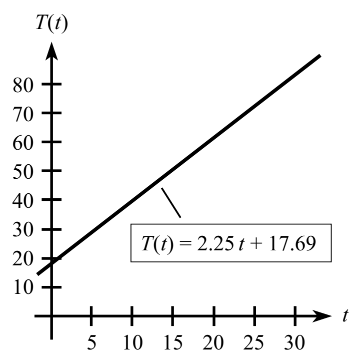
In 240 months, the building will be completely depreciated.

c. (60, 270,000) means that after 60 months the value of the building will be $270,000.

50. a. $T(20) = 2.25(20) + 17.69 = \$62.69$

$t = 20$ represents the year 1980 + 20 = 2000. In the year 2000, the per capita tax burden is expected to be $\$62.69 \cdot 100 = \$6,269$.

b.



51. $y = 5.74x + 14.61$

a. $m = 5.74,\ b = 14.61$

b. In 1995, when $x = 0$, 14.61% of the U.S. population had Internet.

c. The percentage of the U.S. population with Internet is changing at the rate of 5.74% per year.

52. $p = 32.88 - 0.03t$

a. $m = -0.03,\ b = 32.88$

b. The slope represents the annual 0.03% decrease of high school seniors who smoke.

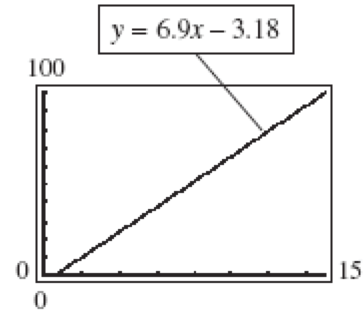
c. The p-intercept represents the 32.88% of high school seniors who smoked in 1975.

53. $y = 6.9x - 3.18$

a. $m = 6.9 \quad b = -3.18$

b. The percent of the U.S. population with internet service is changing at the rate of 6.9% per year.

c.



54. $p = 85.79 - 2.39t$

a. $m = -2.39 \quad b = 85.79$

b. Every year, the percentage of high school seniors who smoke goes down 2.39%.

c. In 1975, 85.79% of high school seniors smoked.

55. $M(x) = -0.762x + 85.284$

a. $m = -0.762 \quad b = 85.284$

b. In 1950, 85% of the unmarried women became married.

c. The annual rate of change is -0.762%. For each passing year the percent of unmarried women who get married decreases by 0.762%.

56. $S = 141.1 - 45.78(1 - H)$

$A = 91.2 + 41.3H$

a. When $H = 0.40$,

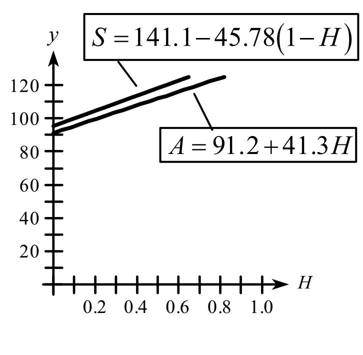
$S = 141.1 - 45.78(1 - 0.4)$

$= 141.1 - 45.78(0.6) \approx 113.6°$

$A = 91.2 + 41.3(0.4) \approx 107.7°\text{F}$

b. According to the Summer Simmer Index, the effect of a relative humidity of 40% is to make 100°F seem like 113.6°F. The other model indicates that the 100°F seems like 107.7°F.

c.



57. $F = 0.78M - 1.316$

a. $m = 0.78$

b. For each $1 increase in male earnings, the female's earning increases by only $0.78.

c. $F = 0.78(60) - 1.316$

= 45.484 thousand

=$48,484

58. $p(x) = 56.009 - 0.758x$

a. $m = -0.758$

b. The percentage of U.S. high school seniors who used marijuana decreases at a rate of 0.758% per year.

59. $y = 16.37 + 0.0838x$

60. $y = 9.19 + 0.9191x$

61. a. $m = \frac{105}{100} = 1.05$ and $(676, 527)$

$B - 527 = 1.05(W - 676)$

$B = 1.05W - 182.80$

b. $B(850) = 1.05(850) + 182.80$

$= \$709.70$

62. a. $p = 0.025(80,000)y = 2000y$

b. $p = 0.025 \cdot c \cdot 30 = 0.75c$

63. a. $(1985, 113.2)$ and $(2005, 324.9)$

$m = \frac{324.9 - 113.2}{2005 - 1985} = \frac{211.7}{20} = 10.585$

$y - 113.2 = 10.585(x - 1985)$

$y = 10.585x - 220,898.025$

b. The consumer price index for urban consumers increases at the rate of $10.59 per year.

64. a. Yes

b. $m = \frac{0.13 - 0.11}{6 - 5} = 0.02$

$y - 0.13 = 0.02(x - 6)$

$y = 0.02x + 0.01$

c. The values in the table fit the model.

65. (x, p) is the reference. (0, 85000) is one point.

$m = \frac{-1700}{1} = -1700$

$p - 85,000 = -1700(x - 0)$ or

$p = -1700x + 85,000$

66. P = (age, hours of sleep) $P_1 = (18, 8)$

Choose $P_2 = (14, 9)$

$m = \frac{9 - 8}{14 - 18} = -\frac{1}{4}$

$y - 8 = -\frac{1}{4}(x - 18)$ or $y - 8 = -\frac{1}{4}x + \frac{9}{2}$

or $y = -\frac{1}{4}x + \frac{25}{2}$

67. (t, R) is the ordered pair.

$P_1 = \left(\frac{7}{2}, 11\right)$, $P_2 = (6, 19)$

$m = \frac{19 - 11}{6 - \frac{7}{2}} = \frac{8}{\frac{5}{2}} = \frac{16}{5} = 3.2$

$R - 19 = 3.2(t - 6)$ or $R = 3.2t - 19.2 + 19$ or

$R = 3.2t - 0.2$

68. Pairs: (0, 960,000) and (240, 0)

$m = \frac{0 - 960,000}{240 - 0} = -4000$

$b = 960,000$

$y = 960,000 - 4000x$

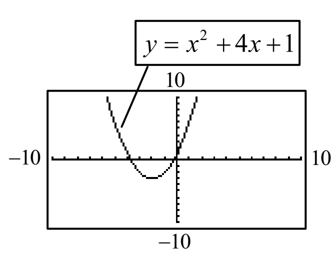
69. $P_1 = (200, 25)$ $P_2 = (250, 49)$

$m = \frac{49 - 25}{250 - 200} = \frac{24}{50} = 0.48$

$y - 25 = 0.48(x - 200)$ or $y = 0.48x - 71$

Exercise 1.4

1.



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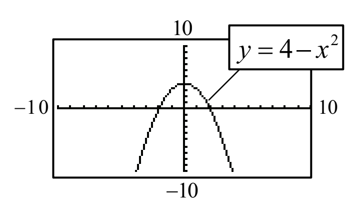

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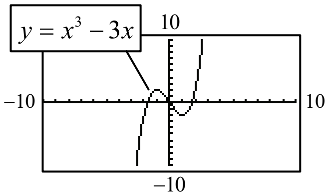

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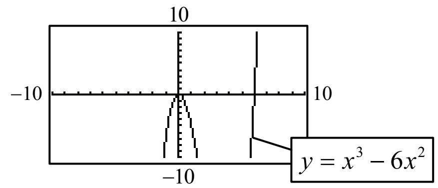

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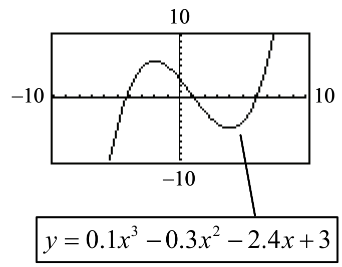

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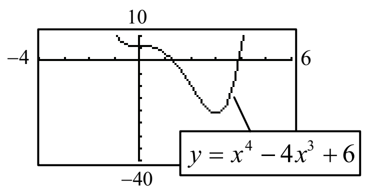

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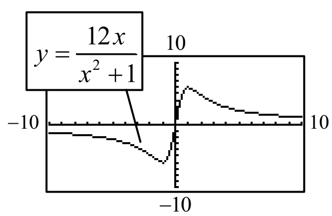

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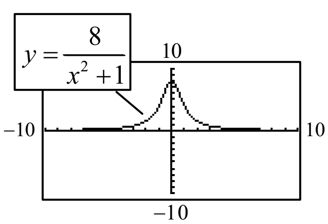
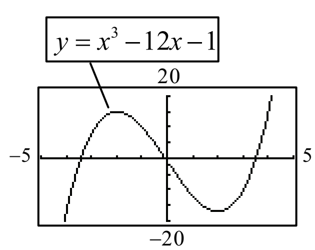
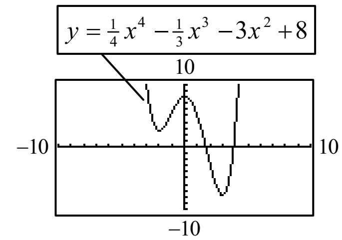


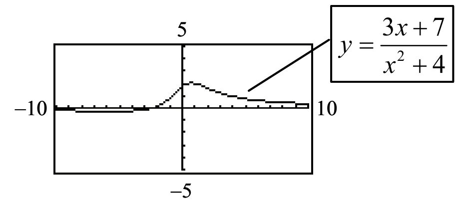
9.



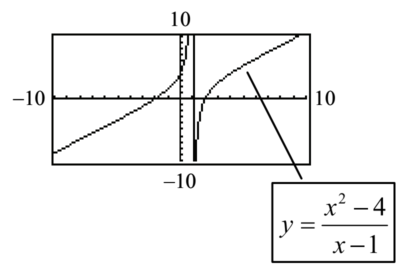
10.



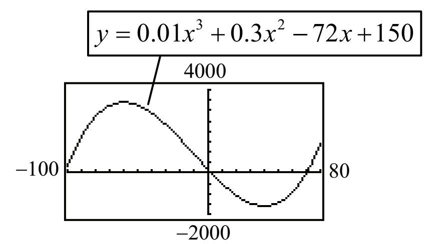
11.



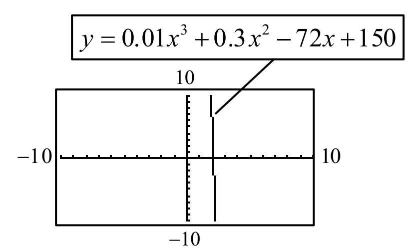
12.



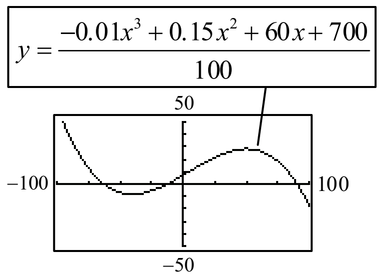
13. a.



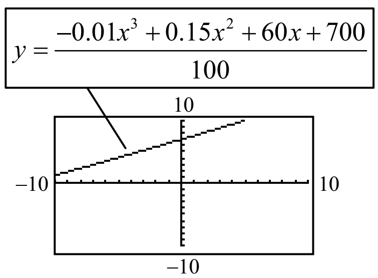
b.



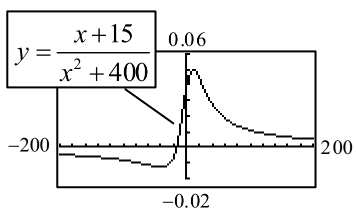
14. a.



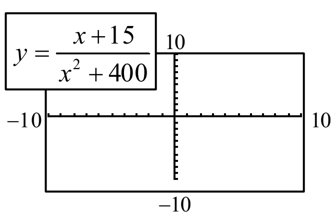
b. Standard viewing window



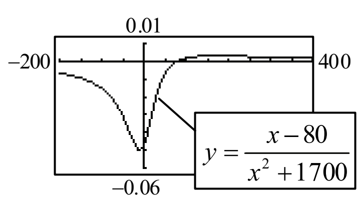
15. a.



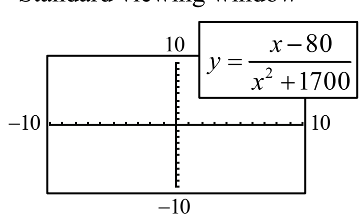
b. Standard Window



16. a.



b. Standard viewing window

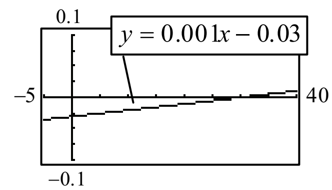


17. a. y-intercept $= -0.03$

x-intercept: $0.001x = 0.03$

$x = 30$

b.



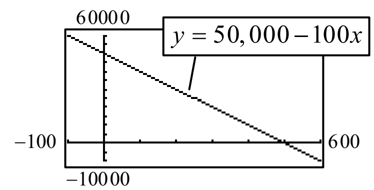
18. $y = 50{,}000 - 100x$

a. The equation is linear, so use the x- and y-intercepts of the line graph to determine an appropriate range. (y-intercept: 50,000; x-intercept: 500)

b. Window: x-min = –100 y-min = –10,000

x-max = 600 y-max = 60,000

Graph using the window in part (b).



19 – 22. Complete graphs can be seen with different windows. A hint is to look at the equation and try to determine the max and/or min of *y*. Also, find the *x*-intercepts.

19. $y = -0.15(x - 10.2)^2 + 10$

There is no min.

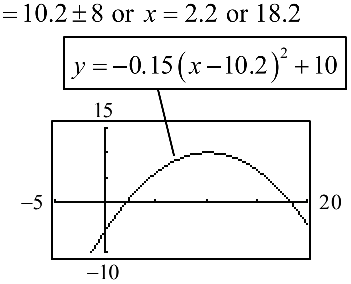
Max value of $y = 10$.

x-intercepts: $0 = -0.15(x - 10.2)^2 + 10$

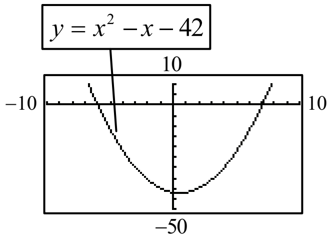
$$(x - 10.2)^2 = \frac{10}{0.15} = 66.66$$

$$x - 10.2 = \pm\sqrt{66.66} \approx \pm 8$$

$x = 10.2 \pm 8$ or $x = 2.2$ or 18.2

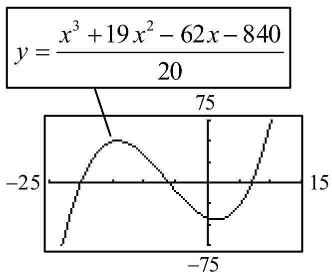


20. A suggested window is shown below.



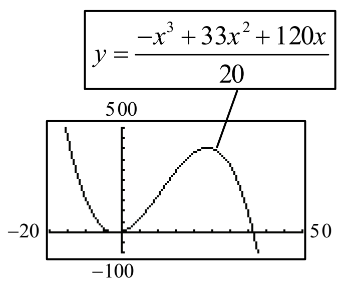
As the graph shows, more of the features of the graph are now visible.

21. If $x = 0$, $y = –42$.
A suggested window is shown below.



22. $y = \dfrac{-x^3 + 33x^2 + 120x}{20}$

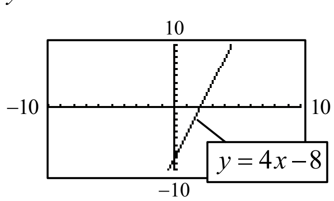
A suggested window is shown below.



As the graph shows, closer detail of the features of the graph are now available.

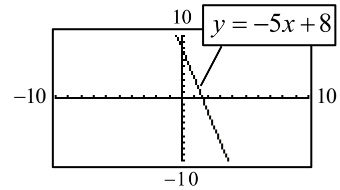
23. $4x - y = 8$

$y = 4x - 8$



24. $5x + y = 8$

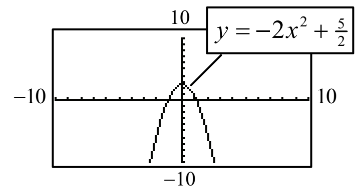
$y = -5x + 8$



25. $4x^2 + 2y = 5$

$$2y = -4x^2 + 5$$

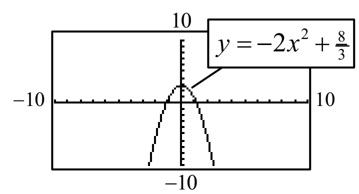
$$y = -2x^2 + \frac{5}{2}$$



26. $6x^2 + 3y = 8$

$$3y = -6x^2 + 8$$

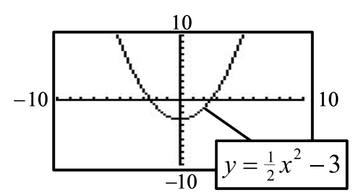
$$y = -2x^2 + \frac{8}{3}$$



27. $x^2 - 6 = 2y$

$$2y = x^2 - 6$$

$$y = \frac{1}{2}x^2 - 3$$

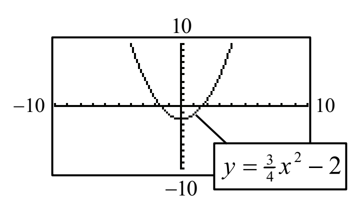


Not linear

28. $3x^2 - 4y = 8$

$$-4y = -3x^2 + 8$$

$$y = \frac{3}{4}x^2 - 2$$



29. $f(x)=x^3-3x^2+2$

$f(-2)=(-2)^3-3(-2)^2+2=-8-12+2=-18$

$f\left(\frac{3}{4}\right)=\left(\frac{3}{4}\right)^3-3\left(\frac{3}{4}\right)^2+2=0.734375$

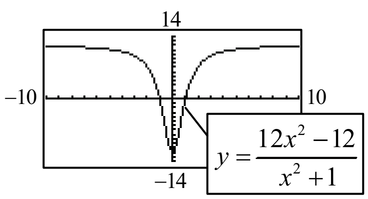
Use your graphing calculator, and evaluate the function at these two points. If either of your answers differ, can you explain the difference?

30. $f(x)=\frac{x^2-2x}{x-1}$

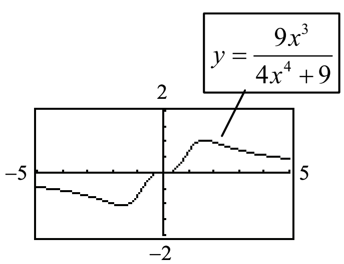
$f(3)=\frac{(3)^2-2(3)}{(3)-1}=\frac{3}{2}$

$f(-4)=\frac{(-4)^2-2(-4)}{-4-1}=-4.8$

31. As x gets large, y approaches 12.
When $x = 0$, $y = -12$, x intercepts at $\pm$ 1.

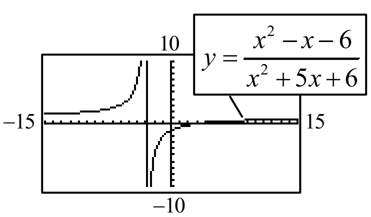


32.

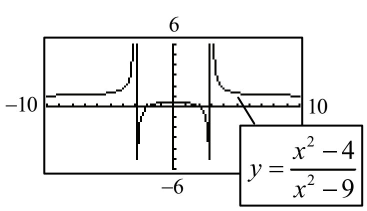


33. $y=\frac{x^2-x-6}{x^2+5x+6}=\frac{(x-3)(x+2)}{(x+3)(x+2)}$

What happens to y as x approaches –3? –2?



34.



35. $6x-21=0$

$6x=21$

$x=\frac{21}{6}=\frac{7}{2}$

36. $12x+28=0$

$x=-\frac{28}{12}=-\frac{7}{3}$

37. $x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=-2$ or 5

38. $6x^2+4x=4$ 6

$6x^2+4x-4=0$

The graphing calculator approximation is
$x=-1.215$, $x=0.549$.

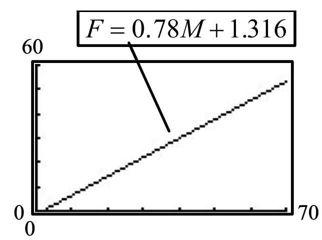
39. *Find the zeros* and *find the x-intercepts* are equivalent statements. Use a graphing calculator's **TRACE** or **ZERO**.

a.-b. Graphing calculator approximation is
$x=-1.1098$, 8.1098 .

40. The x-intercepts and zeros are the same.

a.-b. Graphing calculator approximation is
$x=-1.5495$, 3.5495 .

41. a.

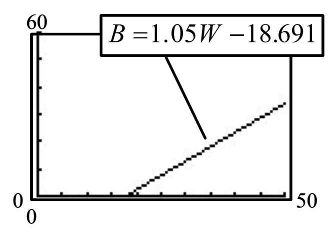


b. If males average \$50,000, then females earn \$37,684.

c. $F(62.5)=0.78(62.5)-1.316$

$=47.434$ thousand $=\$47,434$

42. a.

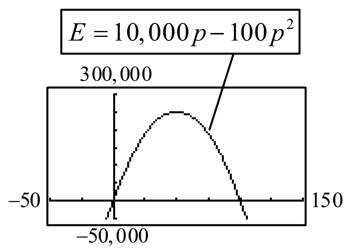


b. When whites average \$50,000 income, blacks only average \$33,809 income.

c. $B(77.5)=1.05(77.5)-18.691$

$=62.684$ thousand = \$62,684

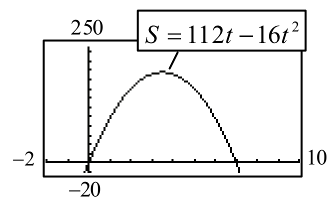
43. a.



b. $E \geq 0$ when $0 \leq p \leq 100$.

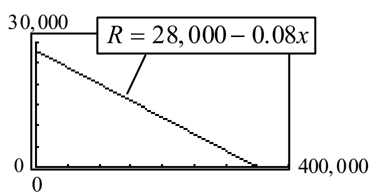
44. $S = 112t - 16t^2$

a.



b. From the graph above, the ball has an estimated maximum height of 196 feet when $t = 3.5$ seconds.

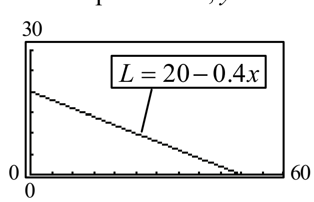
45. a.



b. The rate is -0.08 . As more people become aware of the product, there are fewer to learn about it.

46. $L = 20 - 0.4x$

a. x-intercept: $x = 50$, y-intercept: $y = 20$

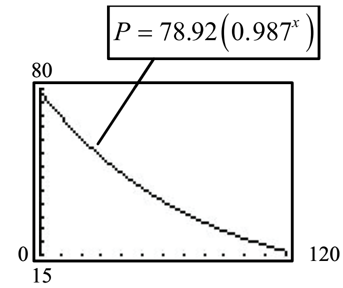


b. The rate is decreasing because the number of words learned is increasing.

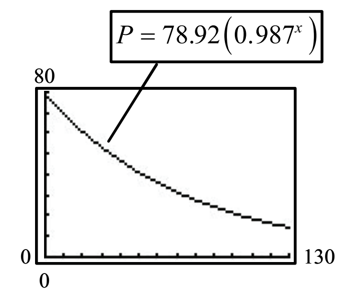
47. a. x-min = 0, x-max = 120

b. y-min = 15, y-max = 80

c.



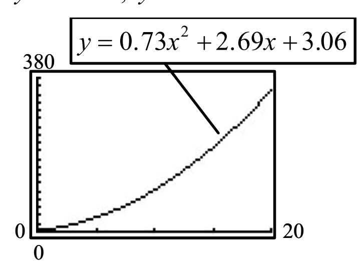
d.



e. The percentage decreases from 78.9% in 1890 to 16.4% in 2010.

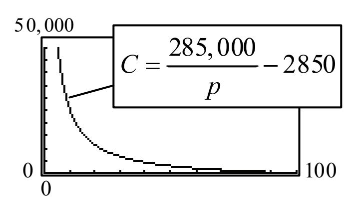
48. a. x-min = 0, x-max = 30
y-min = 0, y-max = 750

b.



c. It continues to increase, but will eventually exceed the population of the U.S.

49. a.

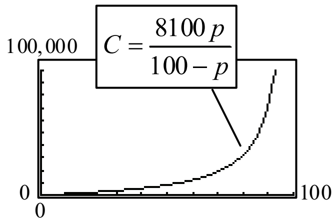


b. Near $p = 0$, cost grows without bound.

c. The coordinates of the point mean that the cost of obtaining stream water with 1% of the current pollution levels would cost \$282,150.

d. The p-intercept means that the cost of stream water with 100% of the current pollution levels would cost \$0.

50. a.

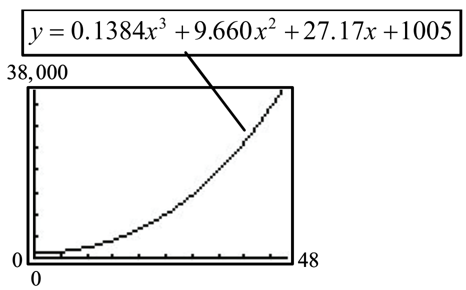


b. C increases rapidly as p gets close to 100.

c. The coordinates (98, 396900) indicate that the cost to remove 98% of the particulate pollution cost $396,900.

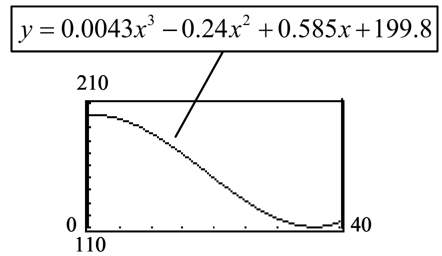
d. The p-intercept is (0, 0). The meaning is that it costs nothing to remove none of the particulate pollution.

51. a.



b. Increasing. The per capita federal tax burden is increasing.

52. a.



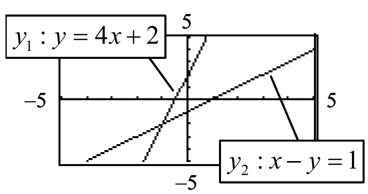
b. $y(30) = 117.39$ millions

$= 117{,}390{,}000$ short tons

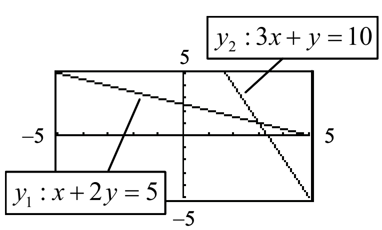
c. It increases dramatically. Planners should account for a huge increase in carbon monoxide.

Exercise 1.5

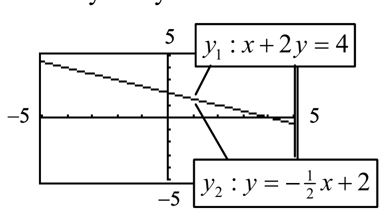
1. Solution: $(-1, -2)$



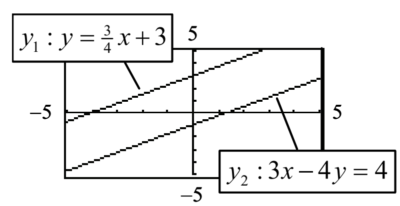
2. Solution: $(3, 1)$



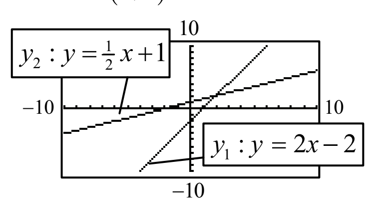
3. Infinitely many solutions.



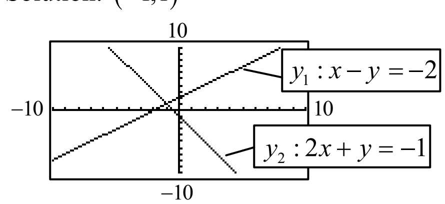
4. No solution, since the graphs do not intersect.



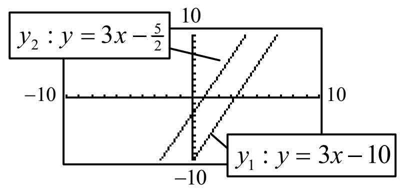
5. Solution: $(2, 2)$



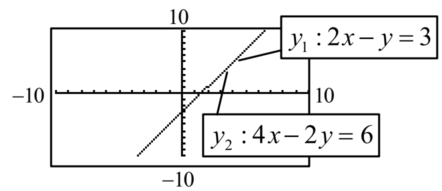
6. Solution: $(-1, 1)$



7. Solution: No solution, since the graphs do not intersect.



8. Solution: Infinitely many solutions.



9. $3x - 2y = 6$

$4y = 8$	Solve for y.	$y = \frac{8}{4} = 2$
	Substitute for this variable in	$3x–2(2)=6$
	first equation and solve for	$3x = 6 + 4 = 10$
	the other variable.	$x = \frac{10}{3}$

The solution of the system is $x = \dfrac{10}{3}$ and $y = 2$, or $\left(\dfrac{10}{3}, 2\right)$.

10. $\begin{cases} 3x = 6 \\ 4x - 3y = 5 \end{cases}$

Substitution: $x = 2$

$$4(2) - 3y = 5$$
$$-3y = -3$$
$$y = 1$$

The solution of the system is $x = 2$, $y = 1$ or $(2,1)$.

11. $2x - y = 2$

$3x + 4y = 6$	Solve for y.	$y = 2x - 2$
	Substitute for this variable in	$3x + 4(2x - 2) = 6$
	second equation and solve for	$3x + 8x - 8 = 6$
	the other variable.	$11x = 14$
		$x = 14/11$

Solve for y: $y = 2\left(\dfrac{14}{11}\right) - 2 = \dfrac{6}{11}$

The solution of the system is $x = \dfrac{14}{11}$ and $y = \dfrac{6}{11}$, or $\left(\dfrac{14}{11}, \dfrac{6}{11}\right)$.

12. $\begin{cases} 4x - y = 3 \\ 2x + 3y = 19 \end{cases}$

Substitution: $y = 4x - 3$

$$2x + 3(4x - 3) = 19$$
$$2x + 12x = 9 + 19$$
$$14x = 28$$
$$x = 2$$
$$y = 4(2) - 3 = 5$$

The solution of the system is $x = 2$ and $y = 5$, or $(2,5)$.

13. $3x+4y=1$ Multiply 1st equation by 3. $9x+12y=3$

$2x-3y=12$ Multiply 2nd equation by 4. $\underline{8x-12y=48}$

Add the two equations. $17x \quad = 51$

Solve for the variable. $x=3$

Substitute for this variable in either original equation and solve for the other variable.

$$3(3)+4y=1$$
$$4y=-8$$
$$y=-2$$

The solution of the system is $x=3$ and $y=-2$, or $(3,-2)$.

14.

$$\begin{cases}5x-2y=4\\2x-3y=5\end{cases} \rightarrow \begin{array}{r}10x-4y=8\\ \underline{-10x+15y=-25}\\ 11y=-17\\ y=-\frac{17}{11}\end{array}$$

$$2x-3\left(-\frac{17}{11}\right)=5 \rightarrow 2x+\frac{51}{11}=5 \rightarrow 2x=\frac{4}{11} \rightarrow x=\frac{2}{11}$$

The solution of the system is $x=\frac{2}{11}$ and $y=-\frac{17}{11}$, or $\left(\frac{2}{11},-\frac{17}{11}\right)$.

15. $-4x+3y=-5$ Multiply first equation by 3. $-12x+9y=-15$

$3x-2y=4$ Multiply second equation by 4. $\underline{12x-8y=16}$

Add the two equations. $y=1$

Substitute for this variable in either original equation and solve for the other variable.

$$-4x+3(1)=-5$$
$$-4x=-8$$
$$x=2$$

The solution of the system is $x=2$ and $y=1$.

16.

$$\begin{cases}x+2y=3\\3x+6y=6\end{cases} \rightarrow \begin{array}{r}3x+6y=9\\ \underline{-3x-6y=-6}\\ 0\neq 3\end{array}$$

No solution.

17. $0.2x-0.3y=4$ $\qquad$ $0.20x-0.3y=4$

$2.3x- \quad y=1.2$ Multiply 2nd equation by 0.3. $\underline{0.69x-0.3y=0.36}$

Subtract the two equations. $-0.49x \quad =3.64$

Solve for the variable. $x=-\frac{52}{7}$

Substitute, solve for y. $y=-\frac{128}{7}$

The solution of the system is $x=-\frac{52}{7}$ and $y=-\frac{128}{7}$, or $\left(-\frac{52}{7},-\frac{128}{7}\right)$.

18.

$$\begin{cases}0.5x+y=3\\0.3x+0.2y=6\end{cases} \rightarrow \begin{array}{r}-0.5x-y=-3\\ \underline{1.5x+y=30}\\ x \quad =27\end{array}$$

$$0.5(27)+y=3$$
$$13.5+y=3$$
$$y=-10.5$$

The solution of the system is $x=27$ and $y=-10.5$, or $(27,-10.5)$.

19. $\frac{5}{2}x - \frac{7}{2}y = -1$ Multiply first equation by 6. $15x - 21y = -6$
$8x + 3y = 11$ Multiply second equation by 7. $\underline{56x + 21y = 77}$
Add the two equations. $71x \quad = 71$
Substitute for this variable in $x = 1$
either original equation and $8(1) + 3y = 11$
solve for the other variable. $3y = 3$
$y = 1$

The solution of the system is $x = 1$ and $y = 1$, or (1, 1).

20.
$$\begin{cases} x - \frac{1}{2}y = 1 \\ \frac{2}{3}x - \frac{1}{3}y = 1 \end{cases} \to \begin{array}{r} -2x + y = -2 \\ \underline{2x - y = 3} \\ 0 \neq 1 \end{array}$$

No solution.

21. $4x + 6y = 4$ $\qquad 4x + 6y = 4$
$2x + 3y = 2$ Multiply second equation by -2. $\underline{-4x - 6y = -4}$
Add the two equations: $0 = 0$

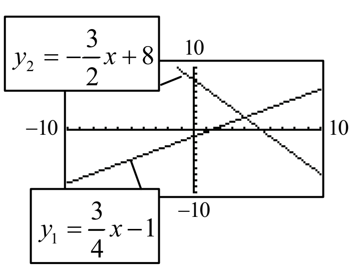
There are infinitely many solutions. The system is dependent. Solve for one of the variables in terms of the remaining variable: $y = \frac{2}{3} - \frac{2}{3}x$. Then a general solution is $\left(c, \frac{2}{3} - \frac{2}{3}c\right)$, where any value of c will give a particular solution.

22.
$$\begin{cases} 6x - 4y = 16 \\ 9x - 6y = 24 \end{cases} \to \begin{array}{r} -36x + 24y = -96 \\ \underline{36x - 24y = 96} \\ 0 = 0 \end{array}$$

There are infinitely many solutions. The system is dependent.

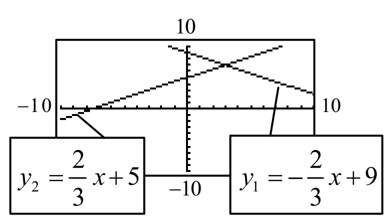
23–26 Use the standard window and graph each equation. Use the TRACE or INTERSECT feature to find the solution.

23.
$$\begin{cases} y = 8 - \frac{3x}{2} \\ y = \frac{3x}{4} - 1 \end{cases}$$



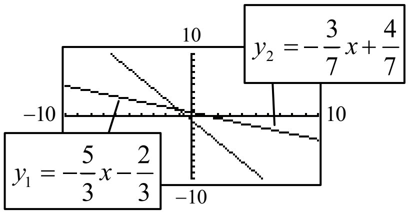
Solution: $(4, 2)$

24.
$$\begin{cases} y = 9 - \frac{2x}{3} \\ y = 5 + \frac{2x}{3} \end{cases}$$



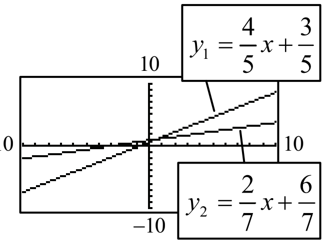
Solution: (3, 7)

25. $\begin{cases} y_1: 5x+3y=-2 \\ y_2: 3x+7y=4 \end{cases}$



Solution: (–1, 1)

26. $\begin{cases} y_1: 4x-5y=-3 \\ y_2: 2x-7y=-6 \end{cases}$



Solution: $\left(\frac{1}{2}, 1\right)$

27. Eq. 1 $\quad x+2y+z=2$ $\quad$ Steps 1, 2, and 3 of the systematic
Eq. 2 $\quad -y+3z=8$ $\quad$ procedure are completed.
Eq. 3 $\quad 2z=10$ $\quad$ Step 4: $z=5$
From Eq. 2 $\quad -y+3(5)=8$ or $y=7$
From Eq. 1 $\quad x+2(7)+5=2$ or $x=-17$
The solution is $x=-17$, $y=7$, $z=5$.

28.
$$\begin{aligned} x-2y+2z&=-10 \\ y+4z&=-10 \\ -3z&=9 \\ z&=-3 \end{aligned} \qquad \begin{aligned} y+4(-3)&=-10 \\ y-12&=-10 \\ y&=2 \end{aligned} \qquad \begin{aligned} x-2(2)+2(-3)&=-10 \\ x-4-6&=-10 \\ x-10&=-10 \\ x&=0 \end{aligned}$$
Solution: (0, 2, –3)

29. Eq. 1 $\quad x-y-8z=0$ $\quad$ Steps 1 and 2 of the systematic
Eq. 2 $\quad y+4z=8$ $\quad$ procedure are completed.
Eq. 3 $\quad 3y+14z=22$
Step 3: $(-3)\times$ Eq 2 added to Eq. 3 gives $2z=-2$ or $z=-1$.
From Eq. 2 $\quad y+4(-1)=8$ or $y=12$
From Eq. 1 $\quad x-12-8(-1)=0$ or $x=4$
The solution is $x=4$, $y=12$, $z=-1$ or (4, 12, –1).

30.
$$\begin{aligned} x+3y-8z=20 &\to x+3y-8z=20 \to x+3y-8z=20 \\ y-3z=11 &\to 2y-6z=22 \to 2y-6z=22 \\ 2y+7z=-4 &\to 2y+7z=-4 \to -13z=26 \to z=-2 \end{aligned}$$
$$\begin{aligned} 2y-6(-2)&=22 \\ 2y+12&=22 \\ 2y&=10 \\ y&=5 \end{aligned} \qquad \begin{aligned} x+3(5)-8(-2)&=20 \\ x+15+16&=20 \\ x+31&=20 \\ x&=-11 \end{aligned}$$
Solution: (–11, 5, –2)

31. Eq. 1 $x+4y-2z=9$ Step 1 is completed.

Eq. 2 $x+5y+2z=-2$

Eq. 3 $x+4y-28z=22$

Step 2:

$x+4y-2z=9$ Eq. 1

Eq. 4 $y+4z=-11$ $(-1)\times$ Eq. 1 added to Eq. 2

Eq. 5 $-26z=13$ $(-1)\times$ Eq. 1 added to Eq. 3

Step 3 is also completed.

Step 4: $z=-\dfrac{1}{2}$ from Eq. 5.

From Eq. 4 $y+4\left(-\dfrac{1}{2}\right)=-11$ or $y=-9$

From Eq. 1 $x+4(-9)-2\left(-\dfrac{1}{2}\right)=9$ or $x=44$

The solution is $x=44$, $y=-9$, $z=-\dfrac{1}{2}$ or $\left(44,-9,-\dfrac{1}{2}\right)$.

32.

$$\begin{array}{rcrcrcr}
x-3y-z=0 & \rightarrow & x-3y-z=0 & \rightarrow & x-3y-z=0 & \rightarrow & x-3y-z=0 \\
x-2y+z=8 & \rightarrow & x-2y+z=8 & \rightarrow & y+2z=8 & \rightarrow & y+2z=8 \\
2x-6y+z=6 & \rightarrow & -2y-z=-10 & \rightarrow & -2y-z=-10 & \rightarrow & 3z=6 \\
 & & & & & \rightarrow & z=2
\end{array}$$

$$\begin{array}{rr}
y+2(2)=8 & x-3(4)-2=0 \\
y+4=8 & x-12-2=0 \\
y=4 & x-14=0 \\
 & x=14
\end{array}$$

Solution: (14, 4, 2)

33. $f(x)=40.74x+742.65,\ h(x)=47.93x+725$

$40.74x+742.65=47.93x+725$

$17.65=7.19x$

$x=2.455$ during 1998

$h(2.455)=f(2.455)=\$842.67$ billion

34. $B(x)=0.081x+8.97,\quad H(x)=0.282x+3.10$

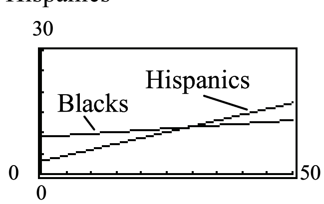
a. $0.081x+8.97=0.282x+3.10$

$5.87=0.201x$

$x=29.2$ in 2000

$H(29.2)=B(29.2)=11.3$ million of each

b. Hispanics



35. **a.** $x + y = 1800$ Total number of tickets

b. $20x =$ revenue from \$20 tickets

c. $30y =$ revenue from \$30 tickets

d. $20x + 30y = 42,000$ Total Revenue

e. Multiply equation from part (a) by -20.

$$\begin{array}{r} -20x - 20y = -36000 \\ \underline{20x + 30y = 42000} \\ 10y = 6000 \\ y = 600 \end{array}$$

Substitution into equation from part (a) gives $x = 1200$.
Sell 1200 of the \$20 tickets and 600 of the \$30 tickets.

36. $x =$ amount invested at 10%, $y =$ amount invested at 12%

a. $x + y = 500,000$

b. $0.10x$

c. $0.12y$

d. $0.10x + 0.12y = 53,000$

e.
$$\begin{array}{r} -x - y = -500,000 \\ \underline{x + 1.2y = 530,000} \\ 0.2y = 30,000 \\ y = \$150,000 \\ x + 150,000 = 500,000 \\ x = \$350,000 \end{array}$$

Invest \$350,000 at 10% and \$150,000 at 12%.

37. x = amount of safe investment.
y = amount of risky investment.
$x + y = 145,600$ Total amount invested
$0.1x + 0.18y = 20,000$ Income from investments
The solution is the solution of the above system of equations.

$$\begin{array}{rl} x + \quad y = 145,600 & \\ \underline{x + 1.8y = 200,000} & (10) \times \text{ second equation} \\ 0.8y = 54,400 & \text{Subtract equations} \\ y = 68,000 & \text{Solve for } y \text{ or amount of risky investment.} \end{array}$$

Substituting $y = 68,000$ into one of the original equations we have $x + 68,000 = 145,600$ or $x = \$77,600$.
Solution: Put \$77,600 in a safe investment and \$68,000 in a risky investment.

38. Let A = loan for product A and
B = loan for product B.

$$\begin{array}{lcll} A + B = 237,000 & \rightarrow & A + B = 237,000 & 153,000 + B = 237,000 \\ A = 69,000 + B & \rightarrow & \underline{A - B = 69,000} & B = \$84,000 \\ & & 2A \quad = 306,000 & \\ & & A \quad = \$153,000 & \end{array}$$

39. x = amount invested at 10%.
y = amount invested at 12%.

$$\begin{array}{lcrl}
x + y = 470{,}000 & \rightarrow & x + \quad y = 470{,}000 & x + 200{,}000 = 470{,}000 \\
0.10x + 0.12y = 51{,}000 & \rightarrow & \underline{-x - 1.2y = -510{,}000} & x = 270{,}000 \\
 & & -0.2y = -40{,}000 & \\
 & & y = \$200{,}000 &
\end{array}$$

40. Let B = amount borrowed from bank and
L = amount borrowed from life insurance.

$$\begin{aligned}
B + L &= 100{,}000 \\
\underline{B + 1.2L} &\underline{= 109{,}000} \\
0.2L &= 9{,}000 \\
L &= \$45{,}000 \\
B + 45{,}000 &= 100{,}000 \\
B &= \$55{,}000
\end{aligned}$$

\$55,000 borrowed from the bank and \$45,000 borrowed from life insurance.

41. A = ounces of substance A.
B = ounces of substance B.

Required ratio $\frac{A}{B} = \frac{3}{5}$ gives $5A - 3B = 0$.

Required nutrition is $5\%A + 12\%B = 100\%$. This gives $5A + 12B = 100$.
The % notation can be trouble. Be careful! Now we can solve the system.

$$\begin{aligned}
5A - \ 3B &= 0 \\
\underline{5A + 12B} &\underline{= 100} \\
15B &= 100 \qquad \text{Subtract first equation from second.} \\
B &= \frac{100}{15} = \frac{20}{3}
\end{aligned}$$

Substituting into the original equation gives $5A - 3\left(\frac{20}{3}\right) = 0$ or $A = 4$.

The solution is 4 ounces of substance A and $6\frac{2}{3}$ ounces of substance B.

42. x = number of glasses of milk
y = number of quarter-pound servings of meat

$$\begin{array}{lclr}
0.1x + 3.4y = 7.15 & \rightarrow & x + 34y = 71.5 & \text{Substitution: } x = 71.5 - 34y \\
8.5x + 22y = 73.75 & \rightarrow & 8.5x + 22y = 73.75 & 8.5(71.5 - 34y) + 22y = 73.75 \\
 & & & 607.75 - 289y + 22y = 73.75 \\
 & & & 607.75 - 267y = 73.75 \\
 & & & -267y = -534 \\
 & & & y = 2
\end{array}$$

$x = 71.5 - 34(2) = 71.5 - 68 = 3.5$
The proper nutrition would be provided with 3.5 glasses of milk and 2 servings of meat.

43. x = population of species A.
y = population of species B.
$2x + y = 10{,}600$ units of first nutrient
$3x + 4y = 19{,}650$ units of second nutrient

$$\begin{array}{ll} 8x+4y=42{,}400 & (4)\times \text{ first equation} \\ \underline{3x+4y=19{,}650} & \\ 5x \quad = 22{,}750 & \text{Subtract} \\ x \quad = 4550 & \text{Solve for } x \end{array}$$

Substituting $x = 4550$ into an original equation we have $2(4550) + y = 10{,}600$.
So, $y = 1500$. Solution is 4550 of species A and 1500 of species B.

44. x = number of cubic centimeters of 40% solution
y = number of cubic centimeters of 10% solution

$$\begin{array}{rclcl} x+y=25 & \to & x+y=25 & & \text{Substitution: } x=25-y \\ 0.40x+0.10y=0.28(25) & \to & 0.40x+0.10y=7 & & 0.40(25-y)+0.10y=7 \\ & & & & 10-0.40y+0.10y=7 \\ & & & & 10-0.30y=7 \\ & & & & -0.30y=-3 \\ & & & & y=10 \end{array}$$

The biologist should mix 10 cc of 10% solution with 15 cc of 40% solution.

45. x = amount of 20% concentration.
y = amount of 5% concentration.
$x + y = 10$ amount of solution
$0.20x + 0.05y = 0.155(10)$ concentration of medicine
Solving this system of equations:

$$\begin{array}{ll} x+\quad y=10 & \\ \underline{x+0.25y=7.75} & (5)\times \text{ second equation} \\ 0.75y=2.25 & \text{Subtract equations} \\ y=3 & \text{Solve for } y \end{array}$$

Substituting into the first equation we have $x + 3 = 10$ or $x = 7$.
The solution is 3 cc of 5% concentration and 7 cc of 20% concentration.

46. A = dosage of medication A
B = dosage of medication B

$$\begin{array}{lclcl} 8A=5B & \to & 8A-5B=0 & \to & 24A-15B=0 \\ 6A+2B=50.6 & \to & 6A+2B=50.6 & \to & \underline{24A+\ 8B=202.4} \\ & & & & 23B=202.4 \\ & & & & B=8.8 \end{array}$$

$$\begin{aligned} 8A-5(8.8)&=0 \\ 8A-44&=0 \\ 8A&=44 \\ A&=5.5 \end{aligned}$$

Each dosage of medication should be 5.5 mg of A and 8.8 mg of B.

47. x = number of \$40 tickets.
y = number of \$60 tickets.

$$\begin{array}{lll} x+y=16{,}000 & \rightarrow & -40x-40y=-640{,}000 \quad x+6{,}000=16{,}000 \\ 40x+60y=760{,}000 & \rightarrow & \underline{\;40x+60y=\;\;760{,}000} \qquad\qquad x=10{,}000 \\ & & \qquad\quad 20y=\;\;120{,}000 \\ & & \qquad\quad\;\; y=\;\;6{,}000 \end{array}$$

48. x = lbs of peanuts
y = lbs of cashews

$$\begin{array}{ll} x+y=100 & \text{Substitution: } x=100-y \\ 2.80x+5.30y=3.30(100) & 2.80(100-y)+5.30y=330 \\ & 280-2.80y+5.30y=330 \\ & 2.50y=50 \\ & y=20 \end{array}$$

The wholesaler should mix 20 pounds of cashews with 80 pounds of peanuts.

49. x = amount of 20% solution to be added.
$0.20x$ = concentration of nutrient in 20% solution.
0.02(100) = 2 is the concentration of nutrient in 2% solution.

$$0.20x+2=0.10(x+100)$$
$$0.20x+2=0.10x+10$$
$$0.1x=8 \text{ or } x=80\text{cc of 20\% solution is needed.}$$

50. Let x = the number of gallons of 13.5% washer fluid.

$$0.135x+0.11(200)=0.13(x+200)$$
$$0.135x+22=0.13x+26$$
$$0.005x=4$$
$$x=800 \text{ gallons}$$

51. x = ounces of substance A,
y = ounces of substance B, and
z = ounces of substance C.

$$\begin{array}{ll} 5x+15y+12z=100 & \text{Nutrition requirements} \\ x=z & \text{Digestive restrictions} \\ y=\frac{1}{5}z & \text{Digestive restrictions} \end{array}$$

Since both x and y are in terms of z, we can substitute in the first equation and solve for z.
So, $5z + 3z + 12z = 100$ or $20z = 100$. So, $z = 5$. Now, since $x = z$, we have $x = 5$.

Since $y=\dfrac{1}{5}z$, we have $y = 1$. The solution is 5 ounces of substance A, 1 ounce of substance B, and 5 ounces of substance C.

52. Let x = the number of glasses of skim milk

y = the number of $\frac{1}{4}$ lb servings of meat

z = the number of 2-slice servings of bread.

$$\begin{array}{lcl} 0.1x+3.4y+2.2z=10.5 & \to & x+34y+22z=105 & \to & x+34y+22z=105 \\ 8.5x+22y+10z=94.5 & \to & 85x+220y+100z=945 & \to & 14y+10z=44 \\ x+20y+12z=61 & \to & x+20y+12z=61 & \to & -2670y-1770z=-7980 \end{array}$$

$$\begin{array}{lcl} x+34y+22z=105 & & x+34y+22z=105 & & \\ y+\frac{5}{7}z=\frac{22}{7} & \to & y+\frac{5}{7}z=\frac{22}{7} & \to & z=3;\ y=\frac{22}{7}-\frac{5}{7}(3)=1;\ x=105-34-66=5 \\ -2670y-1770z=-7980 & & \frac{960}{7}z=\frac{2880}{7} & & \end{array}$$

The solution is: (5, 1, 3).

The requirements will be met with 5 glasses of milk, 1 serving of meat and 3 servings of bread.

53. A = number of A type clients.
B = number of B type clients.
C = number of C type clients.

$A + B + C = 500$	Total clients
$200A + 500B + 300C = 150{,}000$	Counseling costs
$300A + 200B + 100C = 100{,}000$	Food and shelter

To find the solution we must solve the system of equations.

Eq. 1	$A+B+C=500$	
Eq. 2	$2A+5B+3C=1500$	Original equation divided by 100
Eq. 3	$3A+2B+C=1000$	Original equation divided by 100

	$A+B+C=500$	Eq. 1
Eq. 4	$3B+C=500$	$(-2)\times$ Eq. 1 added to Eq. 2
Eq. 5	$-B-2C=-500$	$(-3)\times$ Eq. 1 added to Eq. 3

$A+B+C=500$	Eq. 1
$3B+C=500$	Eq. 4
$-\frac{5}{3}C=\frac{-1000}{3}$	$\frac{1}{3}\times$ Eq. 4 added to Eq. 5
$C=\frac{1000}{3}\cdot\frac{3}{5}=200$	

Substituting C = 200 into Eq. 4 gives $3B$ + 200 = 500 or $3B$ = 300. So, B = 100.
Substituting C = 200 and B = 100 into Eq. 1 gives A + 100 + 200 = 500. So, A = 200.
Thus, the solution is 200 type A clients, 100 type B clients, and 200 type C clients.

54. A = number of type A clients
B = number of type B clients
C = number of type C clients

$$200A+500B+300C=135{,}000$$

$$300A+200B+100C=90{,}000 \quad \begin{cases} A+B+C=450 \\ 300B+100C=45{,}000 \\ -100B-200C=-45{,}000 \end{cases} \quad \begin{cases} A+B+C=450 \\ B+2C=450 \\ -500C=-90{,}000 \end{cases}$$

$$A+B+C=450 \qquad \to \qquad C=180$$

Substitution gives 180 type A clients, 90 type B clients, and 180 type C clients.

Exercise 1.6

1. **a.** $P(x) = R(x) - C(x)$
$= 68x - (34x + 6800)$
$= 34x - 6800$

b. $P(3000) = 34(3000) - 6800 = \$95,200$

2. **a.** $P(x) = R(x) - C(x)$
$= 430x - (210x + 3300)$
$= 220x - 3300$

b. $P(500) = 220(500) - 3300 = \$106,150$

3. **a.** $P(x) = R(x) - C(x)$
$= 80x - (43x + 1850)$
$= 37x - 1850$

b. $P(30) = 37(30) - 1850 = -\$740$
The total costs are more than the revenue.

c. $P(x) = 0$ or $37x - 1850 = 0$
So, $x = \dfrac{1850}{37} = 50$ units is the break-even point.

4. **a.** $P(x) = R(x) - C(x)$
$= 385x - (85x + 3300)$
$= 300x - 3300$

b. $P(351) = 300(351) - 3300 = \$102,000$

c. To avoid losing money, profit must be at least 0.
$0 = 300x - 3300$
$3300 = 300x$
$x = 11$

5. $C(x) = 5x + 250$
a. $m = 5$, C-intercept: 250
b. $\overline{MC} = 5$ means that each additional unit produced costs \$5.
c. Slope = marginal cost.
C-intercept =fixed costs.
d. \$5, \$5 ($\overline{MC} = 5$ at every point)

6. $C(x) = 27.55x + 5180$
a. $m = 27.55$, $b = 5180$ (C-intercept)
b. Marginal cost = \$27.55. The cost of each additional unit is \$27.55.
c. $m = \overline{MC} = 27.55$
C-intercept = FC = \$5180
d. Regardless of the production level, the cost of each additional unit is \$27.55.

7. $R = 27x$
a. $m = 27$
b. 27; each additional unit sold yields \$27 in revenue.
c. In each case, one more unit yields \$27.

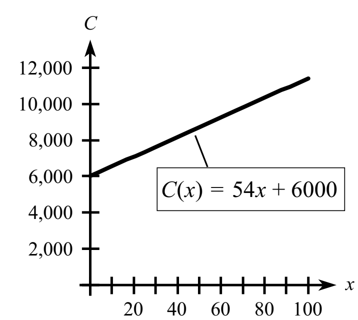
8. $R = 38.95x$
a. $m = 38.95$
b. $\overline{MR} = 38.95$. Each additional unit sold adds \$38.95 to the total revenue.
c. The revenue from each additional unit sold is \$38.95 whether 50 are currently being sold or 100 are currently being sold.

9. $R(x) = 27x$, $C(x) = 5x + 250$
a. $P(x) = 27x - (5x + 250) = 22x - 250$
b. $m = 22$
c. Marginal profit is 22.
d. Each additional unit sold gives a profit of \$22. To maximize profit sell all that you can produce. Note that this is not always true.

10. $P(x) = R(x) - C(x)$
$= 20x - (21.95x + 1400)$
$= -1.95x - 1400$
a. $\overline{MP} = -1.95$ so the company is losing money on every item produced and sold.
b. Stop production, $P(x)$ is never positive.

11. (x, P) is the correct form.
$P_1 = (200,\ 3100)$
$P_2 = (250,\ 6000)$
$m = \dfrac{6000 - 3100}{250 - 200} = 58$
$P - 3100 = 58(x - 200)$ or $P = 58x - 8500$
The marginal profit is 58.

12. $C = 54x + b$, use the fact that (50, 8700) is on the line to solve for b, the fixed costs.
$8700 = 54(50) + b$
$b = 6000$
The cost function is $C(x) = 54x + 6000$.



13. **a.** $TC = 35H + 6600$

b. $TR = 60H$

c. $P = R - C$

$= 60H - (35H + 6600)$

$= 25H - 6600$

d. $C(200) = 35(200) + 6600$

$= \$13,600$ cost of 200 helmets

$R(200) = 60(200)$

$= \$12,000$ revenue from 200 helmets

$P(200) = R(200) - C(200)$

$= \$12,000 - 13,600$

$= -\$1600$ loss from 200 helmets

e. $C(300) = 35(300) + 6600$

$= \$17,100$ cost of 300 helmets

$R(300) = 60(300)$

$= \$18,000$ revenue from 300 helmets

$P(300) = R(300) - C(300)$

$= 18,000 - 17,100$

$= \$900$ profit from 300 helmets

f. The marginal profit is \$25. Each additional helmet sold gives a profit of \$25.

14. **a.** $C(x) = 65x + 9800$

b. $R(x) = 100x$

c. $P(x) = 100x - (65x + 9800) = 35x - 9800$

d. $C(250) = 65(250) + 9800 = \$26,050$

$R(250) = \$25,000$

$P(250) = 35(250) - 9800 = -\$1050$

The sale of 250 units gives revenue of \$25,000 at a cost of \$26,050. This results in a loss of \$1050.

e. $C(400) = 65(400) + 9800 = \$35,800$

$R(400) = \$40,000$

$P(400) = 35(400) - 9800 = \$4200$

The sale of 400 units gives revenue of \$40,000 at a cost of \$35,800. This results in a profit of \$4200.

f. The marginal profit is \$35. Each additional unit sold increases the profit \$35.

15. **a.** The revenue function is the graph that passes through the origin.

b. At a production of zero the fixed costs are \$2000.

c. From the graph, the break-even point is 400 units and \$3000 in revenue or costs.

d. Marginal cost $= \dfrac{3000 - 2000}{400 - 0} = 2.5$

Marginal revenue $= \dfrac{3000 - 0}{400 - 0} = 7.5$

16. $R(x) = 81.50x$, $C(x) = 63x + 1850$
At the break-even point, $R(x) = C(x)$, so

$81.50x = 63x + 1850$

$18.50x = 1850$

$x = 100$ units

17. $R(x) = C(x) = 85x = 35x + 1650$ or $50x = 1650$ or $x = 33$.
Thus, 33 necklaces must be sold to break even.

18. $R(x) = 89x$, $C(x) = 1400 + 75x$
At the break-even point, $R(x) = C(x)$, so

$89x = 1400 + 75x$

$14x = 1400$

$x = 100$ sets of recaps

19. **a.** $R(x) = 12x$, $C(x) = 8x + 1600$

b. $R(x) = C(x)$ if $12x = 8x + 1600$ or $4x = 1600$ or $x = 400$.
It takes 400 units to break even.

20. **a.** $R(x) = 50x$
$C(x) = 30x + 10,000$

b. At the break-even point, $R(x) = C(x)$, so

$50x = 30x + 10,000$

$20x = 10,000$

$x = 500$ watches

21. **a.** $P(x) = R(x) - C(x)$

$= 12x - (8x + 1600)$

$= 4x - 1600$

b. By setting $P(x) = 0$ we get $x = 400$.
Same as 19(b).

22. **a.** $P(x) = R(x) - C(x)$

$= 50x - (30x + 10,000)$

$= 20x - 10,000$

b. $20x - 10,000 = 0$

$20x = 10,000$

$x = 500$

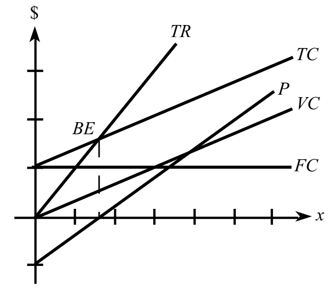
Same as 20(b).

23. **a.** $TC = 4.50x + 1045$
b. $TR = 10x$
c. $P = R - C$
$= 10x - (4.50x + 1045)$
$= 5.50x - 1045$
d. Breakeven also means $P = 0$.
$5.50x - 1045 = 0$
$5.50x = 1045$
$x = 190$ units to break even

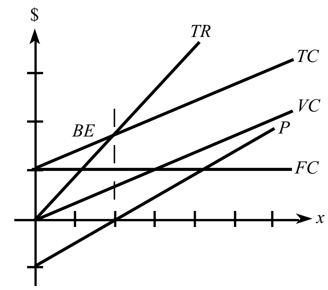
24. **a.** $C(x) = 0.80x + 1245$
b. $R(x) = 4.95x$
c. $P(x) = 4.95x - (0.80x + 1245)$
$= 4.15x - 1245$
d. From $4.95x = 0.80x + 1245$, we have $x = 300$ units to break even.

25. **a.** $R(x) = 54.90x$
b. $P_1 = (2000, 50000)$
$P_2 = (800, 32120)$
$$m = \frac{32{,}120 - 50{,}000}{800 - 2000} = \frac{-17{,}880}{-1200} = 14.90$$
$y - 50{,}000 = 14.90(x - 2000)$ or
$y = 14.90x + 20{,}200 = C(x)$
c. From $54.90x = 14.90x + 20{,}200$ we have $x = 505$ units to break even.

26. **a.** $R(x) = 50x$
b. We have points (100, 4360) and (250, 7060) on the cost function line.
$$m = \frac{7060 - 4360}{250 - 100} = \frac{2700}{150} = 18 = \overline{MC}$$
$C(x) = mx + b$
$4360 = 18(100) + b$
$4360 = 1800 + b$
$2560 = b =$ fixed costs
$C(x) = 18x + 2560$
c. At the break-even point, $R(x) = C(x)$, so
$50x = 18x + 2560$
$32x = 2560$
$x = 80$

27. **a.**


b. TR starts at the origin and intersects TC at the break-even (BE). FC is a horizontal line from the vertical intercept of TC. VC starts at the origin and is parallel to TC.

28. **a.**


b. The upper line must be TC. The horizontal line for FC is drawn from the y-intercept of TC. VC starts at the origin and is parallel to TC. The lower line on the original graph must be P. The x-value of the BE occurs at the point where P crosses x-axis. You can then mark this point on R (using the same x-value) and use it to draw TR (going through the origin and BE).

29. If price increases, then the demand for the product decreases.

30. If the price increases, then the supply will increase.

31. **a.** If $p = \$100$, then $q = 600$ (approximately).
b. If $p = \$100$, then $q = 300$.
c. There is a shortage since more is demanded.

32. **a.** If $p = \$200$, then $q = 400$.
b. If $p = \$200$, then $q = 700$.
c. There will be a surplus since more is supplied.

33. Demand: $2p+5q=200$

$$2(60)+5q=200$$
$$5q=80$$
$$q=16$$

Supply: $p-2q=10$

$$60-2q=10$$
$$2q=50$$
$$q=25$$

There will be a surplus of 9 units at a price of \$60.00.

34. Demand: $p+2q=100$

$$14+2q=100$$
$$2q=86$$
$$q=43$$

Supply: $35p-20q=350$

$$35(14)-20q=350$$
$$-20q=-140$$
$$q=7$$

There will be a shortage at a price of \$14.

35. Remember that (q, p) is the correct form.

$P_1=(240, 900)$

$P_2=(315, 850)$

$$m=\frac{850-900}{315-240}=-\frac{50}{75}=-\frac{2}{3}$$

Note: $m<0$ for demand equations.

$$p-900=-\frac{2}{3}(q-240) \text{ or}$$
$$p=-\frac{2}{3}q+1060$$

36. (q, p) is the correct form.

$P_1=(2500, 1)$

$P_2=(3500, 0.90)$

$$m=\frac{0.90-1}{3500-2500}=\frac{-0.1}{1000}=-0.0001$$
$$p-1=-0.0001(q-2500)$$
$$p-1=-0.0001q+0.25$$
$$p=-0.0001q+1.25$$

37. (q, p) is the correct form.

$P_1=(10000, 1.50)$

$P_2=(5000, 1.00)$

$$m=\frac{1-1.50}{5000-10000}=\frac{-0.50}{-5000}=0.0001$$

Note: $m>0$ for supply equations.

$p-1=0.0001(q-5000)$ or $p=0.0001q+0.5$

38. (q, p) is the correct form.

$P_1=(100,000, 30)$

$P_2=(80,000, 25)$

$$m=\frac{25-30}{80,000-100,000}=0.00025$$
$$p-30=0.00025(q-100,000)$$
$$p-30=0.00025q-25$$
$$p=0.00025q+5$$

39. **a.** The decreasing function is the demand curve. The increasing function is the supply curve.

b. Reading the graph, we have equilibrium at $q=30$ and $p=25$.

40. **a.** 20

b. 40

c. Surplus of 20 (40 – 20 = 20)

41. **a.** Reading the graph, at $p=20$ we have 20 units supplied.

b. Reading the graph, at $p=20$ we have 40 units demanded.

c. At $p=20$ there is a shortage of 20 units.

42. Surplus

43. By observing the graph in the figure, we see that a price below the equilibrium price results in a shortage.

44. At the market equilibrium point, Demand = Supply, so

$$-2q+320=8q+2$$
$$318=10q$$
$$31.8=q$$
$$p=-2q+320$$
$$p=-2(31.8)+320=\$256.40$$

45. $-\frac{1}{2}q+28=\frac{1}{3}q+\frac{34}{3}$ Required condition.

$-3q+168=2q+68$ Multiply both sides by 6 to simplify.

$-5q=-100$

$q=20$

Substituting into one of the original equations gives $p=-\frac{1}{2}(20)+28=18$.

Thus, the equilibrium point is $(q, p) = (20, 18)$.

46. At the market equilibrium point,
Demand = Supply, so

$480-3q=17q+80$

$400=20q$

$20=q$

$p=480-3q$

$p=480-3(20)=\$420$

47. $-4q+220=15q+30$ Required condition.

$190=19q$

$q=10$ Solve for q.

Substituting $q = 10$ into one of the original equations gives $p = 180$.
Thus, the equilibrium point is $(q, p) = (10, 180)$.

48. Demand: (45, 10), (20, 60)

$m=\dfrac{60-10}{20-45}=\dfrac{50}{-25}=-2$

$p-10=-2(q-45)$

$p-10=-2q+90$

$p=-2q+100$

Supply: (35, 30), (70, 50)

$m=\dfrac{50-30}{70-35}=\dfrac{20}{35}=\dfrac{4}{7}$

$p-30=\dfrac{4}{7}(q-35)$

$p=\dfrac{4}{7}q+10$

Supply = Demand

$\dfrac{4}{7}q+10=-2q+100$

$q=35$

$p=\dfrac{4}{7}(35)+10=30$

Market equilibrium point: (35, 30)

49. Demand: (80, 350) and (120, 300) are two points. $m=\dfrac{350-300}{80-120}=-\dfrac{5}{4}$

$p-p_1=m(q-q_1)$ or $p-300=-\dfrac{5}{4}(q-120)$ or $p=-\dfrac{5}{4}q+450$

Supply: (60, 280) and (140, 370) are two points. $m=\dfrac{280-370}{60-140}=\dfrac{9}{8}$

$p-p_1=m(q-q_1)$ or $p-280=\frac{9}{8}(q-60)$ or $p=\frac{9}{8}q+212.5$

Now, set these two equations for p equal to each other and solve for q.

$\frac{9}{8}q+212.5=-\frac{5}{4}q+450$ Required for equilibrium.

$9q+1700=-10q+3600$ Multiply both sides by 8 to simplify.

$19q=1900$

$q=100$

Substituting $q = 100$ into one of the original equations gives $p = 325$.
Thus, the equilibrium point is $(q, p) = (100, 325)$.

50. Demand: (10,75), (30, 25) Supply: (35, 80), (5, 20)

$m=\dfrac{25-75}{30-10}=\dfrac{-50}{20}=-2.5$ $\quad m=\dfrac{20-80}{5-35}=\dfrac{-60}{-30}=2$

$p-75=-2.5(q-10)$ $\quad p-20=2(q-5)$

$p-75=-2.5q+25$ $\quad p-20=2q-10$

$p=-2.5q+100$ $\quad p=2q+10$

Demand = Supply

$-2.5q+100=2q+10$ $\quad p=2(20)+10=50$

$20=q$

Market equilibrium point: (20, 50)

51. **a.** Reading the graph, we have that the tax is $15.
b. From the graph, the original equilibrium was (100, 100).
c. From the graph, the new equilibrium is (50, 110).
d. The supplier suffers because the increased price reduces the demand.

52. **a.** 0 (tax decreases units sold by 50)
b. Yes, because fewer units are demanded.

53. New supply price: $p=15q+30+38=15q+68$

$15q+68=-4q+220$ Required condition

$19q=152$

$q=8$

Substituting $q=8$ into one of the original equations gives $p=188$.
Thus, the new equilibrium point is $(q, p)=(8, 188)$.

54. With the $56 tax/unit, supply becomes

$p=17q+80+56=17q+136$

At the equilibrium point, $480-3q=17q+136$

$344=20q$

$q=17.2$

$p=17(17.2)+136=428.40$. Market equilibrium point: (17.2, 428.40)

55. New supply price: $p=\dfrac{q}{20}+10+5=\dfrac{q}{20}+15$

$\frac{q}{20}+15=-\frac{q}{20}+65$ Required condition

$q+300=-q+1300$

$2q=1000$

$q=500$ $\quad$ Thus, $p=\frac{500}{20}+15=40$.

The new equilibrium point is (500, 40).

56. With the $15 tax/unit, supply becomes

$p=3q+35+15=3q+50$

At the equilibrium point, $3q+50=-8q+2800$

$11q=2750$

$q=250$

$p=3(250)+50=800$. Market equilibrium point: (250, 800)

57. Demand: $p=\dfrac{-q+2100}{60}$ Supply: $p=\dfrac{q+540}{120}$

New supply: $p=\dfrac{q+540}{120}+\dfrac{1}{2}=\dfrac{q+540}{120}+\dfrac{60}{120}=\dfrac{q+600}{120}$

$\frac{q+600}{120}=\frac{-q+2100}{60}$ Required condition

$q+600=-2q+4200$ Multiply both sides by 120

$3q=3600$

$q=1200$ Thus, $p=\frac{1200+600}{120}=15$.

The new equilibrium quantity is 1200.
The new equilibrium price is \$15.

58. With the \$2 tax/unit, supply becomes $p=\dfrac{1}{45}q+8+2=\dfrac{1}{45}q+10$. Demand: $p=-\dfrac{1}{10}q+230$

$\dfrac{1}{45}q+10=-\dfrac{1}{10}q+230$

$\dfrac{11}{90}q=220 \quad\rightarrow\quad q=1800$

$p=-\dfrac{1}{10}(1800)+230=50$. Market equilibrium point: (1800, 50)

Chapter 1 Review Exercises

For this set of exercises we will not give reasons for any steps or list any formulas.

1. $3x - 8 = 23$

$3x = 31$

$x = \dfrac{31}{3}$

2. $2x - 8 = 3x + 5$

$-x = 13$

$x = -13$

3. $\dfrac{6x+3}{6} = \dfrac{5(x-2)}{9}$

$18\left(\dfrac{6x+3}{6}\right) = 18\left(\dfrac{5(x-2)}{9}\right)$

$3(6x+3) = 10(x-2)$

$18x + 9 = 10x - 20$

$8x = -29$

$x = -\dfrac{29}{8}$

4. $2x + \dfrac{1}{2} = \dfrac{x}{2} + \dfrac{1}{3}$

$12x + 3 = 3x + 2$

$9x = -1$

$x = -\dfrac{1}{9}$

5. $\dfrac{6}{3x-5} = \dfrac{6}{2x+3}$

$6(2x+3) = 6(3x-5)$

$2x + 3 = 3x - 5$

$3 + 5 = 3x - 2x$

$x = 8$

6. $\dfrac{2x+5}{x+7} = \dfrac{1}{3} + \dfrac{x-11}{2(x+7)}$

$6(2x+5) = 2(x+7) + 3(x-11)$

$12x + 30 = 2x + 14 + 3x - 33$

$12x - 2x - 3x = 14 - 33 - 30$

$7x = -49$

$x = -7$

There is no solution since we have division by zero when $x = -7$.

7. $3y - 6 = -2x - 10$

$3y = -2x - 4$

$y = \dfrac{-2x-4}{3}$

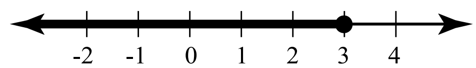
$y = -\dfrac{2}{3}x - \dfrac{4}{3}$

8. $3x - 9 \le 4(3 - x)$

$3x - 9 \le 12 - 4x$

$7x \le 21$

$x \le 3$



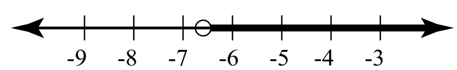
9. $\dfrac{2}{5}x \le x + 4$

$5\left(\dfrac{2}{5}x\right) \le 5(x+4)$

$2x \le 5x + 20$

$-3x \le 20$

$x \ge -\dfrac{20}{3}$



10. $5x + 1 \ge \dfrac{2}{3}(x-6)$

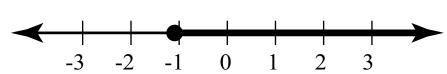
$3(5x+1) \ge 3 \cdot \dfrac{2}{3}(x-6)$

$15x + 3 \ge 2(x-6)$

$15x + 3 \ge 2x - 12$

$13x \ge -15$

$x \ge -\dfrac{15}{13}$



11. Yes.

12. $y^2 = 9x$, is not a function of x. If $x = 1$, then $y = \pm 3$.

13. Yes.

14. $y = \sqrt{9 - x}$

Domain: $9 - x \ge 0$ or $9 \ge x$ or $x \le 9$.

Range: Positive square root means $y \ge 0$.

15. $f(x) = x^2 + 4x + 5$

a. $f(-3) = (-3)^2 + 4(-3) + 5 = 9 - 12 + 5 = 2$

b. $f(4) = (4)^2 + 4(4) + 5 = 16 + 16 + 5 = 37$

c. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 5 = \frac{1}{4} + 2 + 5 = \frac{29}{4}$

16. $g(x) = x^2 + \frac{1}{x}$

a. $g(-1) = (-1)^2 + \frac{1}{-1} = 1 - 1 = 0$

b. $g\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \frac{1}{\frac{1}{2}} = \frac{1}{4} + 2 = 2\frac{1}{4}$

c. $g(0.1) = (0.1)^2 + \frac{1}{0.1} = 0.01 + 10 = 10.01$

17. $f(x) = 9x - x^2$

$$\begin{aligned} f(x+h) &= 9(x+h) - (x+h)^2 \\ &= 9x + 9h - x^2 - 2xh - h^2 \end{aligned}$$

$$f(x) = 9x - x^2$$

$$\begin{aligned} f(x+h) - f(x) &= 9h - 2xh - h^2 \\ &= h(9 - 2x - h) \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = 9 - 2x - h$$

18. y is a function of x. (Use vertical line test.)

19. No, the graph fails vertical line test.

20. $f(2) = 4$

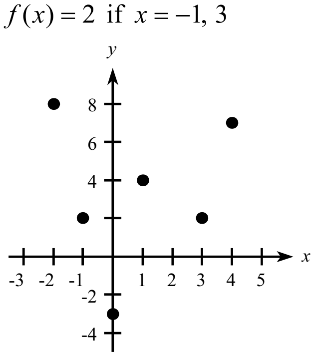
21. $x = 0, x = 4$

22. a. $D = \{-2, -1, 0, 1, 3, 4\}$, $R = \{-3, 2, 4, 7, 8\}$

b. $f(4) = 7$

c. $f(x) = 2$ if $x = -1, 3$

d.



e. No. For $y = 2$, there are two values of x.

23. $f(x) = 3x + 5,\ g(x) = x^2$

a. $(f + g)x = (3x + 5) + x^2 = x^2 + 3x + 5$

b. $\left(\frac{f}{g}\right)x = \frac{3x+5}{x^2}$ or $\frac{3x}{x^2} + \frac{5}{x^2} = \frac{3}{x} + \frac{5}{x^2}$

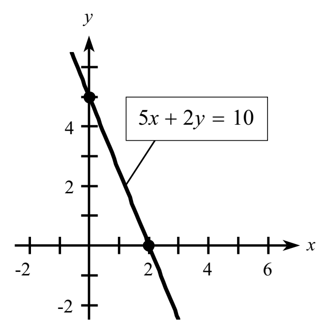
c. $f(g(x)) = f(x^2) = 3x^2 + 5$

d.
$$\begin{aligned} (f \circ f)x &= f(3x+5) \\ &= 3(3x+5) + 5 \\ &= 9x + 20 \end{aligned}$$

24. $5x + 2y = 10$

x-intercept: If $y = 0$, $x = 2$

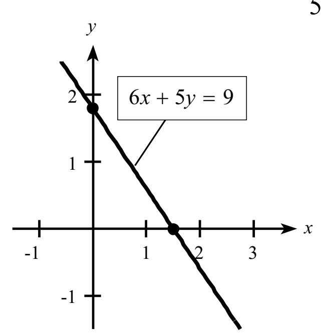
y-intercept: If $x = 0$, $y = 5$



25. $6x + 5y = 9$

x-intercept: If $y = 0$, $x = \frac{9}{6} = \frac{3}{2}$

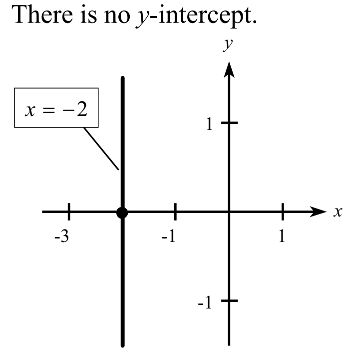
y-intercept: If $x = 0$ or $y = \frac{9}{5}$



26. $x = -2$

x-intercept: $x = -2$

There is no y-intercept.



27. $P_1(2, -1);\ P_2(-1, -4)$

$$m = \frac{-4-(-1)}{-1-2} = \frac{-3}{-3} = 1$$

28. (–3.8, –7.16) and (–3.8, 1.16)

$$m = \frac{-7.16-1.16}{-3.8-(-3.8)} = \frac{-8.32}{0}$$

Slope is undefined.

29. $2x + 5y = 10$

$$y = -\frac{2}{5}x+2,\ m = -\frac{2}{5},\ b = 2$$

30. $x = -\frac{3}{4}y+\frac{3}{2}$ or $y = -\frac{4}{3}x+2$

$$m = -\frac{4}{3},\ b = 2$$

31. $m = 4$, $b = 2$, $y = 4x + 2$

32. $m = -\frac{1}{2},\ b = 3,\ y = -\frac{1}{2}x+3$

33. $P = (-2, 1),\ m = \frac{2}{5}$

$$y-1 = \frac{2}{5}(x+2) \text{ or } y = \frac{2}{5}x+\frac{9}{5}$$

34. (–2, 7) and (6, –4)

$$m = \frac{-4-7}{6-(-2)} = \frac{-11}{8}$$

$$y-7 = \frac{-11}{8}(x-(-2)) \text{ or}$$

$$y = \frac{-11}{8}x+\frac{17}{4}$$

35. $P_1(-1, 8);\ P_2(-1, -1)$

The line is vertical since the x-coordinates are the same. Equation: $x = -1$

36. Parallel to $y = 4x - 6$ means $m = 4$.
$y - 6 = 4(x - 1)$ or $y = 4x + 2$

37. P(–1, 2); $\perp$ to $3x+4y=12$

or

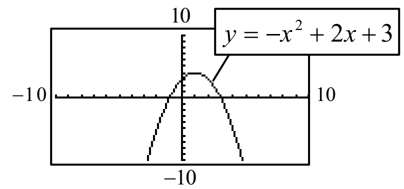
$$y = -\frac{3}{4}x+3$$

$$m = \frac{4}{3}$$

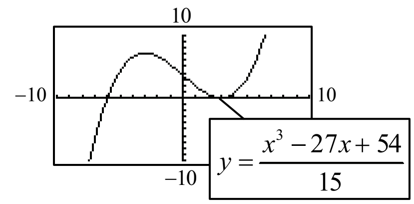
$$y-2 = \frac{4}{3}(x+1) \text{ or}$$

$$y = \frac{4}{3}x+\frac{10}{3}$$

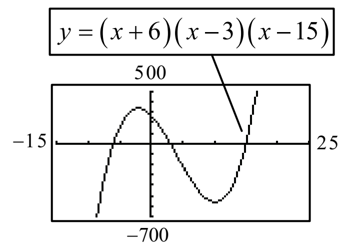
38. $x^2 + y - 2x - 3 = 0;\ y = -x^2 + 2x + 3$



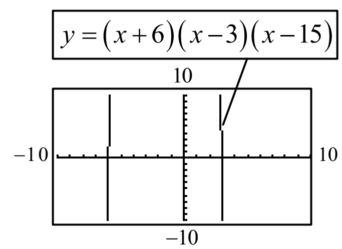
39.



40. a.



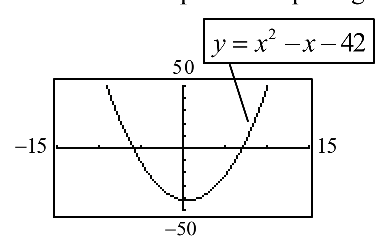
b.



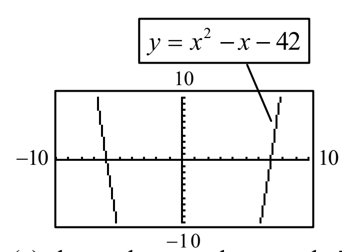
c. The graph in (a) shows the complete graph. The graph in (b) shows a piece that rises toward the high point and a piece between the high and low points.

41. $y = x^2 - x - 42$ is a parabola opening upward.

a.



b.



c. (a) shows the complete graph. The y-min is too large in absolute value for (b) to get a complete graph.

42. $y = \dfrac{\sqrt{x+3}}{x}$

$x \neq 0$; $x+3 \geq 0$ or $x \geq -3$;

Domain: $x \neq 0$, $x \geq -3$

43. Trace approximates $x = -7.2749$, $x = 0.2749$

44.
$$\begin{aligned} 4x - 2y &= 6 \\ 3x + 3y &= 9 \end{aligned}$$
Then,
$$\begin{aligned} 12x - 6y &= 18 \\ \underline{6x + 6y} &\underline{= 18} \\ 18x \quad &= 36 \\ x &= 2 \\ 4(2) - 2y &= 6 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$
Solution: (2, 1)

45.
$$\begin{aligned} 2x + \ \ y &= 19 \\ x - 2y &= 12 \end{aligned}$$
Then,
$$\begin{aligned} 4x + 2y &= 38 \\ \underline{x - 2y} &\underline{= 12} \\ 5x \quad &= 50 \\ x &= 10 \\ 2(10) + y &= 19 \\ y &= -1 \end{aligned}$$
Solution: (10, –1)

46.
$$\begin{aligned} 3x + 2y &= 5 \\ 2x - 3y &= 12 \end{aligned}$$
Then,
$$\begin{aligned} 9x + 6y &= 15 \\ \underline{4x - 6y} &\underline{= 24} \\ 13x \quad &= 39 \\ x &= 3 \\ 3(3) + 2y &= 5 \\ 2y &= -4 \\ y &= -2 \end{aligned}$$
Solution: (3, –2)

47.
$$\begin{aligned} 6x + 3y &= 1 \\ y &= -2x + 1 \\ 6x + 3(-2x + 1) &= 1 \\ 6x - 6x + 3 &= 1 \\ 3 &= 1 \end{aligned}$$
No solution.

48.
$$\begin{aligned} 4x - 3y &= 253 \\ 13x + 2y &= -12 \end{aligned} \qquad \begin{aligned} 8x - 6y &= 506 \\ \underline{39x + 6y} &\underline{= -36} \\ 47x \quad &= 470 \\ x \quad &= 10 \end{aligned} \qquad \begin{aligned} 4(10) - 3y &= 253 \\ -3y &= 213 \\ y &= -71 \end{aligned}$$
Solution: (10, –71)

49.
$$\begin{aligned} x + 2y + 3z &= 5 \\ y + 11z &= 21 \\ 5y + \ \ 9z &= 13 \end{aligned}$$
Steps 1 and 2: Nothing to be done.

Step 3:
$$\begin{aligned} x + 2y + \ \ 3z &= 5 \\ y + 11z &= 21 \\ -46z &= -92 \end{aligned}$$
Step 4: $z = 2$
$$\begin{aligned} y + 11(2) &= 21 \\ y &= -1 \end{aligned} \qquad \begin{aligned} x + 2(-1) + 3(2) &= 5 \\ x &= 1 \end{aligned}$$
Solution is $x = 1$, $y = -1$, $z = 2$.

50. $$\begin{aligned} x + y - z &= 12 \\ 2y - 3z &= -7 \\ 3x + 3y - 7z &= 0 \\ x + y - z &= 12 \\ 2y - 3z &= -7 \\ -4z &= -36 \end{aligned}$$

Thus $z = 9$
$2y - 27 = -7$
$2y = 20$ or $y = 10$
$x + 10 - 9 = 12$
$x = 11$
Solution: (11, 10, 9)

51. **a.** 1980+17=1997

b. $x = 2007 - 1980 = 27$

c. $$\begin{aligned} 461 &= 9.78x + 167.90 \\ 293.1 &= 9.78x \\ x &= 29.97 \end{aligned}$$

1980+29.97=2009.97 in the year 2009.

52. Student has total points of 91 + 82 + 88 + 50 + 42 + 42 = 395.
Total of possible points is 300 + 150 + 200 = 650.
To earn an A students need at least 0.9(650) = 585 points.
Student must earn 585 – 395 = 190 points on the final. This is the same as 95%.

53. Diesel: $C = 0.24x + 38,000$
Gas: $C = 0.30x + 35,600$

$$\begin{aligned} 0.24x + 38,000 &= 0.30x + 35,600 \\ 0.06x &= 2400 \\ x &= 40,000 \end{aligned}$$

Costs are equal at 40,000 miles. A truck is used more than 40,000 miles in 5 years. Buy the diesel.

54. **a.** Yes

b. No

c. $f(300) = 4$

55. **a.** $f(80) = 565.44$

b. The monthly payment on a $70,000 loan is $494.75.

56. $P(x) = 180x - \dfrac{x^2}{100} - 200 \qquad x = q(t) = 1000 + 10t$

a. $(P \circ q)(t) = P(1000 + 10t) = 180(1000 + 10t) - \dfrac{(1000 + 10t)^2}{100} - 200$

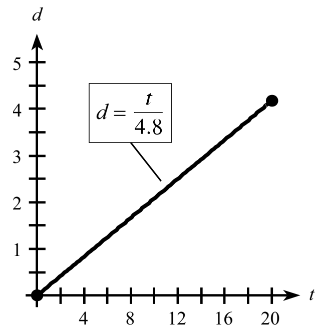
b. $x = q(15) = 1000 + 10(15) = 1150$ units produced

$$P(1150) = 180(1150) - \frac{(1150)^2}{100} - 200 = \$193,575$$

57. $W(L) = kL^3,\ L(t) = 50 - \dfrac{(t-20)^2}{10},\ 0 \le t \le 20$

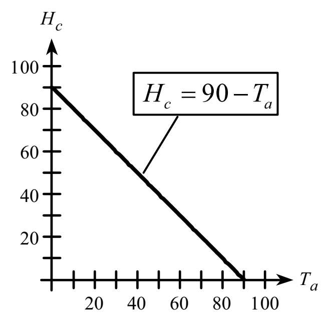
$$(W \circ L)(t) = W\left(50 - \frac{(t-20)^2}{10}\right) = 0.02\left(50 - \frac{(t-20)^2}{10}\right)^3$$

58. a.



b. $(9.6, 2)$ means that the thunderstorm is two miles away if the flash and thunder are 9.6 seconds apart.

59.



60. a. (*x, P*) is the required form.

$P_1 = (200, 3100),\ P_2 = (250, 6000)$

$$m = \frac{6000-3100}{250-200} = \frac{2900}{50} = 58$$

$P - 3100 = 58(x - 200)$ or

$P(x) = 58x - 8500$

b. For each additional unit sold the profit increases by \$58.

61. $A = 71.72x + 1401.36$

a. Yes.

b. $m = 71.72$, A-intercept is 1401.36

c. In 1990 (the year that corresponds to $x = 0$), the average annual cost per consumer was \$1401.36

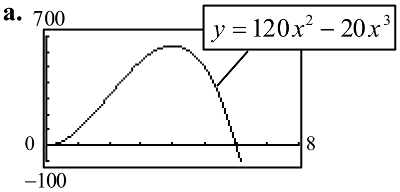
d. The average annual cost per consumer rises by about \$71.72 per year.

62. $(C, F): (0, 32)$ and $(100, 212)$

$$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

Using $y = mx + b$, $F = \frac{9}{5}C + 32.$

63. a.



b. Algebraically, $y \ge 0$ if

$120x^2 - 20x^3 = 20x^2(x-6) \ge 0.$

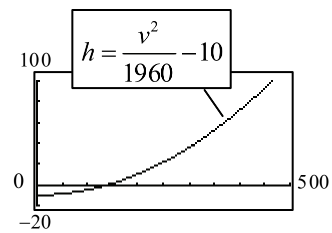
Answer: $0 \le x \le 6$

64. a.

$$v^2 = 1960(h+10)$$

$$h + 10 = \frac{v^2}{1960}$$

$$h = \frac{v^2}{1960} - 10$$



b. $h(210) = \frac{210^2}{1960} - 10 = 12.5$ cm

65. x = amount of safer investment and y = amount of other investment.

$$x + y = 150000$$
$$0.095x + 0.11y = 15000$$

Solving the system:

$$0.11x + 0.11y = 16500$$
$$0.095x + 0.11y = 15000$$
$$0.015x = 1500$$
$$x = 100000$$

Then y = 50000. Thus, invest \$100,000 at 9.5% and \$50,000 at 11%.

66. x = liters of 20% solution
y = liters of 70% solution

$$x + y = 4$$
$$0.2x + 0.7y = 1.4$$

$$x + y = 4$$
$$x + 3.5y = 7$$
$$2.5y = 3 \qquad y = 1.2$$
$$x + 1.2 = 4$$
$$x = 2.8$$

Answer: 2.8 liters of 20%, 1.2 of 70%.

67. $S: p = 4q + 5,\ D: p = -2q + 81$

a. $S: 53 = 4q + 5 \qquad D: 53 = -2q + 81$

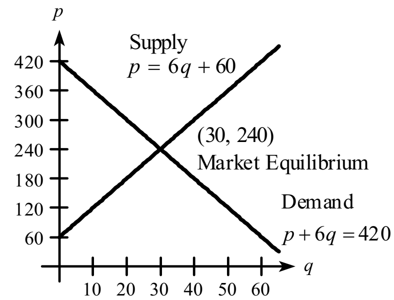
$4q = 48 \qquad 2q = 28$

$q = 12 \qquad q = 14$

b. Demand is greater.
There is a shortfall.

c. Price is likely to increase.

68. a. – c.



69. $C(x) = 38.80x + 4500$, $R(x) = 61.30x$

a. Marginal cost is $38.80.

b. Marginal revenue is $61.30.

c. Marginal profit is $61.30 – 38.80 = $22.50.

d. $61.30x = 38.80x + 4500$

$22.50x = 4500$

$x = 200$ units to break even.

70. $FC = \$1500$, $VC = \$22$ per unit, $R = \$52$ per unit

a. $C(x) = 22x + 1500$

b. $R(x) = 52x$

c. $P = R - C = 30x - 1500$

d. $\overline{MC} = 22$

e. $\overline{MR} = 52$

f. $\overline{MP} = 30$

g. Break even means $30x - 1500 = 0$ or $x = 50$.

71. Supply: $m = \dfrac{100-200}{200-400} = \dfrac{1}{2}$ Demand: $m = \dfrac{200-0}{200-600} = -\dfrac{1}{2}$

$p - 100 = \frac{1}{2}(q - 200) \qquad p - 0 = -\frac{1}{2}(q - 600)$

$p = \frac{1}{2}q \qquad p = -\frac{1}{2}q + 300$

So, $\frac{1}{2}q = -\frac{1}{2}q + 300$ or $q = 300$. The equilibrium price is $p = \frac{1}{2}(300) = \$150$.

72. New supply equation: $p = \dfrac{q}{10} + 8 + 2 = \dfrac{q}{10} + 10$

Demand:
$$p = \frac{-q + 1500}{10} = -\frac{q}{10} + 150$$

$$\frac{q}{10} + 10 = -\frac{q}{10} + 150$$

$$\frac{2q}{10} = 140 \text{ or } q = 700$$

$$p = \frac{700}{10} + 10 = 80$$

Solution: (700, 80)

Chapter 1 Test

1. $4x-3=\frac{x}{2}+6$

$8x-6=x+12$

$7x=18$

$x=\frac{18}{7}$

2. $\frac{3}{x}+4=\frac{4x}{x+1}$

$3(x+1)+4x(x+1)=4x(x)$

$3x+3+4x^2+4x=4x^2$

$7x=-3$

$x=-\frac{3}{7}$

3. $\frac{3x-1}{4x-9}=\frac{5}{7}$

$7(3x-1)=5(4x-9)$

$21x-7=20x-45$

$x=-38$

4. $f(x)=7+5x-2x^2$

$f(x+h)=7+5(x+h)-2(x+h)^2$

$=7+5x+5h-2x^2-4xh-2h^2$

$f(x)=7+5x-2x^2$

$f(x+h)-f(x)=5h-4xh-2h^2$

$\frac{f(x+h)-f(x)}{h}=5-4x-2h$

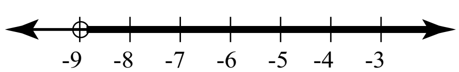
5. $1+\frac{2}{3}t\le 3t+22$

$3\left(1+\frac{2}{3}t\right)\le 3(3t+22)$

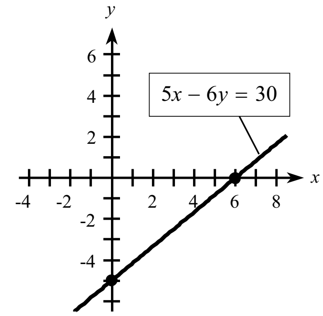
$3+2t\le 9t+66$

$-7t\le 63$

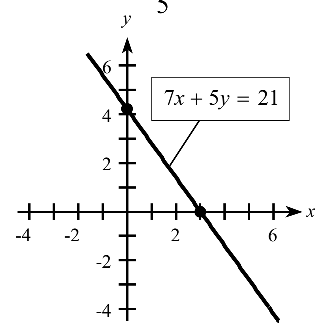
$t\ge -9$



6. $5x-6y=30$
x-intercept: 6
y-intercept: –5



7. $7x+5y=21$
x-intercept: 3
y-intercept: $\frac{21}{5}$



8. $f(x)=\sqrt{4x+16}$

a. $4x+16\ge 0$
$4x\ge -16$
Domain: $x\ge -4$; Range: $y\ge 0$
For range, note square root is positive.

b. $f(3)=\sqrt{12+16}=2\sqrt{7}$

c. $f(5)=\sqrt{20+16}=6$

9. (–1, 2) and (3, –4)

$m=\frac{-4-2}{3-(-1)}=\frac{-6}{4}=\frac{-3}{2}$

$y-2=\frac{-3}{2}(x-(-1))$

$y=\frac{-3}{2}x+\frac{1}{2}$

10. $5x+4y=15$

$y=-\frac{5}{4}x+\frac{15}{4}$

$m=-\frac{5}{4},\ b=\frac{15}{4}$

11. Point (–3, –1)

a. Undefined slope means vertical line. $x = -3$

b. $\perp$ to $y = \frac{1}{4}x + 2$ means $m = -4$.
Thus, $y + 1 = -4(x + 3)$ or $y = -4x - 13$.

12. a. is not a function since for some x-values there are two y's.

b. is a function since for each x there is only one y.

c. is not a function for same reason as (a).

13.
$$\begin{aligned} 3x + 2y &= -2 \\ 4x + 5y &= 2 \\ 12x + \ \ 8y &= -8 \\ \underline{12x + 15y} &\underline{= 6} \\ -7y &= -14 \\ y &= 2 \\ 3x + 2(2) &= -2 \\ 3x &= -6 \\ x &= -2 \end{aligned}$$
Solution: (–2, 2)

14. $f(x) = 5x^2 - 3x,\ g(x) = x + 1$

a. $(fg)(x) = (5x^2 - 3x)(x + 1)$

b. $g(g(x)) = g(x + 1) = (x + 1) + 1 = x + 2$

c.
$$\begin{aligned} (f \circ g)(x) &= f(x + 1) \\ &= 5(x + 1)^2 - 3(x + 1) \\ &= 5x^2 + 10x + 5 - 3x - 3 \\ &= 5x^2 + 7x + 2 \end{aligned}$$

15. $R(x) = 38x$, $C(x) = 30x + 1200$

a. $\overline{MC} = \$30$

b.
$$\begin{aligned} P(x) &= 38x - (30x + 1200) \\ &= 8x - 1200 \end{aligned}$$

c. Break-even means $P(x) = 0$.
$8x = 1200$ or $x = 150$ units

d. $\overline{MP} = \$8$. Each additional unit sold increases the profit by \$8.

16. a. $R(x) = 50x$

b.
$$\begin{aligned} C(100) &= 10(100) + 18000 \\ &= \$19{,}000 \end{aligned}$$
It costs \$19,000 to make 100 units.

c.
$$\begin{aligned} 50x &= 10x + 18000 \\ 40x &= 18000 \\ x &= 450 \text{ units} \end{aligned}$$

17. $S : p = 5q + 1500,\ D : p = -3q + 3100$
$$5q + 1500 = -3q + 3100$$
$$8q = 1600 \text{ or } q = 200$$
$p(200) = 5(200) + 1500 = \$2500$

18. $y = 720{,}000 - 2000x$

a. $b = 720{,}000$
The original value is \$720,000.

b. $m = -2000$.
The building is depreciating \$2000 each month.

19. x = number of reservations
$$\begin{aligned} 0.90x &= 360 \\ x &= 400 \end{aligned}$$
Accept 400 reservations.

20. x = amount invested at 9%
y = amount invested at 6%
$x + y = 20000$ Amount
$0.09x + 0.06y = 1560$ Interest
$$\begin{aligned} 0.09x + 0.09y &= 1800 \\ \underline{0.09x + 0.06y} &\underline{= 1560} \\ 0.03y &= 240 \\ y &= \$8000 \end{aligned}$$
Invest \$8000 at 6% and \$12000 at 9%.

Chapter 1 Extended Applications

I. 1. Revenue per case = \$1000
Annual fixed costs = \$180,000 + 270,000 = \$450,000
Annual variable costs = $(\$380+15+20\cdot\frac{1}{4})x=\$400x$, where x is the number of operations per year.

2. Break-even occurs when Revenue = Total Costs

$$1000x = 450,000 + 400x$$
$$600x = 450,000$$
$$x = 750$$

The hospital must perform 750 operations per year to break even.

3. We have (70 operations/month)(12 months/year) gives 840 operations/year with a savings of (840 operations)(\$50 savings) = \$42,000 on supplies. However, leasing the machine would cost \$50,000. Thus adding the machine would reduce the hospital's profits by \$8000 a year at the current level of operations. (Note that 1000 operations must be performed each year to cover the cost of the machine: [(\$50)100) = \$50,000].)

4. Profit = Revenue – Cost

$$P(x) = 1000x - (450,000 + 400x)$$
$$= 600x - 450,000$$

At current level of operations, the annual profit is:

$$P(840) = 600(840) - 450,000$$
$$= 504,000 - 450,000$$
$$= \$54,000$$

With (40 new operations/month)(12 months/year) = 480 new operations/year, the new level of operations is 840 + 480 = 1320. The advertising costs are (\$10,000/month)(12 months/year) = \$120,000 per year.
At the new level of operations, the profit would be: $P(1320) = 600(1320) - 450,000 - 120,000$

$$= 792,000 - 570,000$$
$$= \$222,000$$

The increase in profit is \$222,000 – 54,000 = \$168,000.

5. Each extra operation adds \$1000 – 400 = \$600 of profit. If the ad campaign costs \$10,000 per month it must generate $\dfrac{\$10,000 \text{ per month}}{\$600 \text{ per operation}} = 16\frac{2}{3}$ operations/month to cover its cost.

6. Recall that the break-even point for leasing the machine is 1000 operations per year. If the ad campaign meets its projections, 1320 operations per year will be performed, with a savings of (320)(\$50) = \$16,000 on medical supplies by leasing the machine. They should reconsider their decision. (Note that this example illustrates that if the assumptions on which a decision was made change, it may be time to take another look at the decision.)

II. Answers will vary.