

MULTIPLE CHOICE

1. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

Find the distance traveled in 16 seconds by an object traveling at a constant velocity of 20 feet per second.

- a. calculus, 320 ft
- b. calculus, 340 ft
- c. precalculus, 320 ft
- d. calculus, 640 ft
- e. precalculus, 640 ft

ANS: C PTS: 1 DIF: Easy REF: 1.1.1
OBJ: Recognize problems requiring precalculus and find the solution
MSC: Skill NOT: Section 1.1

2. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

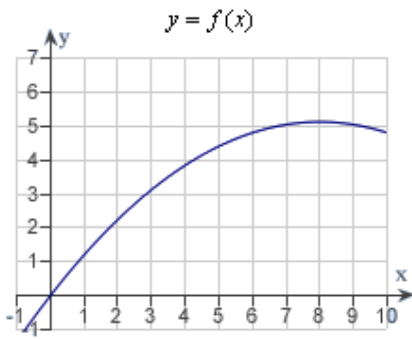
Find the distance traveled in 20 seconds by an object moving with a velocity of $v(t) = 8 + 6 \cos t$ feet per second.

- a. calculus, 162.4485 ft
- b. precalculus, 163.7985 ft
- c. calculus, 165.4777 ft
- d. precalculus, 165.4777 ft
- e. precalculus, 162.4485 ft

ANS: C PTS: 1 DIF: Medium REF: 1.1.2
OBJ: Recognize problems requiring calculus and estimate solutions
MSC: Skill NOT: Section 1.1

3. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

A cyclist is riding on a path whose elevation is modeled by the function $f(x) = 0.08(16x - x^2)$ where x and $f(x)$ are measured in miles. Find the rate of change of elevation when $x = 4$.



-
- a. precalculus, 0.08
 - b. calculus, 0.2
 - c. calculus, 0.64
 - d. calculus, 0.08
 - e. precalculus, 0.2

ANS: C

PTS: 1

DIF: Medium

REF: 1.1.3

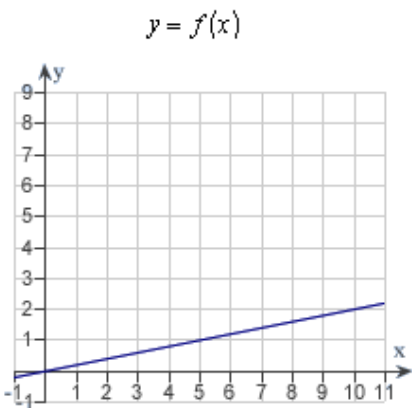
OBJ: Recognize problems requiring calculus and estimate solutions

MSC: Skill

NOT: Section 1.1

4. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

A cyclist is riding on a path whose elevation is modeled by the function $f(x) = 0.2x$ where x and $f(x)$ are measured in miles. Find the rate of change of elevation when $x = 5$.

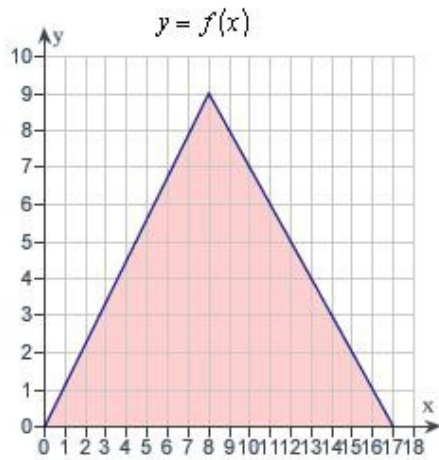


-
- a. calculus, 2
 - b. precalculus, 0.2
 - c. calculus, 0.2
 - d. precalculus, 2
 - e. precalculus, 0.45

ANS: B PTS: 1 DIF: Easy REF: 1.1.4
OBJ: Recognize problems requiring precalculus and find the solution
MSC: Skill NOT: Section 1.1

5. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

Find the area of the shaded region bounded by the triangle with vertices $(0,0)$, $(8,9)$, $(17,0)$.

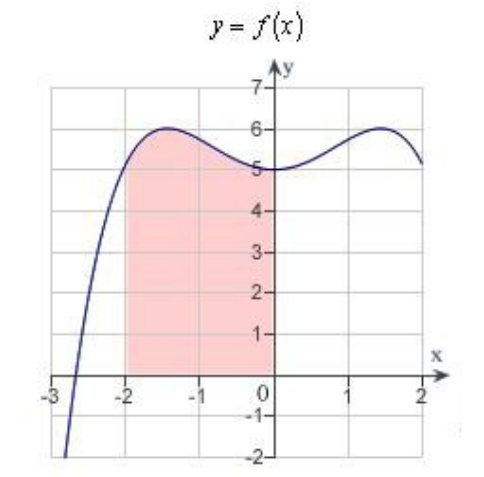


- a. precalculus , 153
- b. calculus , 229.5
- c. precalculus , 76.5
- d. precalculus , 229.5
- e. calculus , 153

ANS: C PTS: 1 DIF: Easy REF: 1.1.5a
OBJ: Recognize problems requiring precalculus and find the solution
MSC: Skill NOT: Section 1.1

6. Decide whether the following problem can be solved using precalculus, or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, use a graphical or numerical approach to estimate the solution.

Find the area of the shaded region.



- a. calculus , 11
- b. precalculus , 11
- c. precalculus , 13
- d. calculus , 16
- e. precalculus , 16

ANS: A

PTS: 1

DIF: Medium

REF: 1.1.5b

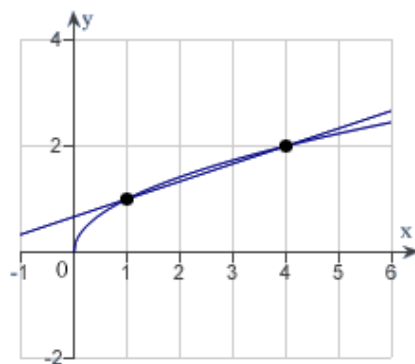
OBJ: Recognize problems requiring calculus and estimate solution

MSC: Skill

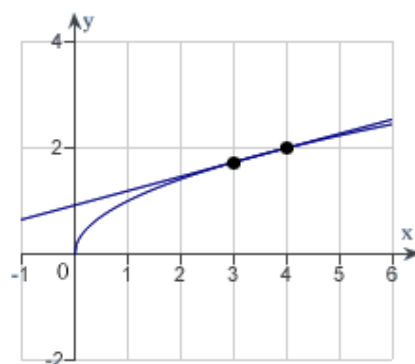
NOT: Section 1.1

7. Consider the function $f(x) = \sqrt{x}$ and the point $P(4, 2)$ on the graph of f . Graph f and the secant line passing through $P(4, 2)$ and $Q(x, f(x))$ for $x = 3$.

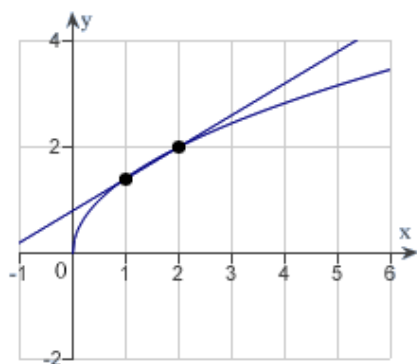
a.



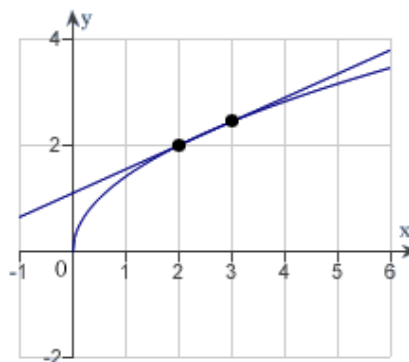
d.



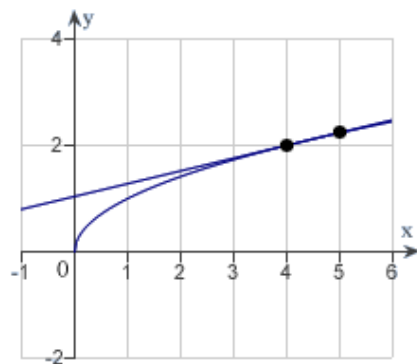
b.



e.



c.



ANS: D PTS: 1 DIF: Easy REF: 1.1.6a
 OBJ: Graph a function and the secant line passing through given points
 MSC: Skill NOT: Section 1.1

8. Consider the function $f(x) = \sqrt{x}$ and the point $P(81, 9)$ on the graph of f . Find the slope of the secant line passing through $P(81, 9)$ and $Q(x, f(x))$ for $x = 1$. Round your answer to four decimal places.

- a. $m=0.1000$
- b. $m=0.0122$
- c. $m=0.0122$
- d. $m=0.3133$
- e. $m=0.1000$

ANS: A PTS: 1 DIF: Easy REF: 1.1.6b
 OBJ: Calculate the slope of a secant line passing through given points
 MSC: Skill NOT: Section 1.1

9. Consider the function $f(x) = \sqrt{x}$ and the point $P(64, 8)$ on the graph of f .

Consider the secant lines passing through $P(64, 8)$ and $Q(x, f(x))$ for x values of 61, 63, and 65. Find the slope of each secant line to four decimal places.

(Think about how you could use your results to estimate the slope of the tangent line of f at $P(64,8)$, and how to improve your approximation of the slope.)

- a. $0.0633, -0.0627, 0.0623$
- b. $0.0633, 0.0627, 0.0623$
- c. $0.0317, 0.0314, 0.0312$
- d. $0.0633, -0.0627, -0.0623$
- e. $-0.0317, -0.0314, -0.0312$

ANS: D PTS: 1 DIF: Medium REF: 1.1.6b
 OBJ: Calculate the slopes of secant lines MSC: Skill NOT: Section 1.1

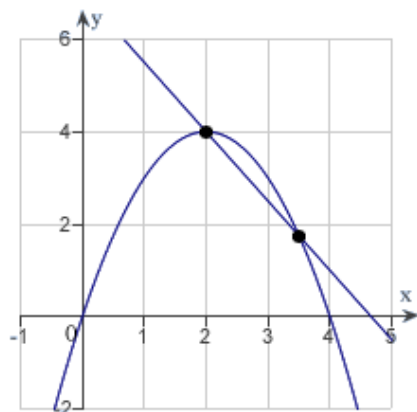
10. Consider the function $f(x) = \sqrt{x}$ and the point $P(9,3)$ on the graph of f . Estimate the slope m of the tangent line of f at $P(9,3)$. Round your answer to four decimal places.

- a. $m=0.1667$
- b. $m=0.0832$
- c. $m=0.3800$
- d. $m=0.0556$
- e. $m=0.0833$

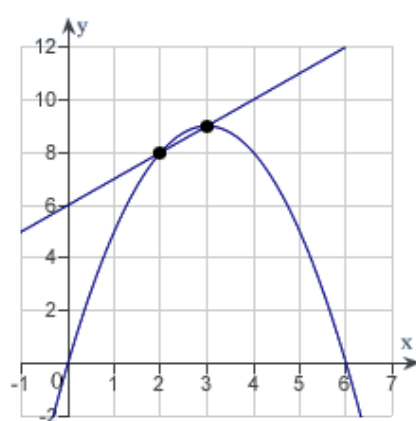
ANS: A PTS: 1 DIF: Medium REF: 1.1.6c
 OBJ: Estimate the slope of a tangent line MSC: Skill
 NOT: Section 1.1

11. Consider the function $f(x) = 6x - x^2$ and the point $P(2,8)$ on the graph of f . Graph f and the secant line passing through $P(2,8)$ and $Q(x, f(x))$ for $x = 3$.

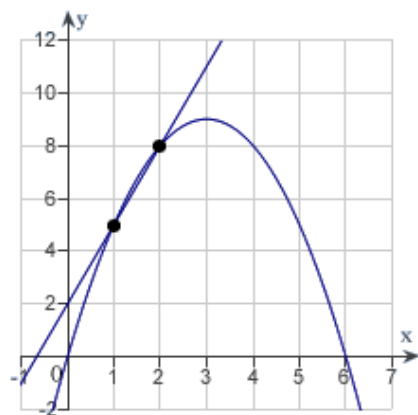
a.



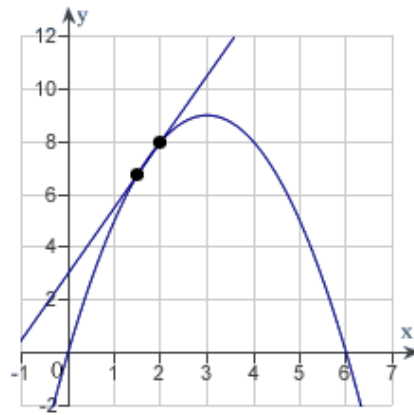
d.



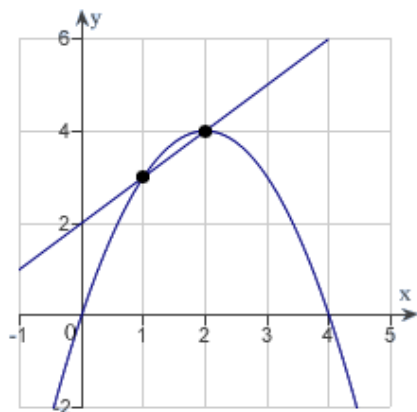
b.



e.



c.



ANS: D

PTS: 1

DIF: Easy

REF: 1.1.7a

OBJ: Graph a function and the secant line passing through given points

MSC: Skill

NOT: Section 1.1

12. Consider the function $f(x) = 11x - x^2$ and the point $P(4, 28)$ on the graph of f . Find the slope of the secant line passing through $P(4, 28)$ and $Q(x, f(x))$ for $x = 5$. Round your answer to one decimal place.

- a. 3.5
- b. 2.0
- c. 3.0
- d. 4.5
- e. 9.0

ANS: B

PTS: 1

DIF: Easy

REF: 1.1.7b

OBJ: Calculate the slope of a secant line passing through given points

MSC: Skill

NOT: Section 1.1

13. Consider the function $f(x) = 8x - x^2$ and the point $P(3, 15)$ on the graph of f . Estimate the slope of the tangent line of f at $P(3, 15)$.

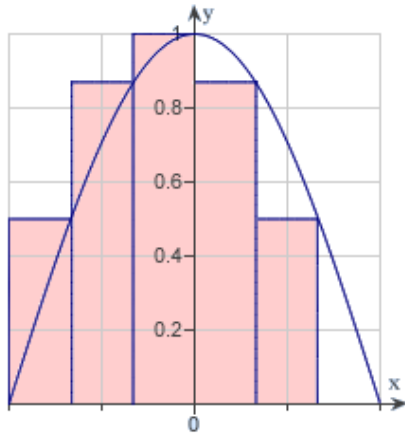
- a. 10
- b. 3

- c. 8
- d. 2
- e. 9

ANS: D PTS: 1 DIF: Medium REF: 1.1.7c
 OBJ: Calculate the slope of secant line passing through the given points
 MSC: Skill NOT: Section 1.1

14. Use the rectangles in the following graph to approximate the area of the region bounded by

$y = \cos x, y = 0, x = -\frac{\pi}{2}, \text{ and } x = \frac{\pi}{2}.$

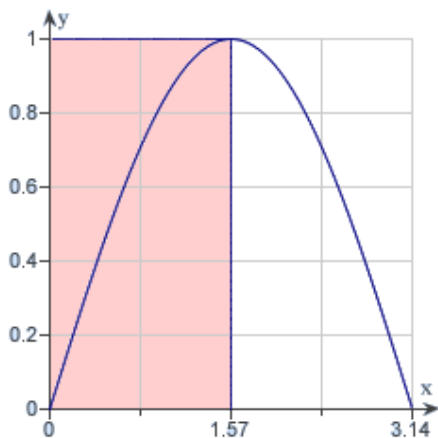


- a. 3.9082
- b. 2.6055
- c. 1.9541
- d. 1.4656
- e. 0.9770

ANS: C PTS: 1 DIF: Medium REF: 1.1.8a
 OBJ: Estimate the area of a region using rectangles
 MSC: Skill
 NOT: Section 1.1

15. Use the rectangles in the following graph to approximate the area of the region bounded by

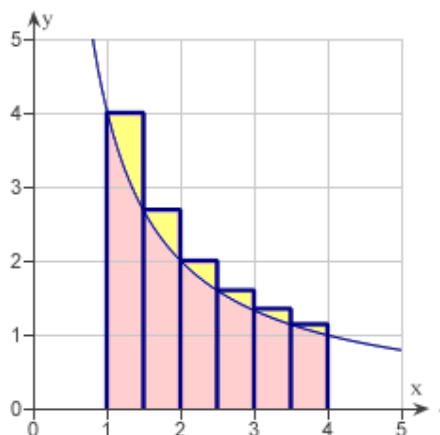
$y = \sin x, y = 0, x = 0, \text{ and } x = \pi.$



- a. 0.7850
- b. 1.5700
- c. 3.1400
- d. 1.1775
- e. 1.0519

ANS: B PTS: 1 DIF: Medium REF: 1.1.8a
 OBJ: Estimate the area of a region using rectangles MSC: Skill
 NOT: Section 1.1

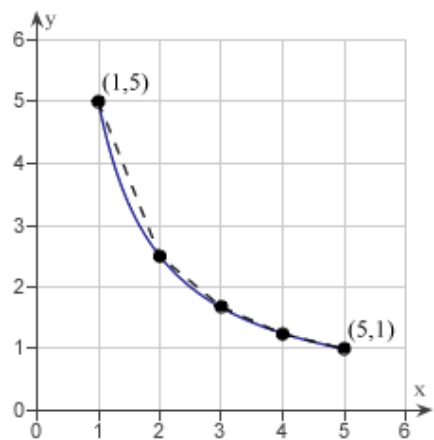
16. Use the rectangles in the graph given below to approximate the area of the region bounded by $y = 4/x$, $y = 0$, $x = 1$, and $x = 4$ Round your answer to three decimal places.



- a. 2.481 units²
- b. 6.371 units²
- c. 3.585 units²
- d. 6.872 units²
- e. 6.903 units²

ANS: B PTS: 1 DIF: Medium REF: 1.1.9a
 OBJ: Estimate the area of a region using rectangles MSC: Skill
 NOT: Section 1.1

17. Consider the length of the graph of $f(x) = 5/x$ from $(1, 5)$ to $(5, 1)$ Approximate the length of the curve by finding the sum of the lengths of four line segments, as shown in following figure. Round your answer to two decimal places.



- a. 6.11
- b. 8.12
- c. 5.66
- d. 8.49
- e. 7.11

ANS: A PTS: 1 DIF: Medium REF: 1.1.11b
OBJ: Estimate the length of the curve using a piecewise linear function
MSC: Skill NOT: Section 1.1

MULTIPLE CHOICE

1. Complete the table and use the result to estimate the limit.

$$\lim_{x \rightarrow 3} \frac{x-3}{x^2-16x+39}$$

| | | | | | | |
|--------|-----|------|-------|-------|------|-----|
| x | 2.9 | 2.99 | 2.999 | 3.001 | 3.01 | 3.1 |
| $f(x)$ | | | | | | |

- a. 0.525000
- b. 0.275000
- c. -0.100000
- d. 0.400000
- e. -0.475000

ANS: C PTS: 1 DIF: Medium REF: 1.2.1
 OBJ: Estimate a limit from a table of values MSC: Skill
 NOT: Section 1.2

2. Complete the table and use the result to estimate the limit.

$$\lim_{x \rightarrow -10} \frac{\sqrt{-6x-54} - \sqrt{6}}{x+10}$$

| | | | | | | |
|--------|-------|--------|---------|--------|-------|------|
| x | -10.1 | -10.01 | -10.001 | -9.999 | -9.99 | -9.9 |
| $f(x)$ | | | | | | |

- a. 0.974745
- b. -1.099745
- c. -1.224745
- d. 1.058078
- e. 1.224745

ANS: C PTS: 1 DIF: Medium REF: 1.2.4
 OBJ: Estimate a limit from a table of values MSC: Skill
 NOT: Section 1.2

3. Complete the table and use the result to estimate the limit.

$$\lim_{x \rightarrow 7} \frac{\frac{1}{x-3} - \frac{1}{4}}{x-7}$$

| | | | | | | |
|--------|-----|------|-------|-------|------|-----|
| x | 6.9 | 6.99 | 6.999 | 7.001 | 7.01 | 7.1 |
| $f(x)$ | | | | | | |

- a. -0.062500
- b. 0.067500
- c. -0.192500
- d. 0.047500
- e. -0.172500

ANS: A PTS: 1 DIF: Medium REF: 1.2.5
 OBJ: Estimate a limit from a table of values MSC: Skill
 NOT: Section 1.2

4. Complete the table and use the result to estimate the limit.

$$\lim_{x \rightarrow 0} \frac{\sin^3 x}{x^3}$$

| | | | | | | |
|--------|------|-------|--------|-------|------|-----|
| x | -0.1 | -0.01 | -0.001 | 0.001 | 0.01 | 0.1 |
| $f(x)$ | | | | | | |

- a. -0.5
- b. 0
- c. 1
- d. 0.5
- e. -1

ANS: C PTS: 1 DIF: Medium REF: 1.2.7
 OBJ: Estimate a limit from a table of values MSC: Skill
 NOT: Section 1.2

5. Complete the table and use the result to estimate the limit.

$$\lim_{x \rightarrow 0} \frac{\cos(3x) - 1}{3x}$$

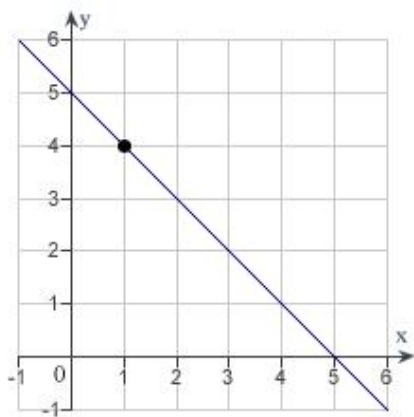
| | | | | | | |
|--------|------|-------|--------|-------|------|-----|
| x | -0.1 | -0.01 | -0.001 | 0.001 | 0.01 | 0.1 |
| $f(x)$ | | | | | | |

- a. -1
- b. -0.5
- c. 0
- d. 0.5
- e. 1

ANS: C PTS: 1 DIF: Medium REF: 1.2.8
 OBJ: Estimate a limit from a table of values MSC: Skill
 NOT: Section 1.2

6. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 1} (5 - x)$$

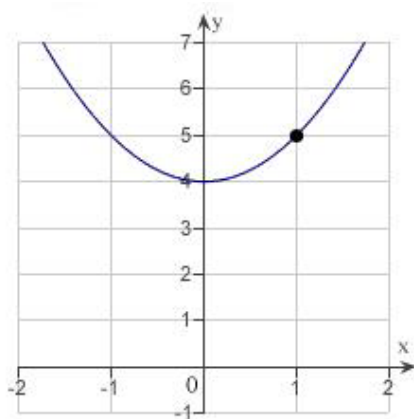


- a. 6
- b. 1
- c. 5
- d. 4
- e. does not exist

ANS: D PTS: 1 DIF: Easy REF: 1.2.15
 OBJ: Estimate the limit of a function from its graph MSC: Skill
 NOT: Section 1.2

7. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 1} (x^2 + 4)$$



- a. 5
- b. 1
- c. 0
- d. 4
- e. does not exist

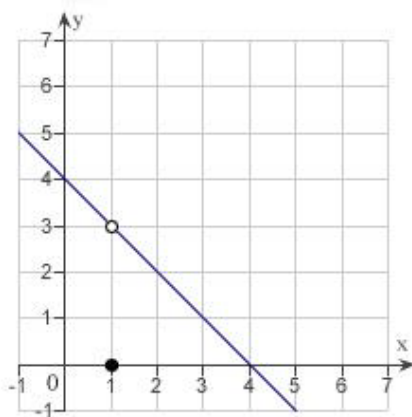
ANS: A PTS: 1 DIF: Medium REF: 1.2.16
 OBJ: Estimate the limit of a function from its graph MSC: Skill

NOT: Section 1.2

8. Let $f(x) = \begin{cases} 4 - x, & x \neq 1 \\ 0, & x = 1 \end{cases}$.

Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 1} f(x)$$



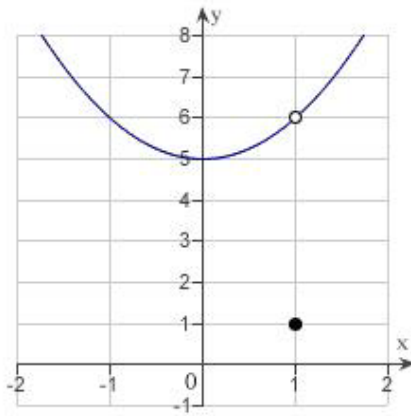
-
- a. 5
 - b. 4
 - c. 3
 - d. 0
 - e. does not exist

ANS: C PTS: 1 DIF: Medium REF: 1.2.17
OBJ: Estimate the limit of a function from its graph MSC: Skill
NOT: Section 1.2

9. Let $f(x) = \begin{cases} x^2 + 5, & x \neq 1 \\ 1, & x = 1 \end{cases}$.

Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 1} f(x)$$

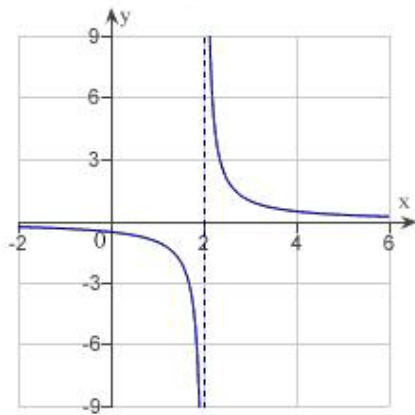


- a. 6
- b. 25
- c. 1
- d. 5
- e. does not exist.

ANS: A PTS: 1 DIF: Medium REF: 1.2.18
 OBJ: Estimate the limit of a function from its graph MSC: Skill
 NOT: Section 1.2

10. Determine the following limit. (Hint: Use the graph to calculate the limit.)

$$\lim_{x \rightarrow 2} \frac{1}{x - 2}$$



- a. -2
- b. 0
- c. -4
- d. 2
- e. does not exist

ANS: E PTS: 1 DIF: Medium REF: 1.2.20
 OBJ: Estimate the limit of a function from its graph MSC: Skill

MULTIPLE CHOICE

1. Find the limit.

$$\lim_{x \rightarrow -4} 9x^2 + 36x$$

- a. 108
- b. -108
- c. 288
- d. -288
- e. 0

ANS: E PTS: 1 DIF: Easy
OBJ: Evaluate a limit using properties of limits
NOT: Section 1.3

REF: 1.3.9
MSC: Skill

2. Find the limit.

$$\lim_{x \rightarrow 6} \frac{x}{x^2 + 8}$$

- a. 1
- b. 14
- c. 10
- d. 3
- e. 22

ANS: C PTS: 1 DIF: Easy
OBJ: Evaluate a limit using properties of limits
NOT: Section 1.3

REF: 1.3.19
MSC: Skill

3. Find the limit.

$$\lim_{x \rightarrow 4} \frac{\sqrt{x+5}}{x-1}$$

- a. 3
- b. -1
- c. -3
- d. 1
- e. 2

ANS: D PTS: 1 DIF: Medium REF: 1.3.22
OBJ: Evaluate a limit using properties of limits MSC: Skill
NOT: Section 1.3

4. Let $f(x) = 4x - 2$ and $g(x) = x^3$. Find the limit.

$$\lim_{x \rightarrow 1} g(f(x))$$

- a. 2
- b. $\frac{1}{8}$
- c. 8
- d. -8
- e. $\frac{-4}{8}$

ANS: C PTS: 1 DIF: Medium REF: 1.3.24c
OBJ: Evaluate the limit of composite functions MSC: Skill
NOT: Section 1.3

5. Let $f(x) = -x^2 - 5$ and $g(x) = 2x$. Find the limit.

$$\lim_{x \rightarrow -2} g(f(x))$$

- a. -18
- b. 25
- c. 21
- d. 8
- e. 9

ANS: A PTS: 1 DIF: Medium REF: 1.3.24c
OBJ: Evaluate the limit of composite functions MSC: Skill
NOT: Section 1.3

6. Let $f(x) = 3 + 2x^2$ and $g(x) = \sqrt{x + 3}$. Find the limit.

$$\lim_{x \rightarrow 2} g(f(x))$$

- a. $\frac{\sqrt{6}}{2}$
- b. $\frac{\sqrt{14}}{2}$
- c. $\frac{\sqrt{11}}{2}$
- d. $\frac{\sqrt{10}}{2}$
- e. $\frac{\sqrt{2}}{2}$

ANS: B PTS: 1 DIF: Medium REF: 1.3.25c
OBJ: Evaluate the limit of composite functions MSC: Skill
NOT: Section 1.3

7. Let $f(x) = x^2 - x - 5$ and $g(x) = \sqrt[3]{x + 14}$. Find the limits.

$$\lim_{x \rightarrow 3} g(f(x))$$

- a. $-\sqrt[3]{1}$
- b. $\sqrt[3]{29}$
- c. $-\sqrt[3]{15}$
- d. $\sqrt[3]{15}$
- e. $\sqrt[3]{1}$

ANS: D

PTS: 1

DIF: Medium

REF: 1.3.26c

OBJ: Evaluate the limit of composite functions

MSC: Skill

NOT: Section 1.3

8. Find the limit.

$$\lim_{x \rightarrow \pi} \tan\left(\frac{x}{3}\right)$$

- a. $\frac{-1}{\sqrt{3}}$
- b. $\frac{\sqrt{3}}{\sqrt{3}}$
- c. $\frac{-\sqrt{3}}{\sqrt{3}}$
- d. $\frac{1}{\sqrt{3}}$
- e. does not exist

ANS: B

PTS: 1

DIF: Medium

REF: 1.3.28

OBJ: Evaluate the limit of the function

MSC: Skill

NOT: Section 1.3

9. Find the limit.

$$\lim_{x \rightarrow 2} \cos \frac{\pi x}{3}$$

- a. $\frac{1}{2}$
- b. $-\frac{1}{2}$
- c. $-\frac{\sqrt{3}}{2}$
- d. $\frac{\sqrt{3}}{2}$
- e. 0

ANS: B

PTS: 1

DIF: Easy

REF: 1.3.29

OBJ: Evaluate a limit using properties of limits

MSC: Skill

NOT: Section 1.3

10. Find the limit.

$$\lim_{x \rightarrow 5} \cos\left(\frac{\pi x}{6}\right)$$

- a. $-\frac{1}{2}$
- b. 0
- c. $\frac{1}{2}$
- d. $-\frac{\sqrt{3}}{2}$
- e. $\frac{\sqrt{3}}{2}$

ANS: D PTS: 1 DIF: Medium REF: 1.3.29
 OBJ: Evaluate a limit using properties of limits MSC: Skill
 NOT: Section 1.3

11. Find the limit.

$$\lim_{x \rightarrow \frac{3\pi}{4}} \sin x$$

- a. $\frac{\sqrt{3}}{2}$
- b. $-\frac{\sqrt{2}}{2}$
- c. $-\frac{1}{2}$
- d. $\frac{\sqrt{2}}{2}$
- e. does not exist

ANS: D PTS: 1 DIF: Medium REF: 1.3.33
 OBJ: Evaluate a limit using properties of limits MSC: Skill
 NOT: Section 1.3

12. Suppose that $\lim_{x \rightarrow c} f(x) = -13$ and $\lim_{x \rightarrow c} g(x) = -10$. Find the following limit.

$$\lim_{x \rightarrow c} [f(x) + g(x)]$$

- a. 0
- b. -10
- c. -3
- d. -23
- e. 130

ANS: D PTS: 1 DIF: Medium REF: 1.3.37b

OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
NOT: Section 1.3

13. Suppose that $\lim_{x \rightarrow c} f(x) = -11$ and $\lim_{x \rightarrow c} g(x) = -3$. Find the following limit.

$$\lim_{x \rightarrow c} [f(x) - g(x)]$$

- a. -11
- b. -8
- c. 33
- d. -14
- e. 0

ANS: B PTS: 1 DIF: Medium REF: 1.3.37b
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
NOT: Section 1.3

14. Suppose that $\lim_{x \rightarrow c} f(x) = -15$ and $\lim_{x \rightarrow c} g(x) = -10$. Find the following limit.

$$\lim_{x \rightarrow c} [f(x)g(x)]$$

- a. 10
- b. -5
- c. -25
- d. -15
- e. 150

ANS: E PTS: 1 DIF: Medium REF: 1.3.37c
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
NOT: Section 1.3

15. Suppose that $\lim_{x \rightarrow c} f(x) = 7$ and $\lim_{x \rightarrow c} g(x) = 3$. Find the following limit.

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$$

- a. 21
- b. $\frac{3}{7}$
- c. -21
- d. $\frac{7}{3}$
- e. does not exist

ANS: D PTS: 1 DIF: Medium REF: 1.3.37c
OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
NOT: Section 1.3

16. Suppose that $\lim_{x \rightarrow c} f(x) = 5$. Find the following limit.

$$\lim_{x \rightarrow c} [f(x)^3]$$

- a. 2
- b. 125
- c. 8
- d. 0
- e. 15

ANS: B PTS: 1 DIF: Medium REF: 1.3.39a
 OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
 NOT: Section 1.3

17. Suppose that $\lim_{x \rightarrow c} f(x) = -5$. Find the following limit.

$$\lim_{x \rightarrow c} 3f(x)$$

- a. -5
- b. 15
- c. -15
- d. 3c
- e. 3

ANS: C PTS: 1 DIF: Medium REF: 1.3.39c
 OBJ: Evaluate the limit of a function using properties of limits MSC: Skill
 NOT: Section 1.3

18. Find the following limit (if it exists). Write a simpler function that agrees with the given function at all but one point.

$$\lim_{x \rightarrow -4} \frac{8x^2 + 40x + 32}{x + 4}$$

- a. 40
- b. -24
- c. 24
- d. -40
- e. does not exist

ANS: B PTS: 1 DIF: Medium REF: 1.3.46
 OBJ: Evaluate the limit of the function and simplify it to an identical function except at the discontinuity point MSC: Skill NOT: Section 1.3

19. Find the limit (if it exists).

$$\lim_{x \rightarrow -8} \frac{x + 8}{x^2 - 64}$$

- a. $-\frac{1}{16}$
- b. $-\frac{1}{32}$
- c. -32
- d. -8
- e. $\frac{1}{16}$

ANS: A PTS: 1 DIF: Medium REF: 1.3.51
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

20. Find the limit (if it exists).

$$\lim_{x \rightarrow 5} \frac{\sqrt{x+4} - 3}{x-5}$$

- a. 6
- b. $\frac{1}{6}$
- c. 0
- d. $\frac{1}{6}$
- e. Limit does not exist.

ANS: D PTS: 1 DIF: Medium REF: 1.3.55
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

21. Find the limit (if it exists).

$$\lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - 9(x + \Delta x) + 2 - (x^2 - 9x + 2)}{\Delta x}$$

- a. $\frac{1}{3}x^3 - \frac{9}{2}x^2 + 2x$
- b. $\frac{2x-9}{x^3 - 9x^2 + 2x}$
- c. $\frac{2x-9}{x^3 - 9x^2 + 2x}$
- d. $\frac{x^2 - 9x + 2}{x^3 - 9x^2 + 2x}$
- e. does not exist

ANS: B PTS: 1 DIF: Medium REF: 1.3.63
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

22. Determine the limit (if it exists).

$$\lim_{x \rightarrow 0} \frac{12(1 - \cos x)}{x^2}$$

- a. 6
- b. 48
- c. 10
- d. 24
- e. does not exist

ANS: A PTS: 1 DIF: Medium REF: 1.3.66
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

23. Determine the limit (if it exists).

$$\lim_{x \rightarrow 0} \frac{\sin x(1 - \cos x)}{2x^8}$$

- a. 8
- b. $\frac{1}{2}$
- c. $\frac{0}{2}$
- d. 2
- e. does not exist

ANS: E PTS: 1 DIF: Medium REF: 1.3.67
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

24. Determine the limit (if it exists).

$$\lim_{x \rightarrow 0} \frac{\sin^4 x}{x^3}$$

- a. 1
- b. 0
- c. $\frac{2}{3}$
- d. ∞
- e. does not exist

ANS: B PTS: 1 DIF: Medium REF: 1.3.69
 OBJ: Evaluate the limit of a function analytically MSC: Skill
 NOT: Section 1.3

25. Find $\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$ where $f(x) = 4x - 3$.

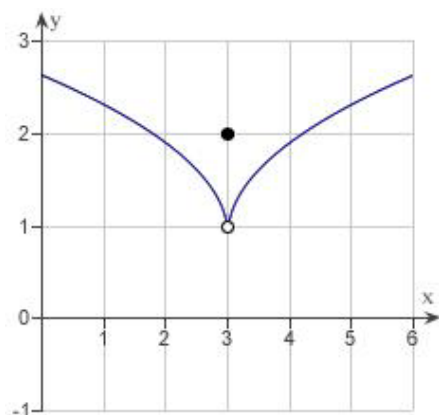
- a. 1
- b. 4
- c. -3
- d. 0
- e. Limit does not exist.

ANS: B PTS: 1 DIF: Medium REF: 1.3.85
 OBJ: Evaluate the limit of a difference quotient MSC: Skill
 NOT: Section 1.3

MULTIPLE CHOICE

1. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = 3$.

(i) $\lim_{x \rightarrow 3^+} f(x)$ (ii) $\lim_{x \rightarrow 3^-} f(x)$ (iii) $\lim_{x \rightarrow 3} f(x)$

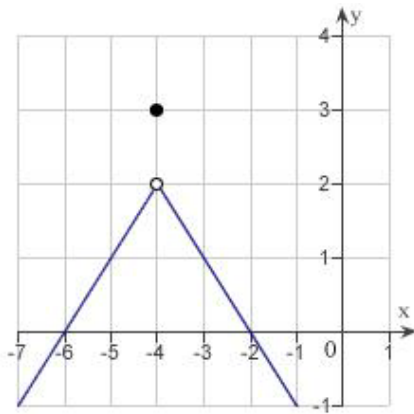


- a. 1, 1, 1, not continuous
 b. 2, 2, 2, continuous
 c. 4, 4, 4, not continuous
 d. 2, 2, 2, not continuous
 e. 1, 1, 1, continuous

ANS: A PTS: 1 DIF: Medium REF: 1.4.3a
 OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill
 NOT: Section 1.4

2. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = -4$.

(i) $\lim_{x \rightarrow -4^+} f(x)$ (ii) $\lim_{x \rightarrow -4^-} f(x)$ (iii) $\lim_{x \rightarrow -4} f(x)$

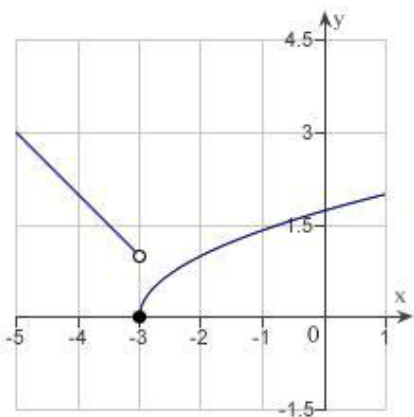


- a. 3, 3, 3, continuous
 b. 2, 2, 2, not continuous
 c. 3, 3, 3, not continuous
 d. -4, -4, -4, continuous
 e. 2, 2, 2, continuous

ANS: B PTS: 1 DIF: Medium REF: 1.4.4b
 OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill
 NOT: Section 1.4

3. Use the graph to determine the following limits, and discuss the continuity of the function at $x = -3$.

- (i) $\lim_{x \rightarrow -3^+} f(x)$ (ii) $\lim_{x \rightarrow -3^-} f(x)$ (iii) $\lim_{x \rightarrow -3} f(x)$



- a. 1, -1, does not exist, not continuous
 b. 1, 0, does not exist, not continuous
 c. 0, 1, does not exist, not continuous
 d. -3, 0, does not exist, not continuous
 e. 0, 1, 0, continuous

ANS: C PTS: 1 DIF: Medium REF: 1.4.6c
 OBJ: Estimate a limit and points of discontinuity from a graph MSC: Skill

NOT: Section 1.4

4. Find the limit (if it exists).

$$\lim_{x \rightarrow 11^+} \frac{11 - x}{x^2 - 121}$$

- a. $\frac{1}{22}$
- b. $\frac{0}{0}$
- c. Limit does not exist.
- d. $-\frac{1}{22}$
- e. $\frac{1}{242}$

ANS: D PTS: 1
OBJ: Evaluate one-sided limits

DIF: Easy
MSC: Skill

REF: 1.4.10
NOT: Section 1.4

5. Find the limit (if it exists).

$$\lim_{x \rightarrow 36^-} \frac{\sqrt{x} - 6}{x - 36}$$

- a. $\frac{0}{0}$
- b. $-\frac{1}{12}$
- c. $\frac{1}{72}$
- d. $\frac{1}{12}$
- e. Limit does not exist.

ANS: D PTS: 1
OBJ: Evaluate one-sided limits

DIF: Medium
MSC: Skill

REF: 1.4.12
NOT: Section 1.4

6. Find the limit (if it exists).

$$\lim_{x \rightarrow 1^-} f(x), \text{ where } f(x) = \begin{cases} x^3 + 10, & x < 1 \\ x + 10, & x \geq 1 \end{cases}$$

- a. Limit does not exist.
- b. $\frac{0}{0}$
- c. $\frac{10}{10}$
- d. $\frac{11}{11}$
- e. $\frac{30}{30}$

ANS: D PTS: 1
OBJ: Evaluate one-sided limits

DIF: Medium
MSC: Skill

REF: 1.4.19
NOT: Section 1.4

7. Find the limit (if it exists). Note that $f(x) = \lfloor |x| \rfloor$ represents the greatest integer function.

$$\lim_{x \rightarrow -6^+} (-3\lfloor |x| \rfloor - 8)$$

- a. 13
- b. -10
- c. 10
- d. -13
- e. does not exist

ANS: A

PTS: 1

DIF: Medium

REF: 1.4.23

OBJ: Evaluate one-sided limits

MSC: Skill

NOT: Section 1.4

8. Find the limit (if it exists). Note that $f(x) = \lfloor |x| \rfloor$ represents the greatest integer function.

$$\lim_{x \rightarrow 5^+} (2x - \lfloor |x| \rfloor)$$

- a. 6
- b. Limit does not exist.
- c. 5
- d. 0
- e. 4

ANS: C

PTS: 1

DIF: Medium

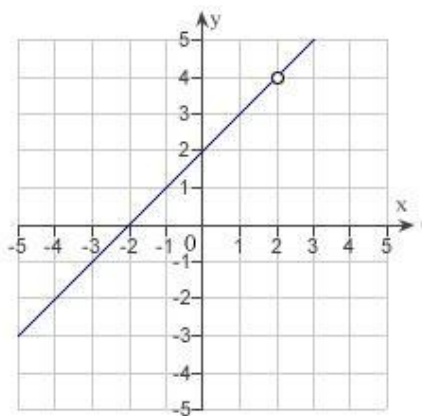
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OBJ: Evaluate one-sided limits

MSC: Skill

NOT: Section 1.4

9. Discuss the continuity of the function $f(x) = \frac{x^2 - 4}{x - 2}$.



- a. $f(x)$ is discontinuous at $x = -2$.
- b. $f(x)$ is discontinuous at $x = -2, 2$.
- c. $f(x)$ is discontinuous at $x = 2$.
- d. $f(x)$ is continuous for all real x .
- e. $f(x)$ is continuous at $x = 4$.

ANS: C PTS: 1 DIF: Easy REF: 1.4.28
 OBJ: Identify the discontinuities of a function if any exist MSC: Skill
 NOT: Section 1.4

10. Find the x -values (if any) at which the function $f(x) = 13x^2 - 15x - 15$ is not continuous. Which of the discontinuities are removable?

- a. $x = 4$, removable
- b. $x = 0$, removable
- c. $x = \frac{15}{26}$, not removable.
- d. continuous everywhere
- e. $x = \frac{15}{26}$, removable.

ANS: D PTS: 1 DIF: Medium REF: 1.4.38
 OBJ: Identify the removable discontinuities of a function MSC: Skill
 NOT: Section 1.4

11. Find the x -values (if any) at which $f(x) = \frac{x}{x^2 - 2x}$ is not continuous.

- a. $f(x)$ is not continuous at $x = 0$ and $f(x)$ has a removable discontinuity at $x = 0$.
- b. $f(x)$ is not continuous at $x = 0, 2$ and both the discontinuities are nonremovable.
- c. $f(x)$ is not continuous at $x = 2$ and $f(x)$ has a removable discontinuity at $x = 2$.
- d. $f(x)$ is not continuous at $x = 0, 2$ and $f(x)$ has a removable discontinuity at $x = 0$.
- e. $f(x)$ is continuous for all real x .

ANS: D PTS: 1 DIF: Easy REF: 1.4.43
 OBJ: Identify the removable discontinuities of a function MSC: Skill
 NOT: Section 1.4

12. Find the x -values (if any) at which the function $f(x) = \frac{x}{x^2 - 100}$ is not continuous. Which of the discontinuities are removable?

- a. 10 and -10, removable
- b. discontinuous everywhere
- c. continuous everywhere
- d. 10 and -10, not removable
- e. 0, removable

ANS: D PTS: 1 DIF: Medium REF: 1.4.44
 OBJ: Identify the removable discontinuities of a function MSC: Skill
 NOT: Section 1.4

13. Find the x -values (if any) at which the function $f(x) = \frac{x+2}{x^2 + 6x + 8}$ is not continuous. Which of the discontinuities are removable?

- a. no points of discontinuity

- b. $x = -2$ (not removable), $x = -4$ (removable)
- c. $x = -2$ (removable), $x = -4$ (not removable)
- d. no points of continuity
- e. $x = -2$ (not removable), $x = -4$ (not removable)

ANS: C PTS: 1 DIF: Medium REF: 1.4.47
 OBJ: Identify the removable discontinuities of a function MSC: Skill
 NOT: Section 1.4

14. Find the x -values (if any) at which $f(x) = \frac{|x-3|}{x-3}$ is not continuous.

- a. $f(x)$ is not continuous at $x = 3$ and the discontinuity is nonremovable.
- b. $f(x)$ is not continuous at $x = 0$ and the discontinuity is removable.
- c. $f(x)$ is continuous for all real x .
- d. $f(x)$ is not continuous at $x = 3$ and the discontinuity is removable.
- e. $f(x)$ is not continuous at $x = 0$, -3 and $x = 0$ is a removable discontinuity.

ANS: A PTS: 1 DIF: Medium REF: 1.4.49
 OBJ: Identify the removable discontinuities of a function MSC: Skill
 NOT: Section 1.4

15. Find the constant a such that the function

$$f(x) = \begin{cases} -4 \cdot \frac{\sin x}{x}, & x < 0 \\ a + 7x, & x \geq 0 \end{cases}$$

is continuous on the entire real line.

- a. 1
- b. $-\frac{7}{4}$
- c. $\frac{7}{4}$
- d. $\frac{4}{7}$
- e. $-\frac{4}{7}$

ANS: E PTS: 1 DIF: Medium REF: 1.4.66
 OBJ: Identify the value of a parameter to ensure a function is continuous
 MSC: Skill NOT: Section 1.4

16. Find the constant a such that the function

$$f(x) = \begin{cases} 6, & x \leq -5 \\ ax + b, & -5 < x < 1 \\ -6, & x \geq 1 \end{cases}$$

is continuous on the entire real line.

- a. $a = 2, b = 0$
- b. $a = 2, b = -4$

- c. $\underline{a = -2, b = -4}$
 d. $\underline{a = -2, b = 4}$
 e. $\underline{a = 2, b = 4}$

ANS: C PTS: 1 DIF: Medium REF: 1.4.67
 OBJ: Identify the value of a parameter to ensure a function is continuous
 MSC: Skill NOT: Section 1.4

17. Find the value of c guaranteed by the Intermediate Value Theorem.

$$\underline{f(x) = x^2 - 2x + 8, [2, 6], f(c) = 11}$$

- a. $\underline{0}$
 b. $\underline{3}$
 c. $\underline{5}$
 d. $\underline{1}$
 e. $\underline{4}$

ANS: B PTS: 1 DIF: Easy REF: 1.4.91
 OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem
 MSC: Skill NOT: Section 1.4

18. Find the value of c guaranteed by the Intermediate Value Theorem.

$$\underline{f(x) = \frac{x^2 - 5x}{x - 3}, \left[\frac{9}{2}, 18 \right], f(c) = 6}$$

- a. $\underline{11}$
 b. $\underline{2}$
 c. $\underline{1}$
 d. $\underline{9}$
 e. $\underline{10}$

ANS: D PTS: 1 DIF: Medium REF: 1.4.94
 OBJ: Identify the value of c guaranteed by the Intermediate Value Theorem
 MSC: Skill NOT: Section 1.4

19. A long distance phone service charges $\underline{\$0.35}$ for the first $\underline{10}$ minutes and $\underline{\$0.1}$ for each additional minute or fraction thereof. Use the greatest integer function to write the cost C of a call in terms of time t (in minutes).

- a.
$$C = \begin{cases} 0.35 & 0 < t \leq 10 \\ 0.35 + 0.1 \lceil |t - 10| \rceil & t > 10, t \text{ is not an integer} \\ 0.35 + 0.1(t - 9) & t > 10, t \text{ is an integer} \end{cases}$$
- b.
$$C = \begin{cases} 0.35 & 0 < t \leq 10 \\ 0.35 + 0.1(t - 10) & t > 10 \end{cases}$$
- c.
$$C = \begin{cases} 0.35 & 0 < t \leq 10 \\ 0.35 + 0.1 \lceil |t - 9| \rceil & t > 10 \end{cases}$$

d.
$$C = \begin{cases} 0.35 & 0 < t \leq 10 \\ 0.35 + 0.1[|t - 10|] & t > 10 \end{cases}$$

e.
$$C = \begin{cases} 0.35 & 0 < t \leq 10 \\ 0.35 + 0.1[|t - 9|] & t > 10, t \text{ is not an integer} \\ 0.35 + 0.1(t - 10) & t > 10, t \text{ is an integer} \end{cases}$$

ANS: E PTS: 1 DIF: Medium REF: 1.4.105
 OBJ: Create functions in applications MSC: Application NOT: Section 1.4

20. Find all values of c such that \underline{f} is continuous on $(-\infty, \infty)$.

$$\underline{f(x) = \begin{cases} 4 - x^2, & x \leq c \\ x, & x > c \end{cases}}$$

- a. $\underline{c = 3}$
 b. $\underline{c = 0}$
 c. $\underline{\frac{-1 + \sqrt{17}}{2}}$
 d. $\underline{\frac{1 + \sqrt{17}}{2}, \frac{1 - \sqrt{17}}{2}}$
 e. $\underline{\frac{-1 + \sqrt{17}}{2}, \frac{-1 - \sqrt{17}}{2}}$

ANS: E PTS: 1 DIF: Medium REF: 1.4.115
 OBJ: Identify the value of a parameter to ensure a function is continuous
 MSC: Skill NOT: Section 1.4

MULTIPLE CHOICE

1. Determine whether $f(x) = \frac{x^{10}}{x^2 - 9}$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the tables below.

| | | | | |
|--------|--------|--------|---------|----------|
| x | -3.5 | -3.1 | -3.01 | -3.001 |
| $f(x)$ | | | | |

| | | | | |
|--------|----------|---------|--------|--------|
| x | -2.999 | -2.99 | -2.9 | -2.5 |
| $f(x)$ | | | | |

- a. $\lim_{x \rightarrow -3^-} f(x) = -\infty, \lim_{x \rightarrow -3^+} f(x) = \infty$
- b. $\lim_{x \rightarrow -3^-} f(x) = \infty, \lim_{x \rightarrow -3^+} f(x) = -\infty$
- c. $\lim_{x \rightarrow -3^-} f(x) = \infty, \lim_{x \rightarrow -3^+} f(x) = \infty$
- d. $\lim_{x \rightarrow -3^-} f(x) = -\infty, \lim_{x \rightarrow -3^+} f(x) = -\infty$

ANS: B PTS: 1 DIF: Medium REF: 1.5.11
 OBJ: Evaluate an infinite limit from a table of values MSC: Skill
 NOT: Section 1.5

2. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \frac{5}{(x-3)^2}$.

- a. $x = -3$
- b. $x = 5$
- c. $x = 3, -3$
- d. $x = 3$
- e. no vertical asymptotes

ANS: D PTS: 1 DIF: Easy REF: 1.5.14
 OBJ: Identify the vertical asymptotes (if any) of the graph of a function
 MSC: Skill NOT: Section 1.5

3. Find the vertical asymptotes (if any) of the function $f(x) = \frac{x^2 - 4}{x^2 + 3x + 2}$.

- a. $x = 2$
- b. $x = -1$
- c. $x = 1$
- d. $x = -2$

e. $x = -2$

ANS: B PTS: 1 DIF: Medium REF: 1.5.19
 OBJ: Identify the vertical asymptotes (if any) of the graph of a function
 MSC: Skill NOT: Section 1.5

4. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \frac{1+x}{x^2(1-x)}$.

- a. $x = -1$
- b. $x = 1$
- c. $x = 0$
- d. $x = 1, x = 0$
- e. no vertical asymptotes

ANS: D PTS: 1 DIF: Medium REF: 1.5.20
 OBJ: Identify the vertical asymptotes (if any) of the graph of a function
 MSC: Skill NOT: Section 1.5

5. Find all the vertical asymptotes (if any) of the graph of the function $f(x) = \frac{x^3+8}{x+2}$.

- a. $x = -2$
- b. $x = 8$
- c. $x = 2$
- d. $x = 2, -2$
- e. no vertical asymptotes

ANS: E PTS: 1 DIF: Medium REF: 1.5.25
 OBJ: Identify the vertical asymptotes (if any) of the graph of a function
 MSC: Skill NOT: Section 1.5

6. Find all vertical asymptotes (if any) of the function $f(x) = \frac{x^2+4x+3}{x^3-4x^2-x+4}$.

- a. $x = 4, 1$
- b. $x = 4, 1, -1$
- c. $x = -4, -1$
- d. $x = 1$
- e. $x = -1$

ANS: A PTS: 1 DIF: Medium REF: 1.5.27
 OBJ: Identify the vertical asymptotes (if any) of the graph of a function
 MSC: Skill NOT: Section 1.5

7. Find the vertical asymptotes (if any) of the function $f(x) = \tan(15x)$.

- a. $x = \frac{k}{15} \pi \quad (k = 0, \pm 1, \pm 2, \dots)$
- b. $x = \frac{2k+1}{30} \pi \quad (k = 0, \pm 1, \pm 2, \dots)$

- c. $x = \frac{2k}{15} \pi \quad (k = 0, \pm 1, \pm 2, \dots)$
- d. $x = \frac{2k+1}{15} \pi \quad (k = 0, \pm 1, \pm 2, \dots)$
- e. no vertical asymptotes

ANS: B PTS: 1 DIF: Medium REF: 1.5.29

OBJ: Identify the vertical asymptotes (if any) of the graph of a function

MSC: Skill NOT: Section 1.5

8. Find the limit.

$$\lim_{x \rightarrow 14^+} \frac{x-3}{x-14}$$

- a. $\frac{1}{20}$
- b. $-\infty$
- c. $\frac{0}{20}$
- d. ∞
- e. $-\frac{1}{20}$

ANS: D PTS: 1 DIF: Medium REF: 1.5.40

OBJ: Evaluate one-sided limits MSC: Skill NOT: Section 1.5

9. Find the limit.

$$\lim_{x \rightarrow -10} \frac{x^2 + 10x}{(x^2 + 100)(x + 10)}$$

- a. $\frac{1}{20}$
- b. $-\frac{1}{20}$
- c. $\frac{20}{20}$
- d. $-\frac{10}{20}$
- e. $-\frac{20}{20}$

ANS: B PTS: 1 DIF: Medium REF: 1.5.45

OBJ: Evaluate the limit of a function MSC: Skill NOT: Section 1.5

10. Find the limit.

$$\lim_{x \rightarrow 0^-} \left(x^2 - \frac{1}{x} \right)$$

- a. $\frac{1}{20}$
- b. $\frac{0}{20}$
- c. $-\frac{1}{20}$
- d. $-\infty$
- e. ∞

ANS: E PTS: 1
OBJ: Evaluate one-sided limits

DIF: Medium
MSC: Skill

REF: 1.5.48
NOT: Section 1.5

11. Find the limit (if it exists).

$$\lim_{x \rightarrow \frac{1}{2}} x \tan \pi x$$

- a. ~~$-\infty$~~
b. ~~$\frac{1}{2}$~~
c. ~~0~~
d. ~~∞~~
e. Limit does not exist

ANS: E PTS: 1
OBJ: Identify a limit that does not exist

DIF: Medium
MSC: Skill

REF: 1.5.53
NOT: Section 1.5

12. Use a graphing utility to graph the function $f(x) = \frac{x^2 - 2x + 4}{x^3 + 8}$ and determine the one-sided limit

$$\lim_{x \rightarrow -2^+} f(x).$$

- a. ~~$-\infty$~~
b. ~~∞~~
c. ~~0~~
d. 12
e. ~~8~~

ANS: B PTS: 1
OBJ: Estimate one-sided limits from a graph
NOT: Section 1.5

DIF: Medium

REF: 1.5.55
MSC: Skill

13. Use a graphing utility to graph the function $f(x) = \csc \frac{\pi x}{2}$ and determine the following one-sided limit.

$$\lim_{x \rightarrow 2^-} f(x)$$

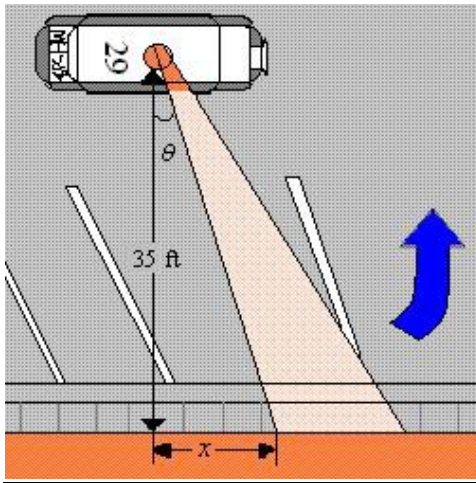
- a. ~~$-\infty$~~
b. 2
c. ~~-2~~
d. ~~∞~~
e. ~~0~~

ANS: D PTS: 1
OBJ: Estimate one-sided limits from a graph
NOT: Section 1.5

DIF: Medium

REF: 1.5.58
MSC: Skill

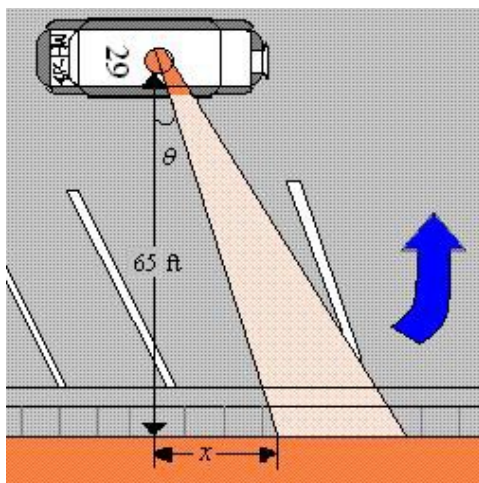
14. A petrol car is parked 35 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is $r = 35\pi \sec^2 \theta$ ft/sec. Find the rate r when θ is $\frac{\pi}{6}$.



- $r = \frac{140}{3}$ ft/sec
- $r = \frac{70\sqrt{3}\pi}{3}$ ft/sec
- $r = \frac{70\sqrt{3}}{3}$ ft/sec
- $r = \frac{140\pi}{3}$ ft/sec
- $r = \frac{70\pi}{3}$ ft/sec

ANS: D PTS: 1 DIF: Easy REF: 1.5.67a
 OBJ: Evaluate functions in applications MSC: Application NOT: Section 1.5

15. A petrol car is parked 65 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is $r = 65\pi \sec^2 \theta$ ft/sec. Find the limit of r as $\theta \rightarrow (\pi/2)^-$.



- ∞
- 65π
- 0
- 65
- $-\infty$

ANS: A

PTS: 1

DIF: Medium

REF: 1.5.67c

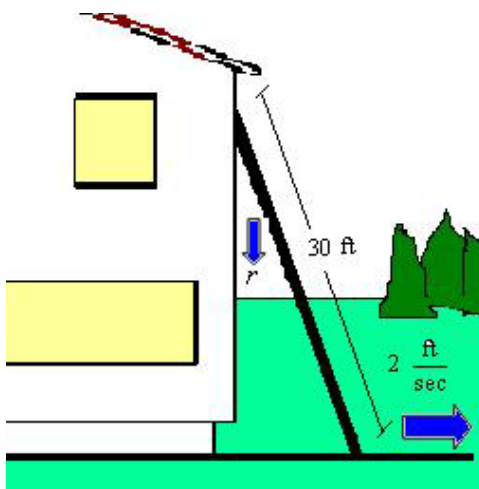
OBJ: Evaluate limits in applications

MSC: Application

NOT: Section 1.5

16. A 30-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

$r = \frac{2x}{\sqrt{900 - x^2}}$ ft/sec, where x is the distance between the base of the ladder and the house. Find the rate r when x is 18 feet.



- $r = \frac{3}{2}$ ft/sec
- $r = \frac{4}{3}$ ft/sec

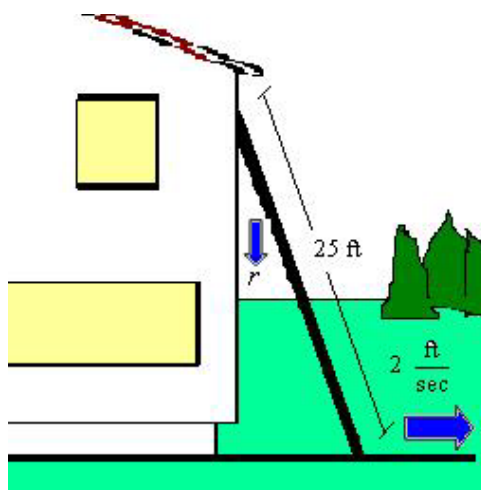
- c. $r = \frac{48}{5} \text{ ft/sec}$
- d. $r = \frac{2}{3} \text{ ft/sec}$
- e. $r = \frac{3}{4} \text{ ft/sec}$

ANS: A PTS: 1 DIF: Easy REF: 1.5.68a
 OBJ: Evaluate functions in applications MSC: Application NOT: Section 1.5

17. A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec where } x \text{ is the distance between the base of the ladder and the house. Find the}$$

limit of r as $x \rightarrow 25^-$.



- a. $-\infty$
- b. 50
- c. 0
- d. ∞
- e. 25

ANS: D PTS: 1 DIF: Medium REF: 1.5.68c
 OBJ: Evaluate limits in applications MSC: Application NOT: Section 1.5