Laboratory 1. Measurement of Length

Comments to the Instructor

For this laboratory in particular it should be strongly emphasized that students read very carefully the General Laboratory Instructions section before reading the laboratory.

Throughout this laboratory manual repeated measurements are called for whenever it is feasible. This demonstrates to students the statistical nature of the measurement process. The main goal of this first laboratory is to introduce the idea that the mean of a set of measurements is the most probable value of the measured quantity, and the standard error is a measure of the precision of the results. In addition the concept of propagation of errors is presented. A distinction is made between the approximation to error propagation represented by the concept of significant figures and the more exact approach using the standard error of the measured quantity when it is known.

It is strongly recommended that students be encouraged to purchase a calculator which is pre-programmed to calculate the mean and standard deviation. For use in later laboratories the calculator should also have pre-programmed routines to perform linear least squares fits to two parameter data.

For this particular laboratory it would be instructive to ask the students to perform the mean and standard deviation calculations both using the pre-programmed routines and directly from the equations given. This will emphasize for the students exactly what operation is being performed by the pre-programmed calculator routine.

The point is for students to observe the statistical variation of random errors. If a mistake is made in reading the meter stick for one or more of the repeated measurements, the resulting error in a single measurement may be so large and non-random that the whole process becomes meaningless. Caution students to be extremely careful and encourage them to check each other in these measurements in order to avoid such personal errors.

Post Laboratory Exam Questions

Fill in the Blank

1. The three types of measurement errors are **personal**, **systematic**, and **random**.
2. The name given to a mistake made by the experimenter either in a measurement or a calculation is a **personal** error.
3. An error that tends to occur in the same direction giving results either consistently above or consistently below the true value is called a **systematic** error.
4. The error produced by unpredictable and unknown variation in the total experimental process is called a **random** error.
5. A voltmeter that is improperly calibrated leads to a **systematic** error.
6. In principle **personal** and **systematic** errors could be eliminated, but there always remains some **random** error.
7. For random errors the probability is **68.3** % that all repeated measurements will be in the range of σ_{n-1} from the mean, where σ_{n-1} is the standard deviation from the mean.
8. For a series of repeated measurements of a quantity, the **mean** of the measurements is the most probable value of that quantity.
9. The **accuracy** of a measurement can be determined only if the true value of the measured quantity is known, but the **precision** can be found from the reproducibility.
10. In this laboratory, if the meter stick had expanded since it was manufactured, the resulting error would be an example of a **systematic** error.

Multiple Choice Questions

1. The type error often caused by faulty equipment is called a (a) random error (b) personal error (c) systematic error (d) calculation error. **Answer (c)**
2. For a normal distribution of random errors what percentage of measurements should fall within $+3\sigma_{n-1}$ of the mean? (a) 50% (b) 68.3% (c) 95.5% (d) 99.7% **Answer (d)**

3. The number of significant figures in the number 0.02340 is (a) 3 (b) 4 (c) 5 (d) 6 **Answer (b)**
4. If the number 46.70 is multiplied by the number 130 the correct answer with the proper number of significant figures is (a) 6100 (b) 6070 (c) 6071 (d) 6071.0**. Answer (a)**
5. The quantity that is a measure of the distribution of repeated measurements about the mean is (a) the standard deviation from the mean (b) the standard error (c) the percentage error (d) the least significant figure. **Answer (a)**
6. The quantity that is a measure of the uncertainty in the mean value of a series of repeated measurements is (a) the standard deviation from the mean (b) the standard error (c) the percentage error (d) the least significant figure **Answer (b)**
7. For a ruler with 1 mm as the smallest marked scale divisions, measurements should be estimated to the nearest (a) 0.01 mm (b) 0.1 mm (c) 1 mm (d) 10 mm. **Answer (b)**
8. A measurement with high precision but poor accuracy is often an indication of the presence of what type of error? (a) random (b) personal (c) systematic (d) statistical **Answer (c)**
9. Even if systematic errors are present the precision of the results are still indicated by the standard deviation from the mean. (a) true (b) false **Answer (a)**
10. Which type of error invalidates all statistical calculations? (a) random (b) personal (c) systematic (d) statistical **Answer (b)**

Pre-Laboratory Assignment

1. State the number of significant figures in each of the following numbers and explain your answer.

   (b) 37.60_____**4**______

   (c) 0.0130____**3**_______

   (d) 13000_____**2**______

   (e) 1.3400____**5**_______

2. Perform the indicated operations to the correct number of significant figures using the rules for significant figures.

(a) 37.60 x 1.23 = **46.2**

(b) 8.975 ÷ 6.7 = **1.3**

(c) 3.765 + 1.2 + 37.21 = **42.2**

Questions 3-6. Three students named Abe, Barb, and Cal make measurements (in m) of the length of a table. Lengths, means, σ_{n-1}, α are tabulated in the table below:

Student	L_1	L_2	L_3	L_4	$\bar{L}$	σ_{n-1}	α
Abe	1.4717	1.4711	1.4722	1.4715	1.4716	0.00046	0.0002
Barb	1.4753	1.4759	1.4756	1.4749	1.4754	0.00043	0.0002
Cal	1.4719	1.4723	1.4727	1.4705	1.4719	0.00096	0.0005

Only one significant figure is kept in the standard error α, and this determines the number of significant figures in the mean. The actual length of the table is 1.4715 m.

3. **Accuracy of a measurement is determined by its difference from the actual value of 1.4715 m. Based on that criterion Abe's value of 1.4716 m is the most accurate.**

4. $$\bar{L} = (1/4)(1.4717 + 1.4711 + 1.4722 + 1.4715) = 1.471625 = 1.4716$$

$$\sigma_{n-1} = \sqrt{(1/(4-1))(1.4717-1.1716)^2 + (1.4711-1.4716)^2 + (1.4722-1.4716)^2 + (1.4715+1.4716)^2}$$
$$= \sqrt{(1/3)(0.00000001 + 0.00000025 + 0.00000036 + 0.00000001)} = 0.00046$$

$$\alpha = (\sigma_{n-1})/\left(\sqrt{4}\right) = 0.00023 = 0.0002 \text{ (one significant figure)}$$

5. **One indication of a systematic error is a measurement with high precision but bad accuracy. Barb has a systematic error because her α = 0.0002 m (high precision),but her $\bar{L}$ = 1.4754 is different from the actual value of 1.4715 by 0.0039 m.**

6. **Abe clearly has the best measurement with a difference from the actual value of only 0.0001 m and high precision noted by α = 0.0002 m.**

Laboratory Report

Data and Calculations Table 1 (nearest 0.0001 m which is 0.1 mm)

Trial	X_1 (cm)	X_1 (m)	X_2 (cm)	X_2 (m)	$L_i = X_2 - X_1$ (m)	$L_i - \bar{L}$ (m)	$(L_i - \bar{L})^2$ (m^2)
1	33.41	0.3341	170.58	1.7058	1.3717	0.0002	0.00000004
2	28.65	0.2865	165.76	1.6576	1.3711	-0.0004	0.00000016
3	36.20	0.3620	173.32	1.7332	1.3712	-0.0003	0.00000009
4	34.74	0.3474	171.91	1.7191	1.3717	0.0002	0.00000004
5	26.23	0.2623	163.40	1.6340	1.3713	-0.0002	0.00000004
6	28.49	0.2849	165.65	1.6565	1.3716	0.0001	0.00000001
7	24.36	0.2436	161.57	1.6157	1.3721	0.0006	0.00000036
8	31.05	0.3105	168.22	1.6822	1.3717	0.0002	0.00000004
9	27.36	0.2736	164.50	1.6450	1.3714	-0.0001	0.00000001
10	33.17	0.3317	170.32	1.7032	1.3715	0.0000	0.00000000
						$\sum_{1}^{n}(L_i - \bar{L})^2 =$	0.00000079

$\bar{L}$ = **1.37153 m**_ σ_{n-1}^{L} = **0.00029 m** $\bar{L} - \sigma_{n-1}^{L}$ = **1.37124 m** $\bar{L} + \sigma_{n-1}^{L}$ = **1.37182 m** α_L = **0.00009 m**

Data and Calculations Table 2.(nearest 0.0001 m which is 0.1 mm)

Trial	Y_1 (cm)	Y_1 (m)	Y_2 (cm)	Y_2 (m)	$W_i = Y_2 - Y_1$ (m)	$W_i - \bar{W}$ (m)	$(W_i - \bar{W})^2$ (m^2)
1	39.61	0.3961	116.01	1.1601	0.7640	0.0002	0.00000004
2	38.09	0.3809	114.49	1.1449	0.7640	0.0002	0.00000004
3	40.63	0.4063	117.02	1.1702	0.7639	0.0001	0.00000001
4	47.39	0.4739	123.78	1.2378	0.7639	0.0001	0.00000001
5	66.17	0.6617	142.58	1.4258	0.7641	0.0003	0.00000009
6	80.15	0.8015	156.51	1.5651	0.7636	-0.0002	0.00000004
7	91.49	0.9149	170.86	1.7086	0.7637	-0.0001	0.00000001
8	41.73	0.4173	118.11	1.1811	0.7638	0.0000	0.00000000
9	64.69	0.6469	141.07	1.4107	0.7638	0.0000	0.00000000
10	74.27	0.7427	150.63	1.5063	0.7636	-0.0002	0.00000004
						$\sum_{1}^{n}(W_i - \bar{W})^2 =$	0.00000028

$\overline{W}$= **0.76384 m** σ_{n-1}^{W}= **0.00017 m** $\overline{W}$ -σ_{n-1}^{W}=**0.76367 m** $\overline{W}$ +σ_{n-1}^{W}=**0.76401 m** α_W = **0.00005 m**

A =$\overline{L}$ × $\overline{W}$ = __**1.0476 m^2**_____ α_A = __**0.0001 m^2**_______

Questions

1. What is the range of these lengths for your data? From _**1.37124**_____m to _**1.37182**__m. How many of your 10 measurements of the length of the table fall in this range? ____**7**_____. **This data agrees as nearly as possible given than 70% of the measurements fall in this range. Generally student data will not be that good but might show between 6 and 8 measurements in that range which would be excellent data.**

2. Answer the same question for the width. Range of $\overline{W}$ -σ_{n-1}^{W} to $\overline{W}$ +σ_{n-1}^{W} is from ____**0.76367**____m to ___**0.76401**___m. The number of measurements that fall in that range is ______**7**______. **Again this data agrees as nearly as possible with 70% of the measurements in the range. Student data will be excellent if 6 to 8 measurements are in the correct range.**

3. Calculate the value of 3σ_{n-1}^{L}. Do any of your measurements of length have a deviation from the mean greater than that value? If so calculate how many times larger than σ_{n-1}^{L} it is. **The value of 3σ_{n-1}^{L} is 3(0.00029) = 0.00087 m. None of the length data have deviations from the mean of greater than this. Student answers should reflect their data.**

4. **The value of 3σ_{n-1}^{W} is 3(0.00017) = 0.00051 m. None of the width data have deviations from the mean of greater. Student answers should reflect their data.**

5. **No statement can be made about the accuracy of these measurements because the actual value for the length and width of the table are unknown. The best statement of the precision of the measurement is that the percentage standard error of the length is 0.007% and the percentage standard error of the width is also 0.007%.**