CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

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Equations, Inequalities, and Mathematical Modeling

Section 1.1 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. circle; (h, k); r

6. numerical

7. (a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$

Yes, the point *is* on the graph.

8. (a) $(1, 2)$: $2 \stackrel{?}{=} \sqrt{5 - 1}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 0)$: $0 \stackrel{?}{=} \sqrt{5 - 5}$

$0 = 0$

Yes, the point *is* on the graph.

9. (a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

10. (a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

11. (a) $(2, 3)$: $3 \stackrel{?}{=} |2 - 1| + 2$

$3 \stackrel{?}{=} 1 + 2$

$3 = 3$

Yes, the point *is* on the graph.

(b) $(-1, 0)$: $0 \stackrel{?}{=} |-1 - 1| + 2$

$0 \stackrel{?}{=} 2 + 2$

$0 \neq 4$

No, the point *is not* on the graph.

12. (a) $(1, 2)$: $2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point *is not* on the graph.

(b) $(1, -1)$: $2(1) - (-1) - 3 \stackrel{?}{=} 0$

$2 + 1 - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

13. (a) $(3, -2)$: $(3)^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + (2)^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

14. (a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$$

$$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$$

$$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$$

$$-\frac{16}{3} = -\frac{16}{3}$$

Yes, the point *is* on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$$

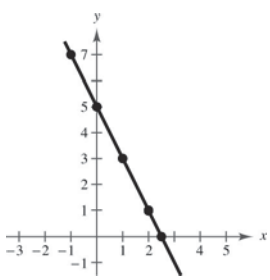
$$-9 - 18 \stackrel{?}{=} 9$$

$$-27 \neq 9$$

No, the point *is not* on the graph.

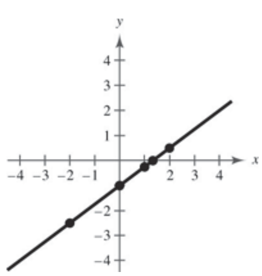
15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



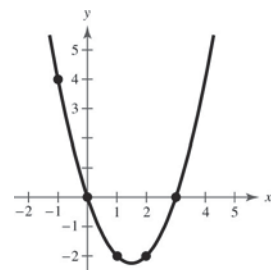
16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



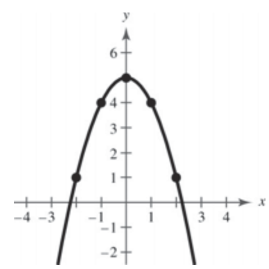
17. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



18. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

22. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

23. x-intercept: $(1, 0)$

y-intercept: $(0, 2)$

24. x-intercepts: $(0, 0)$, $(0, \pm 2)$

y-intercept: $(0, 0)$

25. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

26. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

27. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

28. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

29. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

30. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

31. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

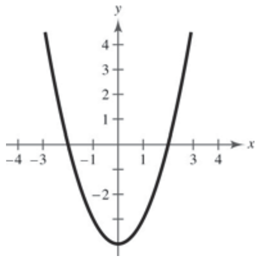
32. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

33.



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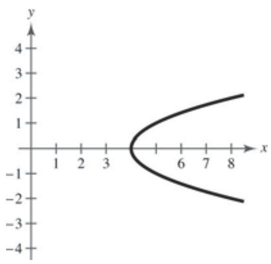

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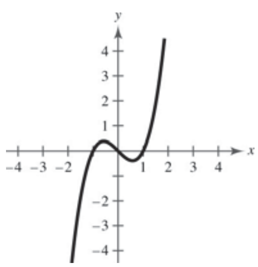

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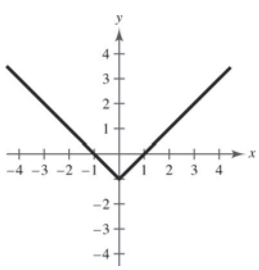
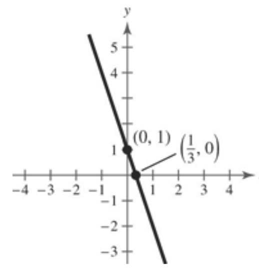


37. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

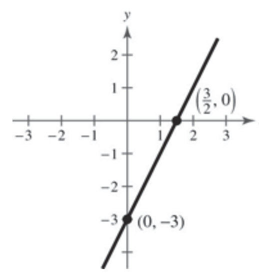


38. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



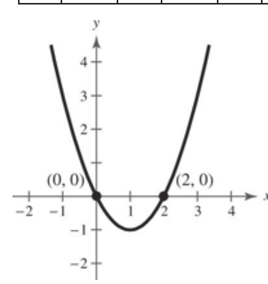
39. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	–1	0	1	2	3
y	3	0	–1	0	3

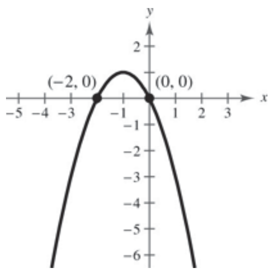


40. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



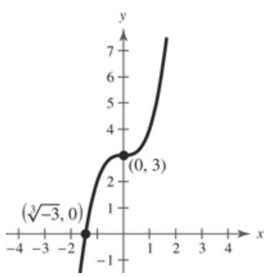
41. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	-2	-1	0	1	2
y	-5	2	3	4	11

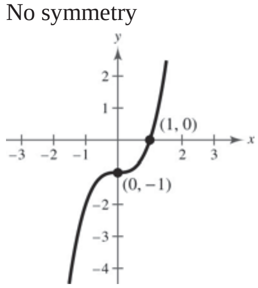


42. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



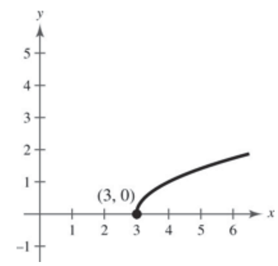
43. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

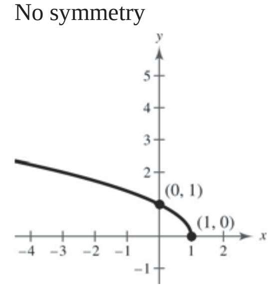


44. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



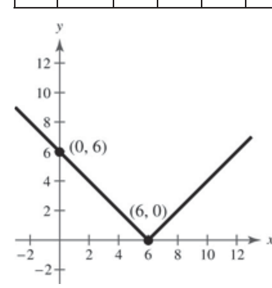
45. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	-2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

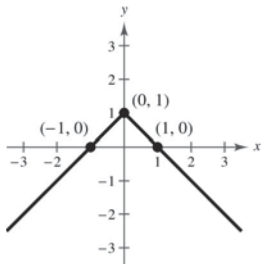


46. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



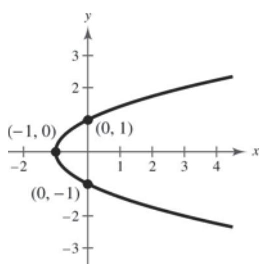
47. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	-1	0	3
y	0	± 1	± 2

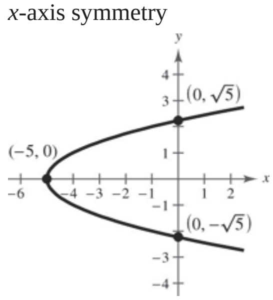


48. $x = y^2 - 5$

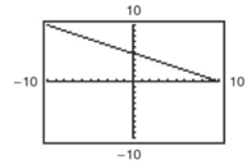
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

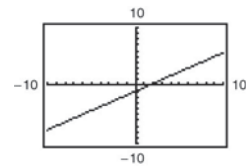


49. $y = 5 - \frac{1}{2}x$



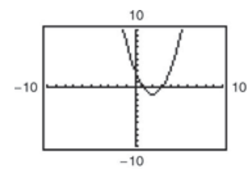
Intercepts: $(10, 0), (0, 5)$

50. $y = \frac{2}{3}x - 1$



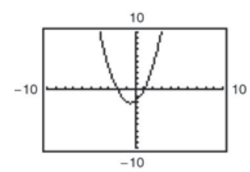
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

51. $y = x^2 - 4x + 3$



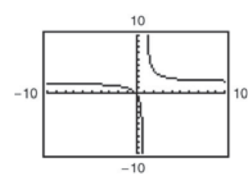
Intercepts: $(3, 0), (1, 0), (0, 3)$

52. $y = x^2 + x - 2$



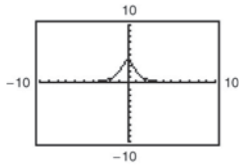
Intercepts: $(-2, 0), (1, 0), (0, -2)$

53. $y = \dfrac{2x}{x - 1}$



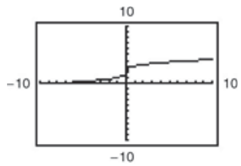
Intercept: $(0, 0)$

54. $y = \dfrac{4}{x^2 + 1}$



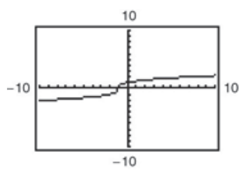
Intercept: $(0, 4)$

55. $y = \sqrt[3]{x} + 2$



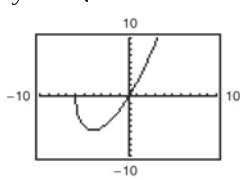
Intercepts: $(-8, 0), (0, 2)$

56. $y = \sqrt[3]{x + 1}$



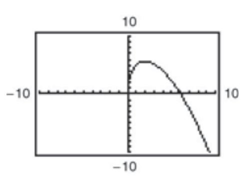
Intercepts: $(-1, 0), (0, 1)$

57. $y = x\sqrt{x + 6}$



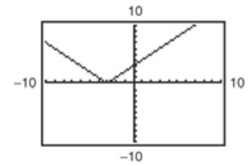
Intercepts: $(0, 0), (-6, 0)$

58. $y = (6 - x)\sqrt{x}$



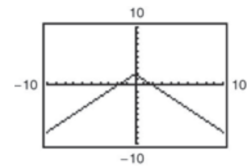
Intercepts: $(0, 0), (6, 0)$

59. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

60. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

61. Center: $(0, 0)$; Radius: 4

$$(x - 0)^2 + (y - 0)^2 = 4^2$$
$$x^2 + y^2 = 16$$

62. Center: $(0, 0)$; Radius: 5

$$(x - 0)^2 + (y - 0)^2 = 5^2$$
$$x^2 + y^2 = 25$$

63. Center: $(2, -1)$; Radius: 4

$$(x - 2)^2 + (y - (-1))^2 = 4^2$$
$$(x - 2)^2 + (y + 1)^2 = 16$$

64. Center: $(-7, -4)$; Radius: 7

$$(x - (-7))^2 + (y - (-4))^2 = 7^2$$
$$(x + 7)^2 + (y + 4)^2 = 49$$

65. Center: $(-1, 2)$; Solution point: $(0, 0)$

$$(x - (-1))^2 + (y - 2)^2 = r^2$$
$$(0 + 1)^2 + (0 - 2)^2 = r^2 \Rightarrow 5 = r^2$$
$$(x + 1)^2 + (y - 2)^2 = 5$$

66. Center: $(3, -2)$; Solution point: $(-1, 1)$

$$r = \sqrt{(3-(-1))^2 + (-2-1)^2}$$
$$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$
$$(x-3)^2 + (y-(-2))^2 = 5^2$$
$$(x-3)^2 + (y+2)^2 = 25$$

67. Endpoints of a diameter: $(0, 0), (6, 8)$

Center: $\left(\dfrac{0+6}{2}, \dfrac{0+8}{2}\right) = (3, 4)$

$$(x-3)^2 + (y-4)^2 = r^2$$
$$(0-3)^2 + (0-4)^2 = r^2 \Rightarrow 25 = r^2$$
$$(x-3)^2 + (y-4)^2 = 25$$

68. Endpoints of a diameter: $(-4, -1), (4, 1)$

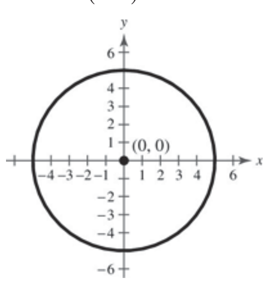
$$r = \frac{1}{2}\sqrt{(-4-4)^2 + (-1-1)^2}$$
$$= \frac{1}{2}\sqrt{(-8)^2 + (-2)^2}$$
$$= \frac{1}{2}\sqrt{64 + 4}$$
$$= \frac{1}{2}\sqrt{68} = \left(\frac{1}{2}\right)(2)\sqrt{17} = \sqrt{17}$$

Midpoint of diameter (center of circle):

$$\left(\frac{-4+4}{2}, \frac{-1+1}{2}\right) = (0, 0)$$
$$(x-0)^2 + (y-0)^2 = \left(\sqrt{17}\right)^2$$
$$x^2 + y^2 = 17$$

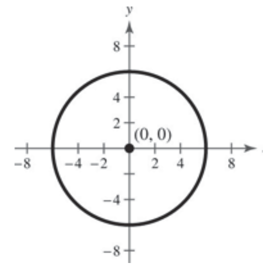
69. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



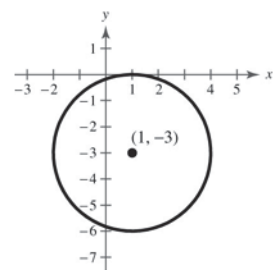
70. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



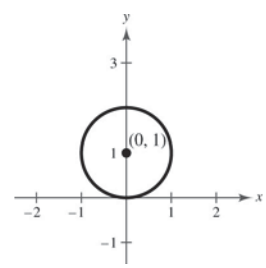
71. $(x-1)^2 + (y+3)^2 = 9$

Center: $(1, -3)$, Radius: 3



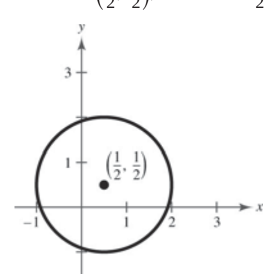
72. $x^2 + (y-1)^2 = 1$

Center: $(0, 1)$, Radius: 1



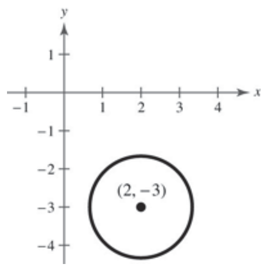
73. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

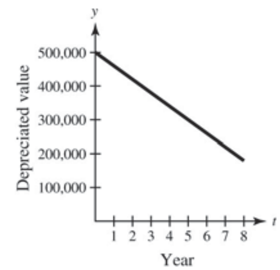


74. $(x-2)^2 + (y+3)^2 = \frac{16}{9}$

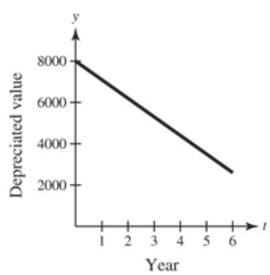
Center: $(2, -3)$, Radius: $\frac{4}{3}$



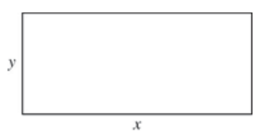
75. $y = 500{,}000 - 40{,}000t,\ 0 \le t \le 8$



76. $y = 8000 - 900t,\ 0 \le t \le 6$



77. (a)



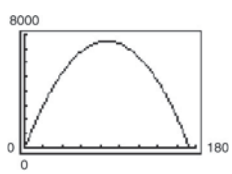
(b) $2x + 2y = \frac{1040}{3}$

$2y = \frac{1040}{3} - 2x$

$y = \frac{520}{3} - x$

$A = xy = x\left(\frac{520}{3} - x\right)$

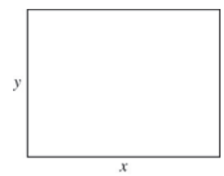
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

78. (a)



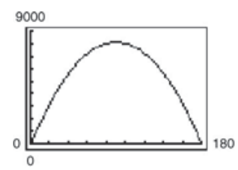
(b) $P = 360$ meters so:

$2x + 2y = 360$

$w = y = 180 - x$

$A = lw = x(180 - x)$

(c)

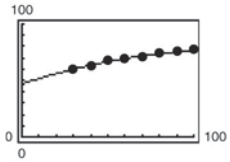


(d) $x = 90$ and $y = 90$

A square will give the maximum area of 8100 square meters.

(e) Answers will vary. *Sample answer:* The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width. A field of length 115 yards and width 75 yards would have an area of 8625 square yards.

79. (a)



Because the line is close to the points, the model fits the data well.

(b) Graphically: The point $(90, 75.4)$ represents a life expectancy of 75.4 years in 1990.

Algebraically:
$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(90)^2 + 0.5(90) + 46.6 \\ &= 75.4 \end{aligned}$$

So, the life expectancy in 1990 was about 75.4 years.

(c) Graphically: The point $(94.6, 76.0)$ represents a life expectancy of 76 years during the year 1994.

Algebraically:
$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ 76.0 &= -0.002t^2 + 0.5t + 46.6 \\ 0 &= -0.002t^2 + 0.5t - 29.4 \end{aligned}$$

Use the quadratic formula to solve.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0.5) \pm \sqrt{(0.5)^2 - 4(-0.002)(-29.4)}}{2(-0.002)} \\ &= \frac{-0.5 \pm \sqrt{0.0148}}{-0.004} \\ &= 125 \pm 30.4 \end{aligned}$$

So, $t = 94.6$ or $t = 155.4$. Since 155.4 is not in the domain, the solution is $t = 94.6$, which is the year 1994.

(d) When $t = 115$:

$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(115)^2 + (0.5)(115) + 46.6 \\ &= 77.65 \end{aligned}$$

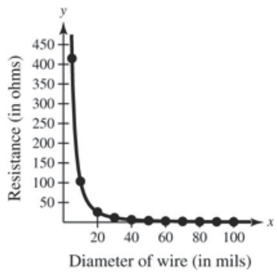
The life expectancy using the model is 77.65 years, which is slightly less than the given projection of 78.9 years.

(e) Answers will vary. *Sample answer:* No. Because the model is quadratic, the life expectancies begin to decrease after a certain point.

80. (a)

x	5	10	20	30	40	50	60	70	80	90	100
y	414.8	103.7	25.9	11.5	6.5	4.1	2.9	2.1	1.6	1.3	1.0

(b)



When $x = 85.5$, the resistance is about 1.4 ohms.

(c) When $x = 85.5$,

$$y = \frac{10{,}370}{(85.5)^2} = 1.4 \text{ ohms.}$$

(d) As the diameter of the copper wire increases, the resistance decreases.

81. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

82. x-axis symmetry:

$$x^2 + y^2 = 1$$
$$x^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

y-axis symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

Origin symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

So, the graph of the equation is symmetric with respect to the x-axis, the y-axis, and the origin.

Section 1.2 Linear Equations in One Variable

1. equation

2. identities; conditional; contradictions

3. $ax + b = 0$

4. equivalent

5. rational

6. extraneous

7. $2(x - 1) = 2x - 2$ is an *identity* by the Distributive Property. It is true for all real values of x.

8. $3(x + 2) = 5x + 4$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 0$).

9. $-6(x - 3) + 5 = -2x + 10$ is *conditional*. There are real values of x for which the equation is not true.

10. $3(x + 2) - 5 = 3x + 1$ is an *identity* by simplification. It is true for all real values of x.

$3(x + 2) - 5 = 3x + 6 - 5 = 3x + 1$

11. $4(x + 1) - 2x = 4x + 4 - 2x = 2x + 4 = 2(x + 2)$

This is an *identity* by simplification. It is true for all real values of x.

12. $-7(x-3)+4x = 3(7-x)$ is an *identity* by simplification. It is true for all real values of x.

$$\begin{aligned} -7(x-3)+4x &= -7x+21+4x \\ &= 21-3x \\ &= 3(7-x) \end{aligned}$$

13. $2(x-1) = 2x-1$ is *conditional*. There are real values of x for which the equation is not true.

14. $\frac{1}{4}(x-4) = \frac{1}{4}x - 4$ is *conditional*. There are real values of x for which the equation is not true.

15.
$$\begin{aligned} x+11 &= 15 \\ x+11-11 &= 15-11 \\ x &= 4 \end{aligned}$$

16.
$$\begin{aligned} 7-x &= 19 \\ 7-x+x &= 19+x \\ 7 &= 19+x \\ 7-19 &= 19+x-19 \\ -12 &= x \end{aligned}$$

17.
$$\begin{aligned} 7-2x &= 25 \\ 7-7-2x &= 25-7 \\ -2x &= 18 \\ \frac{-2x}{-2} &= \frac{18}{-2} \\ x &= -9 \end{aligned}$$

18.
$$\begin{aligned} 7x+2 &= 23 \\ 7x+2-2 &= 23-2 \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

19.
$$\begin{aligned} 3x-5 &= 2x+7 \\ 3x-2x-5 &= 2x-2x+7 \\ x-5 &= 7 \\ x-5+5 &= 7+5 \\ x &= 12 \end{aligned}$$

20.
$$\begin{aligned} 5x+3 &= 6-2x \\ 5x+2x+3 &= 6-2x+2x \\ 7x+3 &= 6 \\ 7x+3-3 &= 6-3 \\ 7x &= 3 \\ \frac{7x}{7} &= \frac{3}{7} \\ x &= \frac{3}{7} \end{aligned}$$

21.
$$\begin{aligned} 4y+2-5y &= 7-6y \\ 4y-5y+2 &= 7-6y \\ -y+2 &= 7-6y \\ -y+6y+2 &= 7-6y+6y \\ 5y+2 &= 7 \\ 5y+2-2 &= 7-2 \\ 5y &= 5 \\ \frac{5y}{5} &= \frac{5}{5} \\ y &= 1 \end{aligned}$$

22.
$$\begin{aligned} 5y+1 &= 8y-5+6y \\ 5y+1 &= 8y+6y-5 \\ 5y+1 &= 14y-5 \\ 5y-5y+1 &= 14y-5y-5 \\ 1 &= 9y-5 \\ 1+5 &= 9y-5+5 \\ 6 &= 9y \\ \frac{6}{9} &= \frac{9y}{9} \\ \frac{2}{3} &= y \end{aligned}$$

23.
$$\begin{aligned} x-3(2x+3) &= 8-5x \\ x-6x-9 &= 8-5x \\ -5x-9 &= 8-5x \\ -5x+5x-9 &= 8-5x+5x \\ -9 &\neq 8 \end{aligned}$$

No solution

24.
$$\begin{aligned} 9x-10 &= 5x+2(2x-5) \\ 9x-10 &= 5x+4x-10 \\ 9x-10 &= 9x-10 \end{aligned}$$

The solution is the set of all real numbers.

25.
$$\begin{aligned} 0.25x + 0.75(10 - x) &= 3 \\ 0.25x + 7.5 - 0.75x &= 3 \\ -0.50x + 7.5 &= 3 \\ -0.50x &= -4.5 \\ x &= 9 \end{aligned}$$

26.
$$\begin{aligned} 0.60x + 0.40(100 - x) &= 50 \\ 0.60x + 40 - 0.40x &= 50 \\ 0.20x &= 10 \\ x &= 50 \end{aligned}$$

27.
$$\begin{aligned} \frac{3x}{8} - \frac{4x}{3} &= 4 \\ \frac{9x}{24} - \frac{32x}{24} &= 4 \\ -\frac{23x}{24} &= 4 \\ -\frac{23x}{24}\left(-\frac{24}{23}\right) &= 4\left(-\frac{24}{23}\right) \\ x &= -\frac{96}{23} \end{aligned}$$

or

$$\begin{aligned} \frac{3x}{8} - \frac{4x}{3} &= 4 \\ 24\left(\frac{3x}{8} - \frac{4x}{3}\right) &= 24(4) \\ 9x - 32x &= 96 \\ -23x &= 96 \\ x &= -\frac{96}{23} \end{aligned}$$

The second method is easier. The fractions are eliminated in the first step.

28.
$$\begin{aligned} \frac{2x}{5} + 5x &= \frac{4}{3} \\ \frac{2x}{5} + \frac{25x}{5} &= \frac{4}{3} \\ \frac{27x}{5} &= \frac{4}{3} \\ \frac{27x}{5}\left(\frac{5}{27}\right) &= \frac{4}{3}\left(\frac{5}{27}\right) \\ x &= \frac{20}{81} \end{aligned}$$

or

$$\begin{aligned} \frac{2x}{5} + 5x &= \frac{4}{3} \\ 15\left(\frac{2x}{5} + 5x\right) &= 15\left(\frac{4}{3}\right) \\ 6x + 75x &= 20 \\ 81x &= 20 \\ x &= \frac{20}{81} \end{aligned}$$

The second method is easier. The fractions are eliminated in the first step.

29.
$$\begin{aligned} \frac{5x}{4} + \frac{1}{2} &= x - \frac{1}{2} \\ 4\left(\frac{5x}{4} + \frac{1}{2}\right) &= 4\left(x - \frac{1}{2}\right) \\ 4\left(\frac{5x}{4}\right) + 4\left(\frac{1}{2}\right) &= 4(x) - 4\left(\frac{1}{2}\right) \\ 5x + 2 &= 4x - 2 \\ x &= -4 \end{aligned}$$

30.
$$\begin{aligned} \frac{x}{5} - \frac{x}{2} &= 3 + \frac{3x}{10} \\ 10\left(\frac{x}{5} - \frac{x}{2}\right) &= 10\left(3 + \frac{3x}{10}\right) \\ 2x - 5x &= 30 + 3x \\ -6x &= 30 \\ x &= -5 \end{aligned}$$

31.
$$\begin{aligned} \frac{5x - 4}{5x + 4} &= \frac{2}{3} \\ 3(5x - 4) &= 2(5x + 4) \\ 15x - 12 &= 10x + 8 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

32.
$$\begin{aligned} \frac{10x + 3}{5x + 6} &= \frac{1}{2} \\ 2(10x + 3) &= 1(5x + 6) \\ 20x + 6 &= 5x + 6 \\ 15x &= 0 \\ x &= 0 \end{aligned}$$

33.
$$\begin{aligned} 10 - \frac{13}{x} &= 4 + \frac{5}{x} \\ \frac{10x - 13}{x} &= \frac{4x + 5}{x} \\ 10x - 13 &= 4x + 5 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

34.
$$\begin{aligned} \frac{15}{x} - 4 &= \frac{6}{x} + 3 \\ \frac{15}{x} - \frac{6}{x} &= 7 \\ \frac{9}{x} &= 7 \\ 9 &= 7x \\ \frac{9}{7} &= x \end{aligned}$$

35.
$$\begin{aligned} 3 &= 2 + \frac{2}{z + 2} \\ 3(z + 2) &= \left(2 + \frac{2}{z + 2}\right)(z + 2) \\ 3z + 6 &= 2z + 4 + 2 \\ z &= 0 \end{aligned}$$

36. $\frac{1}{x} + \frac{2}{x-5} = 0$ Multiply both sides by $x(x-5)$.

$$\begin{aligned} 1(x-5) + 2x &= 0 \\ 3x - 5 &= 0 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

37.
$$\begin{aligned} \frac{x}{x+4} + \frac{4}{x+4} + 2 &= 0 \\ \frac{x+4}{x+4} + 2 &= 0 \\ 1 + 2 &= 0 \\ 3 &\neq 0 \end{aligned}$$

Contradiction; no solution

38. $\frac{7}{2x+1} - \frac{8x}{2x-1} = -4$ Multiply both sides by $(2x+1)(2x-1)$.

$$\begin{aligned} 7(2x-1) - 8x(2x+1) &= -4(2x+1)(2x-1) \\ 14x - 7 - 16x^2 - 8x &= -16x^2 + 4 \\ 6x &= 11 \\ x &= \frac{11}{6} \end{aligned}$$

39. $\frac{2}{(x-4)(x-2)} = \frac{1}{x-4} + \frac{2}{x-2}$ Multiply both sides by $(x-4)(x-2)$.

$$\begin{aligned} 2 &= 1(x-2) + 2(x-4) \\ 2 &= x - 2 + 2x - 8 \\ 2 &= 3x - 10 \\ 12 &= 3x \\ 4 &= x \end{aligned}$$

A check reveals that $x = 4$ is an extraneous solution—it makes the denominator zero. There is no real solution.

40. $\frac{4}{x-1} + \frac{6}{3x+1} = \frac{15}{3x+1}$ Multiply both sides by $(x-1)(3x+1)$.

$$\begin{aligned} (x-1)(3x+1)\frac{4}{x-1} + (x-1)(3x+1)\frac{6}{3x+1} &= (x-1)(3x+1)\frac{15}{3x+1} \\ 4(3x+1) + 6(x-1) &= 15(x-1) \\ 12x + 4 + 6x - 6 &= 15x - 15 \\ 18x - 2 &= 15x - 15 \\ 3x &= -13 \\ x &= -\frac{13}{3} \end{aligned}$$

41.
$$\begin{aligned} \frac{1}{x-3} + \frac{1}{x+3} &= \frac{10}{x^2-9} \\ \frac{1}{x-3} + \frac{1}{x+3} &= \frac{10}{(x+3)(x-3)} \end{aligned}$$

Multiply both sides by $(x+3)(x-3)$.

$$\begin{aligned} 1(x+3) + 1(x-3) &= 10 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

42.
$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6}$$
$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{(x+3)(x-2)} \quad \text{Multiply both sides by } (x+3)(x-2).$$
$$\begin{aligned} (x+3) + 3(x-2) &= 4 \\ x + 3 + 3x - 6 &= 4 \\ 4x - 3 &= 4 \\ 4x &= 7 \\ x &= \frac{7}{4} \end{aligned}$$

43.
$$\frac{3}{x^2 - 3x} + \frac{4}{x} = \frac{1}{x-3}$$
$$\frac{3}{x(x-3)} + \frac{4}{x} = \frac{1}{x-3} \quad \text{Multiply both sides by } x(x-3).$$
$$\begin{aligned} 3 + 4(x-3) &= x \\ 3 + 4x - 12 &= x \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

A check reveals that $x = 3$ is an extraneous solution since it makes the denominator zero, so there is no solution.

44.
$$\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)} \quad \text{Multiply both sides by } x(x+3).$$
$$\begin{aligned} 6(x+3) - 2x &= 3(x+5) \\ 6x + 18 - 2x &= 3x + 15 \\ 4x + 18 &= 3x + 15 \\ x &= -3 \end{aligned}$$

Check: $\frac{6}{-3} - \frac{2}{-3+3} = \frac{3(-3+5)}{-3(-3+3)}$

$$-2 - \frac{2}{0} = \frac{-6}{-3(0)}$$

Division by zero is undefined. Thus, $x = -3$ is not a solution, and the original equation has no solution.

45.
$$\begin{aligned} y &= 12 - 5x \\ 0 &= 12 - 5x \\ 5x &= 12 \\ x &= \tfrac{12}{5} \end{aligned} \qquad \begin{aligned} y &= 12 - 5x \\ y &= 12 - 5(0) \\ y &= 12 \end{aligned}$$

The x-intercept is $\left(\frac{12}{5}, 0\right)$ and the y-intercept is $(0, 12)$.

46.
$$\begin{aligned} y &= 16 - 3x \\ 0 &= 16 - 3x \\ -16 &= -3x \\ \tfrac{16}{3} &= x \end{aligned} \qquad \begin{aligned} y &= 16 - 3x \\ y &= 16 - 3(0) \\ y &= 16 \end{aligned}$$

The x-intercept is $\left(\frac{16}{3}, 0\right)$ and the y-intercept is $(0, 16)$.

47.
$$\begin{aligned} y &= -3(2x+1) \\ 0 &= -3(2x+1) \\ 0 &= 2x + 1 \\ x &= -\tfrac{1}{2} \end{aligned} \qquad \begin{aligned} y &= -3(2x+1) \\ y &= -3(2(0)+1) \\ y &= -3 \end{aligned}$$

The x-intercept is $\left(-\frac{1}{2}, 0\right)$ and the y-intercept is $(0, -3)$.

48.
$$\begin{aligned} y &= 5 - (6 - x) \\ 0 &= 5 - (6 - x) \\ 0 &= -1 + x \\ 1 &= x \end{aligned} \qquad \begin{aligned} y &= 5 - (6 - x) \\ y &= 5 - (6 - 0) \\ y &= -1 \end{aligned}$$

The x-intercept is $(1, 0)$ and the y-intercept is $(0, -1)$.

49.
$$\begin{aligned} 2x + 3y &= 10 \\ 2x + 3(0) &= 10 \\ 2x &= 10 \\ x &= 5 \end{aligned} \qquad \begin{aligned} 2x + 3y &= 10 \\ 2(0) + 3y &= 10 \\ 3y &= 10 \\ y &= \tfrac{10}{3} \end{aligned}$$

The x-intercept is $(5, 0)$ and the y-intercept is $\left(0, \frac{10}{3}\right)$.

50.
$$\begin{aligned} 4x - 5y &= 12 \\ 4x - 5(0) &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned} \qquad \begin{aligned} 4x - 5y &= 12 \\ 4(0) - 5y &= 12 \\ -5y &= 12 \\ y &= -\tfrac{12}{5} \end{aligned}$$

The x-intercept is $(3, 0)$ and the y-intercept is $\left(0, -\frac{12}{5}\right)$.

51.

$$\begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4(0) - 0.75x + 1.2 &= 0 \\ -0.75 + 1.2 &= 0 \\ x = \frac{1.2}{0.75} &= 1.6 \end{aligned} \qquad \begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4y - 0.75(0) + 1.2 &= 0 \\ 4y + 1.2 &= 0 \\ y = \frac{-1.2}{4} &= -0.3 \end{aligned}$$

The x-intercept is $(1.6, 0)$ and the y-intercept is $(0, -0.3)$.

52.

$$\begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3(0)2.5x - 3.4 &= 0 \\ 2.5x &= 3.4 \\ x &= \frac{3.4}{2.5} \\ x &= 1.36 \end{aligned} \qquad \begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3y + 2.5(0) - 3.4 &= 0 \\ 3y &= 3.4 \\ y &= \frac{3.4}{3} \\ y &= 1.1\overline{3} \end{aligned}$$

The x-intercept is $(1.36, 0)$ and the y-intercept is $(0, 1.1\overline{3})$.

53. $\frac{2x}{5} + 8 - 3y = 0 \Rightarrow 2x + 40 - 15y = 0$ Multiply both sides by 5.

$$\begin{aligned} 2x + 40 - 15(0) &= 0 \\ 2x + 40 &= 0 \\ x &= -20 \end{aligned} \qquad \begin{aligned} 2x + 40 - 15y &= 0 \\ 2(0) + 40 - 15y &= 0 \\ 40 - 15y &= 0 \\ y = \frac{40}{15} &= \frac{8}{3} \end{aligned}$$

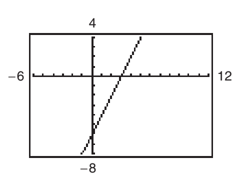
The x-intercept is $(-20, 0)$ and the y-intercept is $\left(0, \frac{8}{3}\right)$.

54.

$$\begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8x}{3} + 5 - 2(0) &= 0 \\ \frac{8x}{3} &= -5 \\ \frac{3}{8} \cdot \frac{8x}{3} &= \frac{3}{8} \cdot (-5) \\ x &= -\frac{15}{8} \end{aligned} \qquad \begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8(0)}{3} + 5 - 2y &= 0 \\ -2y &= -5 \\ y &= \frac{5}{2} \end{aligned}$$

The x-intercept is $\left(-\frac{15}{8}, 0\right)$ and the y-intercept is $\left(0, \frac{5}{2}\right)$.

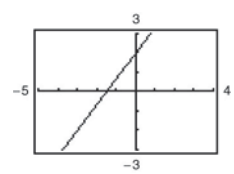
55. $y = 2(x - 1) - 4$



$$\begin{aligned} 0 &= 2(x - 1 - 4) \\ 0 &= 2x - 2 - 4 \\ 0 &= 2x - 6 \\ 6 &= 2x \\ 3 &= x \\ x &= 3 \end{aligned}$$

The x-intercept is at 3. The solution of $0 = 2(x - 1) - 4$ and the x-intercept of $y = 2(x - 1) - 4$ are the same. They are both $x = 3$. The x-intercept is $(3, 0)$.

56. $y = \frac{4}{3}x + 2$

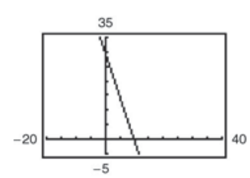


$$\begin{aligned} y &= \tfrac{4}{3}x + 2 \\ -\tfrac{4}{3}x &= 2 \\ \left(-\tfrac{3}{4}\right)\left(-\tfrac{4}{3}x\right) &= \left(-\tfrac{3}{4}\right)(2) \\ x &= -\tfrac{3}{2} \end{aligned}$$

Intercept: $\left(-\frac{3}{2}, 0\right)$

The solution to $0 = \frac{4}{3}x + 2$ is the same as the x-intercept of $y = 10 + 2(x - 2)$. They are both $x = -3$.

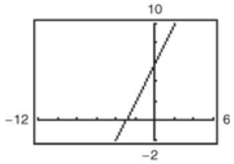
57. $y = 20 - (3x - 10)$



$$\begin{aligned} 0 &= 20 - (3x - 10) \\ 0 &= 20 - 3x + 10 \\ 0 &= 30 - 3x \\ 3x &= 30 \\ x &= 10 \end{aligned}$$

The x-intercept is at 10. The solution of $0 = 2 - (3x - 10)$ and the x-intercept of $y = 20 - (3x - 10)$ are the same. They are both $x = 10$. The x-intercept is $(10, 0)$.

58. $y = 10 + 2(x - 2)$

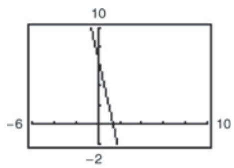


$$\begin{aligned} 0 &= 10 + 2(x - 2) \\ 0 &= 10 + 2x - 4 \\ 0 &= 6 + 2x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

Intercept: $(-3, 0)$

The solution to $0 = 10 + 2(x - 2)$ is the same as the x-intercept of $y = 10 + 2(x - 2)$. They are both $x = -3$.

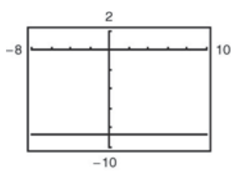
59. $y = -38 + 5(9 - x)$



$$\begin{aligned} 0 &= -38 + 5(9 - x) \\ 0 &= -38 + 45 - 5x \\ 0 &= 7 - 5x \\ 5x &= 7 \\ x &= \tfrac{7}{5} \end{aligned}$$

The x-intercept is at $\frac{7}{5}$. The solution of $0 = -38 + 5(9 - x)$ and the x-intercept of $y = -38 + 5(9 - x)$ are the same. They are both $x = \frac{7}{5}$. The x-intercept is $\left(\frac{7}{5}, 0\right)$.

60. $y = 6x - 6\left(\frac{6}{11} + x\right)$



$$\begin{aligned} 0 &= 6x - 6\left(\tfrac{16}{11} + x\right) \\ 0 &= 6x - \tfrac{96}{11} - 6x \\ 0 &\neq -\tfrac{96}{11} \end{aligned}$$

There is no x-intercept.

61.
$$\begin{aligned} 0.275x + 0.725(500 - x) &= 300 \\ 0.275x + 362.5 - 0.725x &= 300 \\ -0.45x &= -62.5 \\ x &= \frac{62.5}{0.45} \approx 138.889 \end{aligned}$$

62.
$$\begin{aligned} 2.763 - 4.5(2.1x - 5.1432) &= 6.32x + 5 \\ 2.763 - 9.45x + 23.1444 &= 6.32x + 5 \\ 20.9074 &= 15.77x \\ 1.326 &\approx x \end{aligned}$$

63. $\dfrac{2}{7.398} - \dfrac{4.405}{x} = \dfrac{1}{x}$ Multiply both sides by $7.398x$.
$$\begin{aligned} 2x - (4.405)(7.398) &= 7.398 \\ 2x &= (4.405)(7.398) + 7.398 \\ 2x &= (5.405)(7.398) \\ x &= \frac{(5.405)(7.398)}{2} \approx 19.993 \end{aligned}$$

64. $\dfrac{3}{6.350} - \dfrac{6}{x} = 18$ Multiply both sides by $6.350x$.
$$\begin{aligned} 3x - 6(6.350) &= 18(6.350)x \\ 3x - 38.1 &= 114.3x \\ -38.1 &= 111.3x \\ -0.342 &\approx x \end{aligned}$$

65.
$$\begin{aligned} 471 &= 2\pi(25) + 2\pi(5h) \\ 471 &= 50\pi + 10\pi h \\ 471 - 50\pi &= 10\pi h \\ h &= \frac{471 - 50\pi}{10\pi} = \frac{471 - 50(3.14)}{10(3.14)} = 10 \\ h &= 10 \text{ feet} \end{aligned}$$

66.
$$\begin{aligned} 248 &= 2(24) + 2(4x) + 2(6x) \\ 248 &= 48 + 8x + 12x \\ 200 &= 20x \\ x &= 10 \text{ centimeters} \end{aligned}$$

67. Let $y = 18$:
$$\begin{aligned} y &= 0.432x - 10.44 \\ 18 &= 0.432x - 10.44 \\ 28.44 &= 0.432x \\ \frac{28.44}{0.432} &= x \\ 65.8 &\approx x \end{aligned}$$

So, the height of the female is about 65.8 inches or 5 feet 6 inches.

68. Let $y = 21$:
$$\begin{aligned} y &= 0.449x - 12.15 \\ 21 &= 0.449x - 12.15 \\ 33.15 &= 0.449x \\ \frac{33.15}{0.449} &= x \\ 73.8 &\approx x \end{aligned}$$

Because the height of the male is about 73.8 inches or 6 feet 2 inches, it is possible the femur belongs to the missing man.

69. (a) The y-intercept is about $(0, 1480)$.

(b) Let $t = 0$:

$$\begin{aligned} y &= -8.37t + 1480.6 \\ &= -8.37(0) + 1480.6 \\ &= 1480.6 \end{aligned}$$

The y-intercept is (0, 1480.6).

So, there were about 1481 newspapers in the United States in 2000.

(c) Let $y = 1355$:

$$\begin{aligned} y &= -8.37t + 1480.6 \\ 1355 &= -8.37t + 1480.6 \\ -125.6 &= -8.37t \\ \frac{125.6}{8.37} &= t \\ 15.0 &\approx t \end{aligned}$$

In 2015, there will be about 1355 daily newspapers.; Answer will vary. *Sample answer:* Yes. This answer does seem reasonable because the amount of daily newspapers will most likely continue to decline.

70. (a) Let $x = 0$:

$$\begin{aligned} y &= 0.63t + 66.3 \\ &= 0.63(0) + 66.3 \\ &= 66.3 \end{aligned}$$

The y-intercept is (0, 66.3).

So, there were about 66.3 million women in the civilian work force in 2000.

(b) Let $y = 72$:

$$\begin{aligned} y &= 0.63t + 66.3 \\ 72 &= 0.63t + 66.3 \\ 5.7 &= 0.63t \\ \frac{5.7}{0.63} &= t \\ 9.0 &\approx t \end{aligned}$$

The number of women in the civilian work force reached 72 million during 2009.

(c) Answers will vary. *Sample answer:* To find the value graphically, graph both $y = 0.63t + 66.3$ and $y = 72$, and determine where the two lines intersect. To find the value algebraically, solve $72 = 0.63t + 66.3$ for t.

71. Let $c = 10{,}000$:

$$\begin{aligned} c &= 0.37m + 2600 \\ 10{,}000 &= 0.37m + 2600 \\ 7400 &= 0.37m \\ \frac{7400}{0.37} &= m \\ m &= 20{,}000 \end{aligned}$$

So, the number of miles is 20,000.

72.
$$\begin{aligned} y &= -0.25t + 8 \\ 1 &= -0.25t + 8 \\ 0.25t &= 7 \\ t &= 28 \text{ hours} \end{aligned}$$

73. False. $x(3 - x) = 10 \Rightarrow 3x - x^2 = 10$

This is a quadratic equation. The equation cannot be written in the form $ax + b = 0$.

74. False. If both sides of the equation are graphed, you can see that they intersect, which means the equation has a real solution.

75.
$$\begin{aligned} 2(x - 3) + 1 &= 2x - 5 \\ 2x - 6 + 1 &= 2x - 5 \\ 2x - 5 &= 2x - 5 \end{aligned}$$

False. The equation is an identity, so every real number is a solution.

76.
$$\begin{aligned} 3(x - 1) - 2 &= 3x - 6 \\ 3x - 3 - 2 &= 3x - 6 \\ 3x - 5 &= 3x - 6 \\ -5 &= -6 \end{aligned}$$

False. This is a contradiction. The equation has no solution.

77.
$$\begin{aligned} 2 - \frac{1}{x-2} &= \frac{3}{x-2} \\ (x-2)\left(2 - \frac{1}{x-2}\right) &= (x-2)\left(\frac{3}{x-2}\right) \\ 2(x-2) - 1 &= 3 \\ 2x - 4 - 1 &= 3 \\ 2x - 5 &= 3 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

False. $x = 4$ is a solution.

78. Equivalent equations are derived from the substitution principle and simplification techniques. They have the same solution(s).

$2x + 3 = 8$ and $2x = 5$ are equivalent equations.

79. (a)

x	–1	0	1	2	3	4
$3.2x - 5.8$	–9	–5.8	–2.6	0.6	3.8	7

(b) Since the sign changes from negative at 1 to positive at 2, the root is somewhere between 1 and 2.

$1 < x < 2$

(c)

x	1.5	1.6	1.7	1.8	1.9	2
$3.2x - 5.8$	–1	−0.68	−0.36	−0.04	0.28	0.6

(d) Since the sign changes from negative at 1.8 to positive at 1.9, the root is somewhere between 1.8 and 1.9.

$1.8 < x < 1.9$

To improve accuracy, evaluate the expression at subintervals within this interval and determine where the sign changes.

80. $0.3(x - 1.5) - 2 = 0$

x	6	7	8	9	10
$0.3(x - 1.5) - 2$	−0.65	−0.35	−0.05	0.25	0.55

The solution of $0.3(x - 1.5) - 2 = 0$ is in the interval $8 < x < 9$.

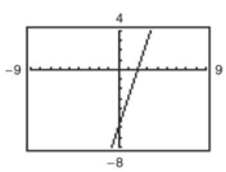
x	8.1	8.2	8.3	8.4
$0.3(x - 1.5) - 2$	−0.02	0.01	0.04	0.07

The solution is in the interval $8.1 < x < 8.2$.

x	8.14	8.15	8.16	8.17	8.18
$0.3(x - 1.5) - 2$	−0.008	−0.005	−0.002	0.001	0.004

The solution is in the interval $8.16 < x < 8.17$.

81. (a)



(b) x-intercept: $(2, 0)$

(c) The x-intercept is the solution of the equation $3x - 6 = 0$.

82. (a) To find the x-intercept, let $y = 0$ and solve for x.

$$0 = ax + b$$
$$-b = ax$$
$$\frac{-b}{a} = x$$

The x-intercept is $\left(-\frac{b}{a}, 0\right)$.

(b) To find the y-intercept, let $x = 0$, and solve for y.

$$y = a(0) + b$$
$$y = b$$

The y-intercept is $(0, b)$.

(c) x-intercept:

$$x = \frac{-b}{a} = \frac{-10}{5} = -2$$

The x-intercept is $(-2, 0)$.

y-intercept:

$$y = b = 10$$

The y-intercept is $(0, 10)$.

83. (a) To find the x-intercept, let $y = 0$, and solve for x.

$$ax + by = c$$
$$ax + b(0) = c$$
$$ax = c$$
$$x = \frac{c}{a}$$

The x-intercept is $\left(\frac{c}{a}, 0\right)$.

(b) To find the y-intercept, let $x = 0$, and solve for y.

$$a(0) + by = c$$
$$by = c$$
$$y = \frac{c}{b}$$

The y-intercept is $\left(0, \frac{c}{b}\right)$.

(c) x-intercept:

$$x = \frac{c}{a} = \frac{11}{2}$$

The x-intercept is $\left(\frac{11}{2}, 0\right)$.

y-intercept:

$$y = \frac{c}{b} = \frac{11}{7}$$

The y-intercept is $\left(0, \frac{11}{7}\right)$.

84. (a) The x-intercept is $(20{,}000, 0)$ and the y-intercept is $(0, 10{,}000)$. The subsidy is \$0 for an earned income of \$20,000. The subsidy is \$10,000 for an earned income of \$0.

(b) Set one of S or E equal to 0 and solve for the other.

(c) The earned income is \$8000.

(d) Set T equal to 14,000, substitute $10{,}000 - \frac{1}{2}E$ in the equation $T = E + S$, and solve for E.

Section 1.3 Modeling with Linear Equations

1. mathematical modeling

2. verbal model; algebraic equation

3. $x + 4$

The sum of a number and 4
A number increased by 4

4. $t - 10$

A number decreased by 10
The difference of a number and 10

5. $\dfrac{u}{5}$

The ratio of a number and 5
The quotient of a number and 5
A number divided by 5

6. $\dfrac{2}{3}x$

The product of $\dfrac{2}{3}$ and a number

Two-thirds of a number

7. $\dfrac{y - 4}{5}$

The difference of a number and 4 is divided by 5.
A number decreased by 4 is divided by 5.

8. $\dfrac{z + 10}{7}$

The sum of a number and 10 is divided by 7.
A number increased by 10 is divided by 7.

9. $-3(b + 2)$

The product of -3 and the sum of a number and 2.
Negative 3 is multiplied by a number increased by 2.

10. $12x(x - 5)$

The difference of a number and 5 is multiplied by 12 times the number.
12 is multiplied by a number and that product is multiplied by the number decreased by 5.

11. $\dfrac{4(p - 1)}{p}$

The product of 4 and the difference of a number and 1 is divided by the number.
A number decreased by 1 is multiplied by 4 and divided by the number.

12. $\dfrac{(q + 4)(3 - q)}{2q}$

The product of the sum of a number and 4 and the difference of 3 and the number is divided by the product of 2 and the number.
A number increased by 4 is multiplied by 3 decreased by the number and the product is divided by 2 multiplied by the number.

13. *Verbal Model:* $(\text{Sum}) = (\text{first number}) + (\text{second number})$

Labels: Sum $= S$, first number $= n$, second number $= n + 1$

Expression: $S = n + (n + 1) = 2n + 1$

14. *Verbal Model:* $\text{Product} = (\text{first number}) \cdot (\text{second number})$

Labels: Product $= P$, first number $= n$, second number $= n + 1$

Expression: $P = n(n + 1) = n^2 + n$

15. *Verbal Model:* $\text{Product} = (\text{first odd integer}) \cdot (\text{second odd integer})$

Labels: Product $= P$, first odd integer $= 2n - 1$, second odd integer $= 2n - 1 + 2 = 2n + 1$

Expression: $P = (2n - 1)(2n + 1) = 4n^2 - 1$

16. *Verbal Model:* $(\text{Sum}) = (\text{first even number})^2 + (\text{second even number})^2$

Labels: Sum $= S$, first even number $= 2n$, second even number $= 2n + 2$

Expression: $S = (2n)^2 + (2n + 2)^2 = 4n^2 + 4n^2 + 8n + 4 = 8n^2 + 8n + 4$

17. *Verbal Model:* $(\text{distance}) = (\text{rate}) \cdot (\text{time})$

Labels: Distance $= d$, rate $= 55$ mph, time $= t$

Expression: $d = 55t$

18. *Verbal Model:* $(\text{time}) = (\text{distance}) \div (\text{rate})$

Labels: time $= t$, distance $= 900$ km, rate $= r$

Expression: $t = \dfrac{900}{r}$

19. *Verbal Model:* $(\text{Amount of acid}) = 20\% \cdot (\text{amount of solution})$

Labels: Amount of acid (in gallons) $= A$, amount of solution (in gallons) $= x$

Expression: $A = 0.20x$

20. *Verbal Model:* $(\text{Sale price}) = (\text{list price}) - (\text{discount})$

Labels: Sale price $= S$, list price $= L$, discount $= 0.33L$

Expression: $S = L - 033L = 0.67L$

21. *Verbal Model:* Perimeter $= 2(\text{width}) + 2(\text{length})$

Labels: Perimeter $= P$, width $= x$, length $= 2(\text{width}) = 2x$

Expression: $P = 2x + 2(2x) = 6x$

22. *Verbal Model:* $(\text{Area}) = \frac{1}{2}(\text{base})(\text{height})$

Labels: Area $= A$, base $= 16$ in., height $= h$

Expression: $A = \frac{1}{2}(16)h = 8h$

23. *Verbal Model:* $(\text{Total cost}) = (\text{unit cost})(\text{number of units}) + (\text{fixed cost})$

Labels: Total cost $= C$, unit cost $= \$40$, number of units $= x$, fixed cost $= \$2500$

Expression: $C = 40x + 2500$

24. *Verbal Model:* $(\text{Revenue}) - (\text{price})(\text{number of units})$

Labels: Revenue $= R$, price $= \$12.99$, number of units $= x$

Expression: $R = 12.99x$

25. *Verbal Model:* Thirty percent of the list price L.

Expression: $0.30L$

26. *Verbal Model:* 28% of q

Expression: $0.28q$

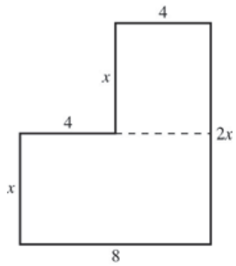
27. *Verbal Model:* Percent of 672 that is represented by the number N

Expression: $N = p(672)$, p is in decimal form

28. *Verbal Model:* $(S_2 - S_1)$ as a percent of S_1:

Expression: $\dfrac{(S_2 - S_1)}{S_1}(100)$

29.



Area = Area of top rectangle + Area of bottom rectangle

$A = 4x + 8x = 12x$

30. Area $= \frac{1}{2}(\text{base})(\text{height})$

$A = \frac{1}{2}\left(\frac{2}{3}b + 1\right) = \frac{1}{3}b^2 + \frac{1}{2}b$

31. *Verbal Model:* Sum = (first number) + (second number)

Labels: Sum = 525, first number = n, second number = $n + 1$

Equation:
$525 = n + (n + 1)$
$525 = 2n + 1$
$524 = 2n$
$n = 262$

Answer: First number = n = 262, second number = $n + 1 = 263$

32. *Verbal Model:* Sum = (first number) + (second number) + (third number)

Labels: Sum = 804, first number = n, second number = $n + 1$, third number = $n + 2$

Equation:
$804 = n + n + 1 + n + 2$
$804 = 3n + 3$
$801 = 3n$
$267 = n$

Answer: $n = 267$, $n + 1 = 268$ (second number), and $n + 2 = 269$ (third number)

33. *Verbal Model:* Difference = (one number) − (another number)

Labels: Difference = 148, one number = $5x$, another number = x

Equation:
$148 = 5x - x$
$148 = 4x$
$x = 37$
$5x = 185$

Answer: The two numbers are 37 and 185.

34. *Verbal Model:* Difference = (number) − (one-fifth of number)

Labels: Difference = 76, number = n, one-fifth of number = $\frac{1}{5}n$

Equation:
$76 = n - \frac{1}{5}n$
$76 = \frac{4}{5}n$
$95 = n$

Answer: The numbers are 95 and $\frac{1}{5} \cdot 95 = 19$.

35. *Verbal Model:* Product $=$ (smaller number) $\cdot$ (larger number) $=$ (smaller number)$^2 - 5$

Labels: Smaller number $= n$, larger number $= n + 1$

Equation:
$$n(n+1) = n^2 - 5$$
$$n^2 + n = n^2 - 5$$
$$n = -5$$

Answer: Smaller number $= n = -5$, larger number $= n + 1 = -4$

36. *Verbal Model:* Difference $=$ (reciprocal of smaller number) $-$ (reciprocal of larger number)

$= \frac{1}{4}$ (reciprocal of smaller number)

Labels: Smaller number $= n$, larger number $= n + 1$, difference $= \frac{1}{4n}$

Equation:
$$\frac{1}{4n} = \frac{1}{n} - \frac{1}{n+1}$$ Multiply both sides by $4n(n+1)$.
$$4n(n+1)\frac{1}{4n} = 4n(n+1)\frac{1}{n} - 4n(n+1)\frac{1}{n+1}$$
$$n + 1 = 4(n+1) - 4n$$
$$n + 1 = 4n + 4 - 4n$$
$$n = 3$$

Answer: The numbers are 3 and $n + 1 = 4$.

37. *Verbal Model:* (first paycheck) + (second paycheck) $=$ total

Labels: second paycheck $= x$, first paycheck $= 0.85x$, total $= \$1125$

Equation:
$$0.85x + x = 1125$$
$$1.85x = 1125$$
$$x \approx 608.11$$
$$0.85x \approx 516.89$$

Answer: The first salesperson's weekly paycheck is \$516.89 and the second salesperson's weekly paycheck is \$608.11.

38. *Verbal Model:* (Sale price) $=$ (list price) $-$ (discount)

Labels: Sale price $-$ \$1210.75, list price $= L$, discount $= 0.165L$

Equation:
$$1210.75 = L - 0.165L$$
$$1210.75 = 0.835L$$
$$1450 = L$$

Answer: The list price of the pool is \$1450.

39. *Verbal Model:* (Loan payments) $=$ (Percent) $\cdot$ (Annual Income)

Labels: Loan payments $= 15{,}680$ (dollars)

Percent $= 0.32$

Annual income $= I$ (dollars)

Equation:
$$15{,}680 = 0.32I$$
$$\frac{15{,}680}{0.32} = \frac{0.32I}{0.32}$$
$$49{,}000 = I$$

Answer: The family's annual income is \$49,000.

40. *Verbal Model:* (Mortgage payment) = (Percent) · (Monthly income)

Labels: Mortgage payment = 760 (dollars)

Percent = 0.16

Monthly income = I (dollars)

Equation:

$$760 = 0.16I$$
$$\frac{760}{0.16} = \frac{0.16I}{0.16}$$
$$4750 = I$$

Answer: The family's monthly income is \$4750.

41. (a)

(b) $l = 1.5w$

$$\begin{aligned} P &= 2l + 2w \\ &= 2(1.5w) + 2w \\ &= 5w \end{aligned}$$

(c) $25 = 5w$

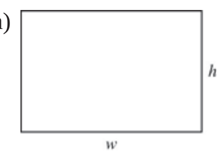
$5 = w$

Width: $w = 5$ meters

Length: $l = 1.5w = 7.5$ meters

Dimensions: 7.5 meters × 5 meters

42. (a)



(b) $h = \frac{2}{3}w$

$$\begin{aligned} P &= 2h + 2w \\ &= 2\left(\frac{2}{3}w\right) + 2w \\ &= \frac{10}{3}w \end{aligned}$$

(c) $3 = \frac{10}{3}w$

$\frac{9}{10} = w$

$h = \frac{2}{3}w = \frac{2}{3}\left(\frac{9}{10}\right) = \frac{3}{5}$

Dimensions: $\frac{9}{10}$ meters × $\frac{3}{5}$ meters

43. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{4}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation: $90 = \dfrac{87 + 92 + 84 + x}{4}$

Answer: You must score 97 or better on test #4 to earn an A for the course.

44. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{5}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation:

$$90 = \frac{87 + 92 + 84 + x}{5}$$
$$450 = 87 + 92 + 84 + x$$
$$450 = 263 + x$$
$$187 = x$$

Answer: You must score 187 out of 200 on the last test to get an A in the course.

45. $\text{Rate} = \dfrac{\text{distance}}{\text{time}} = \dfrac{50 \text{ kilometers}}{\frac{1}{2} \text{ hour}} = 100 \text{ kilometers/hour}$

$\text{Total time} = \dfrac{\text{total distance}}{\text{rate}} = \dfrac{500 \text{ kilometers}}{100 \text{ kilometers/hour}} = 5 \text{ hours}$

46. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time}_1 + \text{time}_2)$

Labels: Distance $= 2 \cdot 200 = 400$ miles, rate $= 2$,

$\text{time}_1 = \dfrac{\text{distance}}{\text{rate}_1} = \dfrac{200}{55}$ hours,

$\text{time}_2 = \dfrac{\text{distance}}{\text{rate}_2} = \dfrac{200}{40}$ hours

Equation:

$$400 = r\left(\frac{200}{55} + \frac{200}{40}\right)$$

$$400 = r\left(\frac{1600}{440} + \frac{2200}{440}\right) = \frac{3800}{440}r$$

$$46.3 \approx r$$

The average speed for the round trip was approximately 46.3 miles per hour.

47. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time})$

Labels: Distance $= 1.5 \times 10^{11}$ (meters)

Rate $= 3.0 \times 10^8$ (meters per second)

Time $= t$

Equation:

$$1.5 \times 10^{11} = (3.0 \times 10^8)t$$

$$500 = t$$

Light from the sun travels to the Earth in 500 seconds or approximately 8.33 minutes.

48. *Verbal Model:* $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

Equation:

$$t = \frac{3.84 \times 10^8 \text{ meters}}{3.0 \times 10^8 \text{ meters per second}}$$

$$t = 1.28 \text{ seconds}$$

The radio wave travels from Mission Control to the moon in 1.28 seconds.

49. *Verbal Model:* $\dfrac{(\text{Height of building})}{(\text{Length of building's shadow})} = \dfrac{(\text{Height of post})}{(\text{Length of post's shadow})}$

Labels: Height of building $= x$ (feet)

Length of building's shadow $= 105$ (feet)

Height of post $= 3 \cdot 12 = 36$ (inches)

Length of post's shadow $= 4$ (inches)

Equation:

$$\frac{x}{105} = \frac{36}{4}$$

$$x = 945$$

One Liberty Place is 945 feet tall.

50. *Verbal Model:* $\dfrac{(\text{height of tree})}{(\text{length of tree's shadow})} = \dfrac{(\text{height of lamppost})}{(\text{length of lamppost's shadow})}$

Labels: height of tree $= h$, height of tree's shadow $= 8$ meters, height of lamppost $= 2$ meters, height of lamppost's shadow $= 0.75$ meter

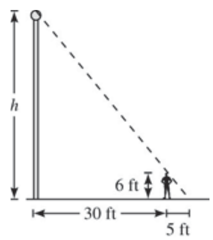
Equation:

$$\frac{h}{8} = \frac{2}{0.75}$$

$$h = \frac{8(2)}{0.75} = 21\frac{1}{3}$$

The tree is $21\frac{1}{3}$ meters tall.

51. (a)



(b) *Verbal Model:* $\dfrac{(\text{height of pole})}{(\text{height of pole's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$

Labels: Height of pole $= h$, height of pole's shadow $= 30 + 5 = 35$ feet, height of person $= 6$ feet, height of person's shadow $= 5$ feet

Equation:

$$\frac{h}{35} = \frac{6}{5}$$

$$h = \frac{6}{5} \cdot 35 = 42$$

The pole is 42 feet tall.

52. *Verbal Model:* $\dfrac{(\text{height of tower})}{(\text{height of tower's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$

Labels: Let $x =$ length of person's shadow.

Equation:

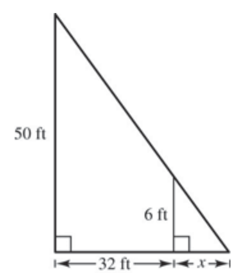
$$\frac{50}{32 + x} = \frac{6}{x}$$

$$50x = 6(32 + x)$$

$$50x = 192 + 6x$$

$$44x = 192$$

$$x \approx 4.36 \text{ feet}$$



53. *Verbal Model:* $\boxed{\text{Interest from } 4\tfrac{1}{2}\%} + \boxed{\text{Interest from } 5\%} = \boxed{\text{Total interest}}$

Labels:

Amount invested at $4\frac{1}{2}\% = x$ dollars

Amount invested at $5\% = 12{,}000 - x$ dollars

Interest from $4\frac{1}{2}\% = x(0.045)$ dollars

Interest from $5\% = (12{,}000 - x)(0.05)$ dollars

Total annual interest $= 580$ dollars

Equation:

$$0.045x + 0.05(12{,}000 - x) = 580$$

$$0.045x + 600 - 0.05x = 580$$

$$-0.005x = -20$$

$$x = 4000$$

So, \$4000 was invested at $4\frac{1}{2}\%$ and $12000 - 4000 = \$8000$ was invested at 5%.

54. *Verbal Model:* $\boxed{\text{Interest from 3\%}} + \boxed{\text{Interest from } 4\tfrac{1}{2}\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at 3% $= x$

Amount invested at $4\frac{1}{2}\% = 25{,}000 - x$

Equation:
$$
\begin{aligned}
0.03x + 0.045(25{,}000 - x) &= 900\\
0.03x + 1125 - 0.045x &= 900\\
-0.015x &= -225\\
x &= 15{,}000
\end{aligned}
$$

So, \$15,000 was invested at 3% and $25{,}000 - 15{,}000 = \$10{,}000$ was invested at 4%.

55. *Verbal Model:* (Profit from dogwood trees) + (profit from red maple trees) = (total profit)

Labels: Inventory of dogwood trees $= x$, inventory of red maple trees $= 40{,}000 - x$, profit from dogwood trees $= 0.25x$, profit from red maple trees $= 0.17(40{,}000 - x)$, total profit $= 0.20(40{,}000) = 8000$

Equation:
$$
\begin{aligned}
0.25x + 0.17(40{,}000) &= 8000\\
0.25x + 6800 - 0.17x &= 8000\\
0.08x &= 1200\\
x &= 15{,}000
\end{aligned}
$$

The amount invested in dogwood trees was \$15,000 and the amount invested in red maple trees was $40{,}000 - 15{,}000 = \$25{,}000$.

56. *Verbal Model:* (Profit from minivans) + (profit from alternative-fueled vehicles) = (total profit)

Labels: Inventory of minivans $= x$, inventory of alternative-fueled vehicles $= 600{,}000 - x$, profit from minivans $= 0.24x$, profit from alternative-fueled vehicles $= 0.28(600{,}000 - x)$, total profit $= 0.25(600{,}000) = 150{,}000$

Equation:
$$
\begin{aligned}
0.24x + 0.28(600{,}000 - x) &= 150{,}000\\
0.24x + 168{,}000 - 0.28x &= 150{,}000\\
-0.04x &= -18{,}000\\
x &= 450{,}000
\end{aligned}
$$

The amount invested in minivans was \$450,000 and the amount invested in alternative-fueled vehicles was. $600{,}000 - 450{,}000 = \$150{,}000$.

57. *Verbal Model:* $\boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{in mixture}\end{array}} + \boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{to add}\end{array}} = \boxed{\begin{array}{l}\text{Amount of gasoline}\\ \text{in final mixture}\end{array}}$

Labels: Amount of gasoline in mixture $= \frac{32}{33}(2)$ (gallons)

Amount of gasoline to add $= x$ (gallons)

Amount of gasoline in final mixture $= \frac{50}{51}(2 + x)$ (gallons)

Equation:
$$
\begin{aligned}
\tfrac{64}{33} + x &= \tfrac{50}{51}(2 + x)\\
\tfrac{64}{33} + x &= \tfrac{100}{51} + \tfrac{50}{51}x\\
3264 + 1683x &= 3300 + 1650x\\
33x &= 36\\
x &\approx 1.09
\end{aligned}
$$

The forester should add about 1.09 gallons of gasoline to the mixture.

58. *Verbal Model:* $\begin{pmatrix}\text{Price per pound}\\\text{of peanuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{peanuts}\end{pmatrix}+\begin{pmatrix}\text{price per pound}\\\text{of walnuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{walnuts}\end{pmatrix}=\begin{pmatrix}\text{price per pound}\\\text{of nut mixture}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{nut mixture}\end{pmatrix}$

Labels: Price per pound of peanuts $= \$1.49$, pounds of peanuts $= x$, price per pound of peanuts $= \$2.69$, pounds of walnuts $= 100 - x$, price per pound of nut mixture $= \$2.21$, pounds of nut mixture $= 100$

Equation:
$$\begin{aligned}1.49x + 2.69(100 - x) &= 2.21(100)\\ 1.49x + 2.69 - 2.69x &= 2.21\\ -1.2x &= -48\\ x &= 40\end{aligned}$$

There were 40 pounds of peanuts and $100 - 40 = 60$ pounds of walnuts in the mixture.

59.
$$\begin{aligned}A &= \frac{1}{2}bh\\ 2A &= bh\\ \frac{2A}{b} &= h\end{aligned}$$

60.
$$\begin{aligned}V &= lwh\\ \frac{V}{wh} &= l\end{aligned}$$

61.
$$\begin{aligned}S &= C + RC\\ S &= C(1 + R)\\ \frac{S}{1 + R} &= C\end{aligned}$$

62.
$$\begin{aligned}S &= L - RL\\ S &= L(1 - R)\\ \frac{S}{1 - R} &= L\end{aligned}$$

63.
$$\begin{aligned}A &= P + Prt\\ A - P &= Prt\\ \frac{A - P}{Pt} &= r\end{aligned}$$

64.
$$\begin{aligned}A &= \frac{1}{2}(a + b)h\\ 2A &= (a + b)h\\ \frac{2A}{h} &= a + b\\ b &= \frac{2A}{h} - a\end{aligned}$$

65.
$$\begin{aligned}W_1x &= W_2(L - x)\\ 50x &= 75(10 - x)\\ 50x &= 750 - 75x\\ 125x &= 750\\ x &= 6 \text{ feet from 50-pound child}\end{aligned}$$

66.
$$\begin{aligned}W_1x &= W_2(L - x)\\ W_1 &= 200 \text{ pounds}\\ W_2 &= 550 \text{ pounds}\\ L &= 5 \text{ feet}\\ 200x &= 550(5 - x)\\ 200x &= 2750 - 550x\\ 750x &= 2750\\ x &= 3\frac{2}{3} \text{ feet from the person}\end{aligned}$$

67.
$$\begin{aligned}V &= \frac{4}{3}\pi r^3\\ 5.96 &= \frac{4}{3}\pi r^3\\ 17.88 &= 4\pi r^3\\ \frac{17.88}{4\pi} &= r^3\\ r &= \sqrt[3]{\frac{4.47}{\pi}} \approx 1.12 \text{ inches}\end{aligned}$$

68.
$$\begin{aligned}V &= \pi r^2h\\ h &= \frac{V}{\pi r^2} = \frac{603.2}{\pi(2)^2} \approx 48 \text{ feet}\end{aligned}$$

69.
$$\begin{aligned}C &= \tfrac{5}{9}(F - 32)\\ &= \tfrac{5}{9}(64.4 - 32)\\ &= \tfrac{5}{9}(32.4)\\ &= 18\end{aligned}$$

The average daily temperature is 18° C.

70.
$$\begin{aligned}C &= \tfrac{5}{9}(F - 32)\\ &= \tfrac{5}{9}(39.1 - 32)\\ &= \tfrac{5}{9}(7.1)\\ &\approx 3.9\end{aligned}$$

The average daily temperature is about 3.9° F.

71. $F = \frac{9}{5}C + 32$
$= \frac{9}{5}(50) + 32$
$= 90 + 32$
$= 122$

The highest recorded temperature was 122°F.

72. $C = \frac{9}{5}C + 32$

When $C = -30°$, $\frac{9}{5}(-30) + 32 = -22°$F.

73. False, it should be written as $\dfrac{z^3 - 8}{z^2 - 9}$.

74. True.

Area of circle: $A = \pi r^2 = \pi(2)^2 = 4\pi \approx 12.56$ in.^{2}

Area of square: $A = s^2 = (4)^2 = 16$ in.^{2}

Because 12.56 in.^{2} < 16 in.^{2} the area of the circle is less than the area of the square.

75. False

Cube: $V = s^3 = 9.5^3 = 857.375$ in.^{3}

Sphere: $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5.9)^3 \approx 860.290$ in.^{3}

76. (a) *Verbal Model:* $\dfrac{\text{Height of building}}{\text{Length of building's shadow}} = \dfrac{\text{Height of post}}{\text{Length of post's shadow}}$

(b) *Equation:* $\dfrac{x}{30} = \dfrac{4}{3}$

Section 1.4 Quadratic Equations and Applications

1. quadratic equation

2. second-degree polynomial

3. factoring; square roots; completing; square; Quadratic Formula

4. discriminant

5. position equation

6. Pythagorean Theorem

7. $2x^2 = 3 - 5x$

General form: $2x^2 + 5x - 3 = 0$

8. $x^2 = 16x$

General form: $x^2 - 16x = 0$

9. $(x - 3)^2 = 3$
$x^2 - 6x + 9 = 3$

General form: $x^2 - 6x + 6 = 0$

10. $13 - 3(x + 7)^2 = 0$
$13 - 3(x^2 + 14x + 49) = 0$
$13 - 3x^2 - 42x - 147 = 0$

General form: $-3x^2 - 42x - 134 = 0$

11. $\frac{1}{5}(3x^2 - 10) = 12x$
$3x^2 - 10 = 60x$

General form: $3x^2 - 60x - 10 = 0$

12. $x(x + 2) = 5x^2 + 1$
$x^2 + 2x = 5x^2 + 1$
$-4x^2 + 2x - 1 = 0$
$(-1)(-4x^2 + 2x - 1) = -1(0)$

General form: $4x^2 - 2x + 1 = 0$

13. $6x^2 + 3x = 0$
$3x(2x + 1) = 0$
$3x = 0$ or $2x + 1 = 0$
$x = 0$ or $x = -\frac{1}{2}$

14. $9x^2 - 1 = 0$
$(3x + 1)(3x - 1) = 0$
$3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$
$3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

15. $x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x - 4 = 0$ or $x + 2 = 0$
$x = 4$ or $x = -2$

16. $x^2 - 10x + 9 = 0$
$(x - 9)(x - 1) = 0$
$x - 9 = 0 \Rightarrow x = 9$
$x - 1 = 0 \Rightarrow x = 1$

17. $$\begin{aligned} x^2 + 10x + 25 &= 0 \\ (x+5)(x+5) &= 0 \\ x + 5 &= 0 \\ x &= -5 \end{aligned}$$

18. $$\begin{aligned} 4x^2 + 12x + 9 &= 0 \\ (2x+3)(2x+3) &= 0 \end{aligned}$$
$$2x + 3 = 0 \Rightarrow x = -\tfrac{3}{2}$$

19. $$\begin{aligned} 3 + 5x - 2x^2 &= 0 \\ (3-x)(1+2x) &= 0 \end{aligned}$$
$$3 - x = 0 \quad \text{or} \quad 1 + 2x = 0$$
$$x = 3 \quad \text{or} \quad x = -\tfrac{1}{2}$$

20. $$\begin{aligned} 2x^2 &= 19x + 33 \\ 2x^2 - 19x - 33 &= 0 \\ (2x+3)(x-11) &= 0 \end{aligned}$$
$$2x + 3 = 0 \Rightarrow x = -\tfrac{3}{2}$$
$$x - 11 = 0 \Rightarrow x = 11$$

21. $$\begin{aligned} x^2 + 4x &= 12 \\ x^2 + 4x - 12 &= 0 \\ (x+6)(x-2) &= 0 \end{aligned}$$
$$x + 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -6 \quad \text{or} \quad x = 2$$

22. $$\begin{aligned} -x^2 + 8x &= 12 \\ -x^2 + 8x - 12 &= 0 \\ (-1)(-x^2 + 8x - 12) &= (-1)(0) \\ x^2 - 8x + 12 &= 0 \\ (x-6)(x-2) &= 0 \end{aligned}$$
$$x - 6 = 0 \Rightarrow x = 6$$
$$x - 2 = 0 \Rightarrow x = 2$$

23. $$\begin{aligned} \tfrac{3}{4}x^2 + 8x + 20 &= 0 \\ 4\left(\tfrac{3}{4}x^2 + 8x + 20\right) &= 4(0) \\ 3x^2 + 32x + 80 &= 0 \\ (3x+20)(x+4) &= 0 \end{aligned}$$
$$3x + 20 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -\tfrac{20}{3} \quad \text{or} \quad x = -4$$

24. $$\begin{aligned} \tfrac{1}{8}x^2 - x - 16 &= 0 \\ x^2 - 8x - 128 &= 0 \\ (x-16)(x+8) &= 0 \end{aligned}$$
$$x - 16 = 0 \Rightarrow x = 16$$
$$x + 8 = 0 \Rightarrow x = -8$$

25. $$\begin{aligned} x^2 &= 49 \\ x &= \pm 7 \end{aligned}$$

26. $$\begin{aligned} x^2 &= 144 \\ x &= \pm 12 \end{aligned}$$

27. $$\begin{aligned} x^2 &= 11 \\ x &= \pm\sqrt{11} \end{aligned}$$

28. $$\begin{aligned} x^2 &= 32 \\ x &= \pm\sqrt{32} = \pm 4\sqrt{2} \end{aligned}$$

29. $$\begin{aligned} 3x^2 &= 81 \\ x^2 &= 27 \\ x &= \pm 3\sqrt{3} \end{aligned}$$

30. $$\begin{aligned} 9x^2 &= 36 \\ x^2 &= 4 \\ x &= \pm\sqrt{4} = \pm 2 \end{aligned}$$

31. $$\begin{aligned} (x-12)^2 &= 16 \\ x - 12 &= \pm 4 \\ x &= 12 \pm 4 \\ x &= 16 \quad \text{or} \quad x = 8 \end{aligned}$$

32. $$\begin{aligned} (x-5)^2 &= 25 \\ x - 5 &= \pm 5 \\ x &= 5 \pm 5 \\ x &= 0 \quad \text{or} \quad x = 10 \end{aligned}$$

33. $$\begin{aligned} (x+2)^2 &= 14 \\ x + 2 &= \pm\sqrt{14} \\ x &= -2 \pm \sqrt{14} \end{aligned}$$

34. $$\begin{aligned} (x+9)^2 &= 24 \\ x + 9 &= \pm\sqrt{24} \\ x &= -9 \pm 2\sqrt{6} \end{aligned}$$

35. $$\begin{aligned}(2x-1)^2 &= 18\\ 2x-1 &= \pm\sqrt{18}\\ 2x &= 1 \pm 3\sqrt{2}\\ x &= \frac{1 \pm 3\sqrt{2}}{2}\end{aligned}$$

36. $$\begin{aligned}(4x+7)^2 &= 44\\ 4x+7 &= \pm\sqrt{44}\\ 4x &= -7 \pm 2\sqrt{11}\\ x &= \frac{-7 \pm 2\sqrt{11}}{4} = -\frac{7}{4} \pm \frac{\sqrt{11}}{2}\end{aligned}$$

37. $$\begin{aligned}(x-7)^2 &= (x+3)^2\\ x-7 &= \pm(x+3)\end{aligned}$$

$$\begin{aligned}x-7 = x+3 \quad &\text{or} \quad x-7 = -x-3\\ -7 \neq 3 \quad &\text{or} \quad 2x = 4\\ &\qquad\quad x = 2\end{aligned}$$

The only solution of the equation is $x = 2$.

38. $$\begin{aligned}(x+5)^2 &= (x+4)^2\\ x+5 &= \pm(x+4)\end{aligned}$$

$$\begin{aligned}x+5 = +(x+4) \quad &\text{or} \quad x+5 = -(x+4)\\ 5 \neq 4 \quad &\text{or} \quad x+5 = -x-4\\ &\qquad\quad 2x = -9\\ &\qquad\quad x = -\tfrac{9}{2}\end{aligned}$$

The only solution of the equation is $x = -\frac{9}{2}$.

39. $$\begin{aligned}x^2+4x-32 &= 0\\ x^2+4x &= 32\\ x^2+4x+2^2 &= 32+2^2\\ (x+2)^2 &= 36\\ x+2 &= \pm 6\\ x &= -2 \pm 6\\ x &= 4 \text{ or } x = -8\end{aligned}$$

40. $$\begin{aligned}x^2-2x-3 &= 0\\ x^2-2x &= 3\\ x^2-2x+(-1)^2 &= 3+(1)^2\\ (x-1)^2 &= 4\\ x-1 &= \pm\sqrt{4}\\ x &= 1 \pm 2\\ x &= 3 \text{ or } x = -1\end{aligned}$$

41. $$\begin{aligned}x^2+6x+2 &= 0\\ x^2+6x &= -2\\ x^2+6x+3^2 &= -2+3^2\\ (x+3)^2 &= 7\\ x+3 &= \pm\sqrt{7}\\ x &= -3 \pm \sqrt{7}\end{aligned}$$

42. $$\begin{aligned}x^2+8x+14 &= 0\\ x^2+8x &= -14\\ x^2+8x+4^2 &= -14+16\\ (x+4)^2 &= 2\\ x+4 &= \pm\sqrt{2}\\ x &= -4 \pm \sqrt{2}\end{aligned}$$

43. $$\begin{aligned}9x^2-18x &= -3\\ x^2-2x &= -\frac{1}{3}\\ x^2-2x+1^2 &= -\frac{1}{3}+1^2\\ (x-1)^2 &= \frac{2}{3}\\ x-1 &= \pm\sqrt{\frac{2}{3}}\\ x &= 1 \pm \sqrt{\frac{2}{3}}\\ x &= 1 \pm \frac{\sqrt{6}}{3}\end{aligned}$$

44. $$\begin{aligned}4x^2-4x &= 1\\ x^2-x &= \frac{1}{4}\\ x^2-x+\left(-\frac{1}{2}\right)^2 &= \frac{1}{4}+\left(-\frac{1}{2}\right)^2\\ \left(x-\frac{1}{2}\right)^2 &= \frac{1}{2}\\ x-\frac{1}{2} &= \pm\frac{\sqrt{2}}{2}\\ x &= \frac{1}{2} \pm \frac{\sqrt{2}}{2}\\ x &= \frac{1 \pm \sqrt{2}}{2}\end{aligned}$$

45.
$$\begin{aligned} 7 + 2x - x^2 &= 0 \\ -x^2 + 2x + 7 &= 0 \\ x^2 - 2x - 7 &= 0 \\ x^2 - 2x &= 7 \\ x^2 - 2x + (-1)^2 &= 7 + (-1)^2 \\ (x-1)^2 &= 8 \\ x - 1 &= \pm 2\sqrt{2} \\ x &= 1 \pm 2\sqrt{2} \end{aligned}$$

46.
$$\begin{aligned} -x^2 + x - 1 &= 0 \\ x^2 - x + 1 &= 0 \\ x^2 - x + \tfrac{1}{4} &= -1 + \tfrac{1}{4} \\ \left(x - \tfrac{1}{2}\right)^2 &= -\tfrac{3}{4} \end{aligned}$$

No real solution

47.
$$\begin{aligned} 2x^2 + 5x - 8 &= 0 \\ 2x^2 + 5x &= 8 \\ x^2 + \frac{5}{2}x &= 4 \\ x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 &= 4 + \left(\frac{5}{4}\right)^2 \\ \left(x + \frac{5}{4}\right)^2 &= \frac{89}{16} \\ x + \frac{5}{4} &= \pm\frac{\sqrt{89}}{4} \\ x &= -\frac{5}{4} \pm \frac{\sqrt{89}}{4} \\ x &= \frac{-5 \pm \sqrt{89}}{4} \end{aligned}$$

48.
$$\begin{aligned} 3x^2 - 4x - 7 &= 0 \\ 3x^2 - 4x &= 7 \\ x^2 - \frac{4}{3}x &= \frac{7}{3} \\ x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 &= \frac{7}{3} + \left(-\frac{2}{3}\right)^2 \\ \left(x - \frac{2}{3}\right)^2 &= \frac{25}{9} \\ x - \frac{2}{3} &= \pm\frac{5}{3} \\ x &= \frac{2}{3} \pm \frac{5}{3} \\ x &= -1 \text{ or } x = \frac{7}{3} \end{aligned}$$

49.
$$\begin{aligned} \frac{1}{x^2 + 2x + 5} &= \frac{1}{x^2 + 2x + 1^2 - 1^2 + 5} \\ &= \frac{1}{(x+1)^2 + 4} \end{aligned}$$

50.
$$\begin{aligned} \frac{1}{x^2 + 6x + 10} &= \frac{1}{x^2 + 6x + (3)^2 - (3)^2 + 10} \\ &= \frac{1}{x^2 + 6x + 9 + 1} \\ &= \frac{1}{(x+3)^2 + 1} \end{aligned}$$

51.
$$\begin{aligned} \frac{4}{x^2 + 10x + 74} &= \frac{4}{x^2 + 10x + (5)^2 - (5)^2 + 74} \\ &= \frac{4}{x^2 + 10x + 25 + 49} \\ &= \frac{4}{(x+5)^2 + 49} \end{aligned}$$

52.
$$\begin{aligned} \frac{5}{x^2 - 18x + 162} &= \frac{5}{x^2 - 18x + (9)^2 - (9)^2 + 162} \\ &= \frac{5}{x^2 - 18x + 81 + 81} \\ &= \frac{5}{(x-9)^2 + 81} \end{aligned}$$

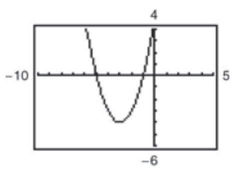
53.
$$\begin{aligned} \frac{1}{\sqrt{3 + 2x - x^2}} &= \frac{1}{\sqrt{-1(x^2 - 2x - 3)}} \\ &= \frac{1}{\sqrt{-1\left[x^2 - 2x + (1)^2 - (1)^2 - 3\right]}} \\ &= \frac{1}{\sqrt{-1(x^2 - 2x + 1) + 4}} \\ &= \frac{1}{\sqrt{4 - (x-1)^2}} \end{aligned}$$

54.
$$\begin{aligned} \frac{1}{\sqrt{9 + 8x - x^2}} &= \frac{1}{\sqrt{-1(x^2 - 8x - 9)}} \\ &= \frac{1}{\sqrt{-1\left[x^2 - 8x + (4)^2 - (4)^2 - 9\right]}} \\ &= \frac{1}{\sqrt{-1\left[(x^2 - 8x + 16) - 25\right]}} \\ &= \frac{1}{\sqrt{25 - (x-4)^2}} \end{aligned}$$

55. $$\frac{1}{\sqrt{12 + 4x - x^2}} = \frac{1}{\sqrt{-1(x^2 - 4x - 12)}}$$
$$= \frac{1}{\sqrt{-1\left[x^2 - 4x + (2)^2 - (2)^2 - 12\right]}}$$
$$= \frac{1}{\sqrt{-1\left[(x^2 - 4x + 4) - 16\right]}}$$
$$= \frac{1}{\sqrt{16 - (x - 2)^2}}$$

56. $$\frac{1}{\sqrt{16 - 6x - x^2}} = \frac{1}{\sqrt{16 - 1\left(x^2 + 6x\right)}}$$
$$= \frac{1}{\sqrt{16 - \left(x^2 + 6x + 3^2\right) + 9}}$$
$$= \frac{1}{\sqrt{25 - \left(x + 3\right)^2}}$$

57. (a) $y = \left(x + 3\right)^2 - 4$

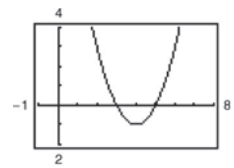


(b) The x-intercepts are $(-1, 0)$ and $(-5, 0)$.

(c) $$\begin{aligned} 0 &= (x + 3)^2 - 4 \\ 4 &= (x + 3)^2 \\ \pm\sqrt{4} &= x + 3 \\ -3 \pm 2 &= x \\ x &= -1 \quad \text{or} \quad x = -5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x + 3)^2 - 4$.

58. (a) $y = (x - 4)^2 - 1$

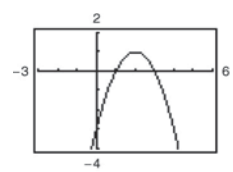


(b) The x-intercepts are $(5, 0)$ and $(3, 0)$.

(c) $$\begin{aligned} 0 &= (x - 4)^2 - 1 \\ (x - 4)^2 &= 1 \\ x - 4 &= \pm\sqrt{1} \\ x &= 4 \pm 1 = 5, 3 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x - 4)^2 - 1$.

59. (a) $y = 1 - (x - 2)^2$

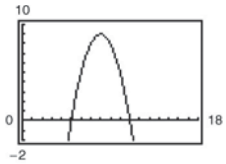


(b) The x-intercepts are $(1, 0)$ and $(3, 0)$.

(c) $$\begin{aligned} 0 &= 1 - (x - 2)^2 \\ (x - 2)^2 &= 1 \\ x - 2 &= \pm 1 \\ x &= 2 \pm 1 \\ x &= 3 \quad \text{or} \quad x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 1 - (x - 2)^2$.

60. (a) $y = 9 - (x - 8)^2$

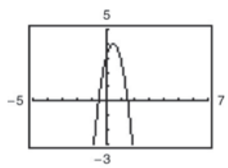


(b) The x-intercepts are $(5, 0)$ and $(11, 0)$.

(c)
$$\begin{aligned} 0 &= 9 - (x - 8)^2 \\ (x - 8)^2 &= 9 \\ x - 8 &= \pm\sqrt{9} \\ x &= 8 \pm 3 = 11, 5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 9 - (x - 8)^2$.

61. (a) $y = -4x^2 + 4x + 3$

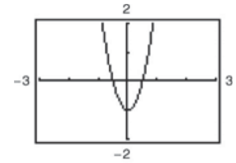


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{3}{2}, 0\right)$.

(c)
$$\begin{aligned} 0 &= -4x^2 + 4x + 3 \\ 4x^2 - 4x &= 3 \\ 4(x^2 - x) &= 3 \\ x^2 - x &= \frac{3}{4} \\ x^2 - x + \left(\frac{1}{2}\right)^2 &= \frac{3}{4} + \left(\frac{1}{2}\right)^2 \\ \left(x - \frac{1}{2}\right)^2 &= 1 \\ x - \frac{1}{2} &= \pm\sqrt{1} \\ x &= \frac{1}{2} \pm 1 \\ x &= \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{2} \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = -4x^2 + 4x + 3$.

62. (a) $y = 4x^2 - 1$

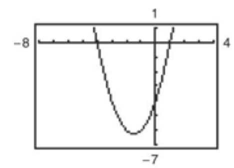


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$.

(c)
$$\begin{aligned} 0 &= 4x^2 - 1 \\ 4x^2 &= 1 \\ x^2 &= \frac{1}{4} \\ x &= \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2} \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 4x^2 - 1$.

63. (a) $y = x^2 + 3x - 4$

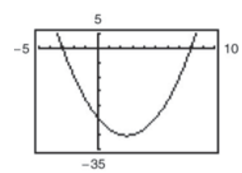


(b) The x-intercepts are $(-4, 0)$ and $(1, 0)$.

(c)
$$\begin{aligned} 0 &= x^2 + 3x - 4 \\ 0 &= (x + 4)(x - 1) \\ x + 4 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = -4 \quad &\text{or} \quad x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 + 3x - 4$.

64. (a) $y = x^2 - 5x - 24$



(b) The x-intercepts are $(8, 0)$ and $(-3, 0)$.

(c)
$$\begin{aligned} 0 &= x^2 - 5x - 24 \\ (x - 8)(x + 3) &= 0 \\ x - 8 = 0 &\Rightarrow x = 8 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 - 5x - 24$.

65. $2x^2 - 5x + 5 = 0$

$b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0$

No real solution

66. $-5x^2 - 4x + 1 = 0$

$b^2 - 4ac = (-4)^2 - 4(-5)(1) = 16 + 20 = 36 > 0$

Two real solutions

67. $2x^2 - z - 1 = 0$

$b^2 - 4ac = (-1)^2 - 4(2)(-1) = 9 > 0$

Two real solutions

68. $x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

One repeated solution

69. $\frac{1}{3}x^2 - 5x + 25 = 0$

$b^2 - 4ac = (-5)^2 - 4\left(\frac{1}{3}\right)(25) = -\frac{25}{3} < 0$

No real solution

70. $\frac{4}{7}x^2 - 8x + 280$

$b^2 - 4ac = (-8)^2 - 4\left(\frac{4}{7}\right)(28) = 64 - 64 = 0$

One repeated solution

71. $0.2x^2 + 1.2x - 8 = 0$

$b^2 - 4ac = (1.2)^2 - 4(0.2)(-8) = 7.84 > 0$

Two real solutions

72. $9 + 2.4x - 8.3x^2 = 0$

$$\begin{aligned} b^2 - 4ac &= (2.4)^2 - 4(-8.3)(9) \\ &= 5.76 + 298.8 = 304.56 > 0 \end{aligned}$$

Two real solutions

73. $2x^2 + x - 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-1 \pm 3}{4} = \frac{1}{2}, -1 \end{aligned}$$

74. $2x^2 - x - 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1 + 8}}{4} \\ &= \frac{1 \pm 3}{4} = 1, -\frac{1}{2} \end{aligned}$$

75. $16x^2 + 8x - 3 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} \\ &= \frac{-8 \pm 16}{32} = \frac{1}{4}, -\frac{3}{4} \end{aligned}$$

76. $25x^2 - 20x + 3 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-20) \pm \sqrt{(-20)^2 - (25)(3)}}{2(25)} \\ &= \frac{20 \pm \sqrt{400 - 300}}{50} \\ &= \frac{20 \pm 10}{50} = \frac{3}{5}, \frac{1}{5} \end{aligned}$$

77. $2 + 2x - x^2 = 0$

$-x^2 + 2x + 2 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} \\ &= \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3} \end{aligned}$$

78. $x^2 - 10x + 22 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)} \\ &= \frac{10 \pm \sqrt{100 - 88}}{2} \\ &= \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3} \end{aligned}$$

79. $x^2 + 12x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4(1)(16)}}{2(1)} = \frac{-12 \pm 4\sqrt{5}}{2} = -6 \pm 2\sqrt{5}$$

80. $4x = 8 - x^2$

$$x^2 + 4x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)} = \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

81. $x^2 + 8x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)} = \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5}$$

82. $2x^2 - 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} = \frac{3 \pm \sqrt{41}}{4} = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$$

83. $12x - 9x^2 = -3$

$$-9x^2 + 12x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4(-9)(3)}}{2(-9)} = \frac{-12 \pm 6\sqrt{7}}{-18} = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$$

84. $9x^2 - 37 = 6x$

$$9x^2 - 6x - 37 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(-37)}}{2(9)} = \frac{6 \pm 6\sqrt{38}}{18} = \frac{1}{3} \pm \frac{\sqrt{38}}{3}$$

85. $9x^2 + 30x + 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-30 \pm \sqrt{30^2 - 4(9)(25)}}{2(9)} = \frac{-30 \pm 0}{18} = -\frac{5}{3}$$

86. $36x^2 + 24x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)} = \frac{-24 \pm \sqrt{576 + 1008}}{72} = \frac{-24 \pm \sqrt{(144)(11)}}{72} = -\frac{1}{3} \pm \frac{\sqrt{11}}{6}$$

87. $4x^2 + 4x = 7$

$$4x^2 + 4x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(4)(-7)}}{2(4)} = \frac{-4 \pm 8\sqrt{2}}{8} = -\frac{1}{2} \pm \sqrt{2}$$

88. $16x^2 - 40x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)} = \frac{40 \pm \sqrt{1600 - 320}}{32} = \frac{40 \pm 16\sqrt{5}}{32} = \frac{5}{4} \pm \frac{\sqrt{5}}{2}$$

89. $$\begin{aligned} 28x - 49x^2 &= 4 \\ -49x^2 + 28x - 4 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)} \\ &= \frac{-28 \pm 0}{-98} = \frac{2}{7} \end{aligned}$$

90. $$\begin{aligned} 3x + x^2 - 1 &= 0 \\ x^2 + 3x - 1 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2} \end{aligned}$$

91. $$\begin{aligned} 8t &= 5 + 2t^2 \\ -2t^2 + 8t - 5 &= 0 \\ t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)} \\ &= \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2} \end{aligned}$$

92. $$\begin{aligned} 25h^2 + 80h + 61 &= 0 \\ h &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)} \\ &= \frac{-80 \pm \sqrt{6400 - 6100}}{50} \\ &= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50} \\ &= -\frac{8}{5} \pm \frac{\sqrt{3}}{5} \end{aligned}$$

93. $$\begin{aligned} (y - 5)^2 &= 2y \\ y^2 - 12y + 25 &= 0 \\ y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(25)}}{2(1)} \\ &= \frac{12 \pm 2\sqrt{11}}{2} = 6 \pm \sqrt{11} \end{aligned}$$

94. $$\begin{aligned} (z + 6)^2 &= -2z \\ z^2 + 12z + 36 &= -2z \\ z^2 + 14z + 36 &= 0 \\ z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-14 \pm \sqrt{14^2 - 4(1)(36)}}{2(1)} \\ &= \frac{-14 \pm \sqrt{52}}{2} = -7 \pm \sqrt{13} \end{aligned}$$

95. $$\begin{aligned} \frac{1}{2}x^2 + \frac{3}{8}x &= 2 \\ 4x^2 + 3x &= 16 \\ 4x^2 + 3x - 16 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4(4)(-16)}}{2(4)} \\ &= \frac{-3 \pm \sqrt{265}}{8} = -\frac{3}{8} \pm \frac{\sqrt{265}}{8} \end{aligned}$$

96. $$\begin{aligned} \left(\frac{5}{7}x - 14\right)^2 &= 8x \\ \frac{25}{49}x^2 - 20x + 196 &= 8x \\ \frac{25}{49}x^2 - 28x + 196 &= 0 \\ 25x^2 - 1372x + 9604 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1372) \pm \sqrt{(-1372)^2 - 4(25)(9604)}}{2(25)} \\ &= \frac{1372 \pm \sqrt{931{,}984}}{50} \\ &= \frac{686 \pm 196\sqrt{6}}{25} \end{aligned}$$

97. $5.1x^2 - 1.7x - 3.2 = 0$

$$x = \frac{1.7 \pm \sqrt{(-1.7)^2 - 4(5.1)(-3.2)}}{2(5.1)}$$
$$\approx 0.976, -0.643$$

98. $2x^2 - 2.50x - 0.42 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-2.50) \pm \sqrt{(-2.50)^2 - 4(2)(-0.42)}}{2(2)}$$
$$= \frac{2.50 \pm \sqrt{9.61}}{4} = 1.400, -0.150$$

99. $-0.067x^2 - 0.852x + 1.277 = 0$

$$x = \frac{-(-0.852) \pm \sqrt{(-0.852)^2 - 4(-0.067)(1.277)}}{2(-0.067)}$$
$$\approx -14.071, 1.355$$

100. $-0.005x^2 + 0.101x - 0.193 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-0.101 \pm \sqrt{(0.101)^2 - 4(-0.005)(-0.193)}}{2(-0.005)}$$
$$= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}$$
$$\approx 2.137, 18.063$$

101. $422x^2 - 506x - 347 = 0$

$$x = \frac{506 \pm \sqrt{(-506)^2 - 4(422)(-347)}}{2(422)}$$
$$\approx 1.687, -0.488$$

102. $1100x^2 + 326x - 715 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-326 \pm \sqrt{(326)^2 - 4(1100)(-715)}}{2(1100)}$$
$$= \frac{-326 \pm \sqrt{3,252,276}}{2200} \approx 0.672, 00.968$$

103. $12.67x^2 + 31.55x + 8.09 = 0$

$$x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)}$$
$$\approx 2.200, -0.290$$

104. $-3.22x^2 - 0.08x + 28.651 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}$$
$$= \frac{0.08 \pm \sqrt{369.031}}{-6.44} \approx -2.995, 2.971$$

105. $x^2 - 2x - 1 = 0$ Complete the square.

$$x^2 - 2x = 1$$
$$x^2 - 2x + 1^2 = 1 + 1^2$$
$$(x-1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

106. $11x^2 + 33x = 0$ Factor.

$$11(x^2 + 3x) = 0$$
$$x(x+3) = 0$$
$$x = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = -3$$

107. $(x+3)^2 = 81$ Extract square roots.

$$x + 3 = \pm 9$$
$$x + 3 = 9 \quad \text{or} \quad x + 3 = -9$$
$$x = 6 \quad \text{or} \quad x = -12$$

108. $x^2 - 14x + 49 = 0$ Extract square roots.

$$(x-7)^2 = 0$$
$$x - 7 = 0$$
$$x = 7$$

109. $x^2 - x - \frac{11}{4} = 0$ Complete the square.

$$x^2 - x = \frac{11}{4}$$
$$x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{11}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{12}{4}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{12}{4}}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$

110. $x^2 + 3x - \frac{3}{4} = 0$ Complete the square.

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$
$$x = -\frac{3}{2} \pm \sqrt{3}$$

111. $(x + 1)^2 = x^2$ Extract square roots.

$$x^2 = (x + 1)^2$$
$$x = \pm(x + 1)$$

For $x = +(x + 1)$:

$0 \neq 1$ No solution

For $x = -(x + 1)$:

$$2x = -1$$
$$x = -\frac{1}{2}$$

112. $3x + 4 = 2x^2 - 7$ Quadratic Formula

$$0 = 2x^2 - 3x - 11$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{97}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{97}}{4}$$

113. (a) $w(w + 14) = 1632$

(b) $w^2 + 14w - 1632 = 0$

$$(w + 48)(w - 34) = 0$$
$$w = -48 \quad \text{or} \quad w = 34$$

Because w must be greater than zero, $w = 34$ feet and the length is $w + 14 = 48$ feet.

114. Total fencing: $4x + 3y = 100$

Total area: $2xy = 350$

$$x = \frac{100 - 3y}{4}$$
$$2\left(\frac{100 - 3y}{4}\right)y = 350$$
$$\frac{1}{2}\left(100y - 3y^2\right) - 350 = 0$$
$$100y - 3y^2 - 700 = 0$$
$$-3y^2 + 100y - 700 = 0$$
$$(3y - 70)(-y + 10) = 0$$
$$3y - 70 = 0 \Rightarrow y = \frac{70}{3}$$
$$-y + 10 = 0 \Rightarrow y = 10$$

For $y = \dfrac{70}{3}$:

$$2x\left(\frac{70}{3}\right) = 350$$
$$x = 7.5$$

For $y = 10$:

$$2x(10) = 350$$
$$x = 17.5$$

There are two solutions: $x = 7.5$ meters and $y = \frac{70}{3}$ meters or $x = 17.5$ meters and $y = 10$ meters.

115. $S = x^2 + 4xh$

$$108 = x^2 + 4x(3)$$
$$0 = x^2 + 12x - 108$$
$$0 = (x + 18)(x - 6)$$
$$x = -18 \quad \text{or} \quad x = 6$$

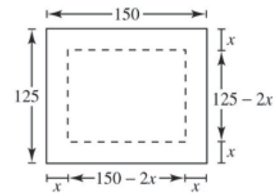
Because x must be positive, $x = 6$ inches. The dimensions of the box are 6 inches $\times$ 6 inches $\times$ 3 inches.

116. Volume: $4x^2 = 576$

$$x^2 = 144$$
$$x = \pm 12$$

Because x must be positive, $x = 12$ centimeters and the side length of the original material is $x + 8 = 20$ centimeters. The dimensions of the original material are 20 centimeters $\times$ 20 centimeters.

117. $(\text{Area mowed by first landscaper}) = \frac{1}{2}(\text{Total area of the lawn})$



$$(125 - 2x)(150 - 2x) = \frac{1}{2}(125)(150)$$
$$18750 - 550x + 4x^2 = 9375$$
$$4x^2 - 550x + 9375 = 0$$

Let $a = 4$, $b = -550$, and $c = 9375$. Use the Quadratic Formula:

$$x = \frac{-(-550) \pm \sqrt{(-550)^2 - 4(4)(93750)}}{2(4)}$$
$$= \frac{550 \pm \sqrt{302500 - 150,000}}{8}$$
$$= \frac{550 \pm \sqrt{152,500}}{8}$$

$x \approx 19.936$ and $x \approx 117.564$

Since the lot is only 125 feet wide, the only possible solution is $x \approx 19.936$ feet.

The first landscaper must go around the lot $\frac{19.936 \text{ feet}}{24 \text{ inches}} = \frac{19.936 \text{ feet}}{2 \text{ feet}} \approx 10$ times.

118. Original arrangement: x rows, y seats per row, $xy = 72$, $y = \frac{72}{x}$

New arrangement: $(x - 2)$ rows, $(y + 3)$ seats per row

$$(x - 2)(y + 3) = 72$$
$$(x - 2)\left(\frac{72}{x} + 3\right) = 72$$
$$x(x - 2)\left(\frac{72}{x} + 3\right) = 72x$$
$$(x - 2)(72 + 3x) = 72x$$
$$72x + 3x^2 - 144 - 6x = 72x$$
$$3x^2 - 6x - 144 = 0$$
$$x^2 - 2x - 48 = 0$$
$$(x - 8)(x + 6) = 0$$

$x - 8 = 0 \Rightarrow x = 8$

$x + 6 = 0 \Rightarrow x = -6$

Originally, there were 8 rows of seats with $\frac{72}{9} = 9$ seats per row.

119. (a) $s = -16t^2 + v_0 t + s_0$

$s = -16t^2 + 550$

Let $s = 0$ and solve for t.

$$0 = -16t^2 + 550$$
$$16t^2 = 550$$
$$t^2 = \frac{550}{16}$$
$$t = \sqrt{\frac{550}{16}}$$
$$t \approx 5.86$$

The supply package will take about 5.86 seconds to reach the ground.

(b) *Verbal Model:* $(\text{Distance}) = (\text{Rate}) \cdot (\text{Time})$

Labels: Distance $= d$

Rate = 138 miles per hour

Time = $\dfrac{5.86 \text{ seconds}}{3600 \text{ seconds per hour}} \approx 0.0016$ hour

Equation: $d = (138)(0.0016)$

≈ 0.22 mile

The supply package will travel about 0.22 mile.

120. (a) $s = -16t^2 + v_0 t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 984$. Thus, $s = -16t^2 + 984$.

(b) $s = -16(4)^2 + 984 = 728$ feet

(c) $$0 = -16t^2 + 984$$
$$16t^2 = 984$$
$$t^2 = \frac{984}{16}$$
$$t = \sqrt{\frac{984}{16}} = \frac{\sqrt{246}}{2} \approx 7.84$$

It will take the coin about 7.84 seconds to strike the ground.

121. (a) $s = -16t^2 + v_0 t + s_0$

$$v_0 = 100 \text{ mph} = \frac{(100)(5280)}{3600} = 146\frac{2}{3} \text{ ft/sec}$$

$$s_0 = 6\frac{1}{4} \text{ feet}$$

$$s = -16t^2 + 146\frac{2}{3}t + 6\frac{1}{4}$$

(b) When $t = 3$: $s(3) = 302.25$ feet

When $t = 4$: $s(4) \approx 336.92$ feet

When $t = 5$: $s(5) \approx 339.58$ feet

During the interval $3 \le t \le 5$, the baseball's speed decreased due to gravity.

(c) The ball hits the ground when $s = 0$.

$$-16t^2 + 146\tfrac{2}{3}t + 6\tfrac{1}{4} = 0$$

By the Quadratic Formula, $t \approx -0.042$ or $t \approx 9.209$. Assuming that the ball is not caught and drops to the ground, it will be in the air for approximately 9.209 seconds.

122. (a) $s = -16t^2 + v_0 t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 1815$. Thus, $s = -16t^2 + 1815$.

(b)

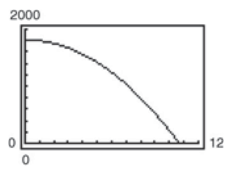
Time, t	0	2	4	6	8	10	12
Height, s	1815	1751	1559	1239	791	215	-489

(c) The object reaches the ground between $t = 10$ seconds and $t = 12$ seconds. Numerical approximation will vary, though 10.7 seconds is a reasonable estimate.

(d)
$$0 = -16t^2 + 1815$$
$$16t^2 = 1815$$
$$t^2 = \frac{1815}{16}$$
$$t = \sqrt{\frac{1815}{16}} = \frac{11\sqrt{15}}{4} \approx 10.65$$

It will take the object about 10.65 seconds to reach the ground.

(e)



The zero of the graph is at $t \approx 10.65$ seconds.

123. (a)

t	5	6	7	8	9	10	11
D	7.7	8.0	8.6	9.5	10.7	12.2	14.0

Sometime during the year 2008, the total public dept reached $10 trillion.

(b) *Algebraically:* $D = 0.157t^2 - 1.46t + 11.1$

$$10 = 0.157t^2 - 1.46t + 11.1$$

$$0 = 0.157t^2 - 1.46t + 1.1$$

Let $a = 0.157$, $b = -1.46$, and $c = 1.1$.

Use the Quadratic Formula:

$$t = \frac{-(-1.46) \pm \sqrt{(-1.46)^2 - 4(0.157)(1.1)}}{2(0.157)}$$

$$= \frac{1.46 \pm \sqrt{1.4408}}{0.314}$$

$t \approx 8.47$ and $t \approx 0.827$

Because the domain of the model is $5 \le t \le 11$, $t \approx 8.47$ is the only solution. So, the total public debt reached $10 trillion during 2008.

Graphically: Use a graphing utility to graph $y_1 = 0.157t^2 - 1.46t + 11.1$ and $y_2 = 10$ in the same viewing window. Then use the intersect feature to find that the graphs intersect when $t \approx 0.827$ and $t \approx 8.47$. Choose $t \approx 8.47$ because it is in the domain.

So, the total public debt reached $10 trillion during 2008.

(c) For 2020, let $t = 20$

$$D = 0.157t^2 - 1.46t + 11.0$$

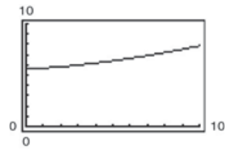
$$= 0.157(20)^2 - 1.46(20) + 11.1$$

$$= 44.7$$

Using the model, in 2020, the total public debt will be $44.7 trillion.

Answers will vary. *Sample Answer*: No, it is unlikely the total public debt will increase nearly $31 trillion over 9 years.

124. (a)



During 2005, the average ticket price reached $6.50.

(b) $$P = 0.0133t^2 + 0.096t + 5.57$$

$$6.50 = 0.0133t^2 + 0.096t + 5.57$$

$$0 = 0.0133t^2 + 0.096t - 0.93$$

Use the Quadratic Formula:

$$t = \frac{-0.096 \pm \sqrt{(0.096)^2 - 4(0.0133)(-0.93)}}{2(0.0133)}$$

$$= \frac{-0.096 \pm \sqrt{0.058692}}{2(0.0133)}$$

$t \approx -12.72$ and $t \approx 5.50$

Because the domain of the model is $1 \le t \le 10$, $t \approx 5.50$ is the only solution. So, the average ticket price reached $6.50 during 2005.

(c) For 2020, let $t = 20$.

$$P = 0.0133t^2 + 0.096t + 5.57$$

$$= 0.0133(20)^2 + 0.096(20) + 5.57$$

$$\approx 12.81$$

Using the model, the average ticket price in the year 2020 will be about $12.81.

125. $L = -0.270t^2 + 3.59t + 83.1$

$$93 = -0.270t^2 + 3.59t + 83.1$$

$$0 = -0.270t^2 + 3.59t - 9.9$$

$$0 = 0.270t^2 - 3.59t + 9.9$$

Using the Quadratic Formula,

$$t = \frac{-(-3.59) \pm \sqrt{(-3.59)^2 - 4(0.270)(9.9)}}{2(0.270)}$$

$$= \frac{3.59 \pm \sqrt{2.1961}}{0.54}$$

$t \approx 3.9$ and $t \approx 9.4$

Because the domain of the model is $2 \le t \le 7$, $t \approx 3.9$ is the only solution. The patient's blood oxygen level was 93% at approximately 4:00 P.M.

126. (a) $150 = 0.45x^2 - 1.65x + 50.75$

$$0 = 0.45x^2 - 1.65x - 99.25$$

$$x = \frac{1.65 \pm \sqrt{(-1.65)^2 - 4(0.45)(-99.25)}}{2(0.45)}$$

$$\approx -13.1, 16.8$$

Because $10 \le x \le 25$, choose $16.8°C$.

$x = 10$: $0.45(10)^2 - 1.65(10) + 50.75 = 79.25$

(b) $x = 20$: $0.45(20)^2 - 1.65(20) + 50.75 = 197.75$

$197.75 \div 79.25 \approx 2.5$

Oxygen consumption is increased by a factor of approximately 2.5.

127. (a) *Model:* $(\text{winch})^2 + (\text{distance to dock})^2 = (\text{length of rope})^2$

Labels: winch $= 15$, distance to dock $= x$, length of rope $= l$

Equation: $15^2 + x^2 = l^2$

(b) When $l = 75$: $15^2 + x^2 = 75^2$

$$x^2 = 5625 - 225 = 5400$$

$$x = \sqrt{5400} = 30\sqrt{5} \approx 73.5$$

The boat is approximately 73.5 feet from the dock when there is 75 feet of rope out.

128.

$$d_N = (3 \text{ hours})(r + 50 \text{ mph})$$

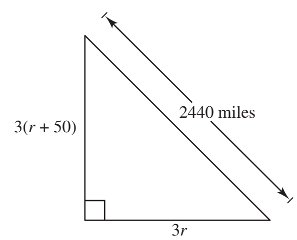
$$d_E = (3 \text{ hours})(r \text{ mph})$$

$$d_N{}^2 + d_E{}^2 = 2440^2$$

$$9(r + 50)^2 + 9r^2 = 2440^2$$

$$18r^2 + 900r - 5{,}931{,}100 = 0$$

$$r = \frac{-900 \pm \sqrt{900^2 - 4(18)(-5{,}931{,}100)}}{2(18)} = \frac{-900 \pm \sqrt{118{,}847}}{36}$$



Using the positive value for r, we have one plane moving northbound at $r + 50 \approx 600$ miles per hour and one plane moving eastbound at $r \approx 550$ miles per hour.

129. False.

$b^2 - 4ac = (-1)^2 - 4(-3)(-10) < 0$,

So, the quadratic equation has no real solutions.

130. False.

The product must equal zero for the Zero Factor Property to be used.

131. *Sample answer:* $(x - 3)(x - (-5)) = 0$

$$(x - 3)(x + 5) = 0$$

$$x^2 + 2x - 15 = 0$$

132. *Sample answer:* $(x - (-6))(x - (-9)) = 0$

$$(x + 6)(x + 9) = 0$$

$$x^2 + 15x + 54 = 0$$

133. One possible equation is:

$$(x - 8)(x - 14) = 0$$

$$x^2 - 22x + 112 = 0$$

Any non-zero multiple of this equation would also have these solutions.

134. $x = \frac{1}{6} \Rightarrow 6x = 1 \Rightarrow 6x - 1$ is a factor.

$x = -\frac{2}{5} \Rightarrow 5x = -2 \Rightarrow 5x + 2$ is a factor.

$$(6x - 1)(5x + 2) = 0$$

$$30x^2 + 7x - 2 = 0$$

135. One possible equation is:

$$\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$$
$$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$$
$$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2x + 1 - 2 = 0$$
$$x^2 - 2x - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

136. $x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$, so:

$$\left(x - \left(-3 + \sqrt{5}\right)\right)\left(x - \left(-3 - \sqrt{5}\right)\right) = 0$$
$$\left(x + 3 - \sqrt{5}\right)\left(x + 3 + \sqrt{5}\right) = 0$$
$$x^2 + 6x + 4 = 0$$

137. Yes, the vertex of the parabola would be on the x-axis.

138. (a) The discriminant is positive because the graph has two x-intercepts.

$$b^2 = 4ac = (-2)^2 - 4(1)(0) = 4$$

(b) The discriminant is zero because the graph has one x-intercept.

$$b^2 = 4ac = (-2)^2 - 4(1)(1) = 0$$

(c) The discriminant is negative because the graph has no x-intercepts.

$$b^2 = 4ac = (-2)^2 - 4(1)(2) = -4$$

In part (c), if the linear term was $2x$, the discriminant would still be zero and the equation would have no solutions.

In part (c), if the constant term was -2, the discriminant would be positive and the equation would have two solutions.

Section 1.5 Complex Numbers

1. real

2. imaginary

3. pure imaginary

4. $\sqrt{-1}$; -1

5. principal square

6. complex conjugates

7. $a + bi = -12 + 7i$
$a = -12$
$b = 7$

8. $a + bi = 13 + 4i$
$a = 13$
$b = 4$

9. $(a - 1) + (b + 3)i = 5 + 8i$
$a - 1 = 5 \Rightarrow a = 6$
$b + 3 = 8 \Rightarrow b = 5$

10. $(a + 6) + 2bi = 6 - 5i$
$a + 6 = 6 \Rightarrow a = 0$
$2b = -5 \Rightarrow b = -\frac{5}{2}$

11. $8 + \sqrt{-25} = 8 + 5i$

12. $5 + \sqrt{-36} = 5 + 6i$

13. $2 - \sqrt{-27} = 2 - \sqrt{27}i$
$= 2 - 3\sqrt{3}i$

14. $1 + \sqrt{-8} = 1 + 2\sqrt{2}i$

15. $\sqrt{-80} = 4\sqrt{5}i$

16. $\sqrt{-4} = 2i$

17. $14 = 14 + 0i = 14$

18. $75 = 75 + 0i = 75$

19. $-10i + i^2 = -10i - 1 = -1 - 10i$

20. $-4i^2 + 2i = -4(-1) + 2i$
$= 4 + 2i$

21. $\sqrt{-0.09} = \sqrt{0.09}i$
$= 0.3i$

22. $\sqrt{-0.0049} = \sqrt{0.0049}i$
$= 0.07i$

23. $(7 + i) + (3 - 4i) = 10 - 3i$

24. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

25. $(9 - i) - (8 - i) = 1$

26. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$= -3 - 11i$

27. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i$
$= 3 - 3\sqrt{2}i$

28. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i$
$= 4$

29. $13i - (14 - 7i) = 13i - 14 + 7i$
$= -14 + 20i$

30. $25 + (-10 + 11i) + 15i = 15 + 26i$

31. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = -\frac{3}{2} - \frac{5}{2}i + \frac{5}{3} + \frac{11}{3}i$
$= -\frac{9}{6} - \frac{15}{6}i + \frac{10}{6} + \frac{22}{6}i$
$= \frac{1}{6} + \frac{7}{6}i$

32. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

33. $(1+i)(3-2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

34. $(7-2i)(3-5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

35. $12i(1-9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

36. $-8i(9+4i) = -72i - 32i^2$
$= 32 - 72i$

37. $(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i) = 14 - 10i^2$
$= 14 + 10 = 24$

38. $(\sqrt{3} + \sqrt{15}i)(\sqrt{3} - \sqrt{15}i) = 3 - 15i^2$
$= 3 + 15 = 18$

39. $(6+7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

40. $(5-4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

41. $(2+3i)^2 + (2-3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2$
$= 4 + 12i - 9 + 4 - 12i - 9$
$= -10$

42. $(1-2i)^2 - (1+2i)^2 = 1 - 4i + 4i^2 - (1 + 4i + 4i^2)$
$= 1 - 4i + 4i^2 - 1 - 4i - 4i^2$
$= -8i$

43. The complex conjugate of $9 + 2i$ is $9 - 2i$.
$(9+2i)(9-2i) = 81 - 4i^2$
$= 81 + 4$
$= 85$

44. The complex conjugate of $8 - 10i$ is $8 + 10i$.
$(8-10i)(8+10i) = 64 - 100i^2$
$= 64 + 100$
$= 164$

45. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.
$(-1 - \sqrt{5}i)(-1 + \sqrt{5}i) = 1 - 5i^2$
$= 1 + 5 = 6$

46. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$(-3 + \sqrt{2}i)(-3 - \sqrt{2}i) = 9 - 2i^2$
$= 9 + 2$
$= 11$

47. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.
$(2\sqrt{5}i)(-2\sqrt{5}i) = -20i^2 = 20$

48. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.
$(\sqrt{15}i)(-\sqrt{15}i) = -15i^2 = 15$

49. The complex conjugate of $\sqrt{6}$ is $\sqrt{6}$.
$(\sqrt{6})(\sqrt{6}) = 6$

50. The complex conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$.
$(1+\sqrt{8})(1+\sqrt{8}) = 1 + 2\sqrt{8} + 8$
$= 9 + 4\sqrt{2}$

51. $\dfrac{3}{i} \cdot \dfrac{-i}{-i} = \dfrac{-3i}{-i^2} = -3i$

52. $-\dfrac{14}{2i} \cdot \dfrac{-2i}{-2i} = \dfrac{28i}{-4i^2} = \dfrac{28i}{4} = 7i$

53. $\dfrac{2}{4-5i} = \dfrac{2}{4-5i} \cdot \dfrac{4+5i}{4+5i}$
$= \dfrac{2(4+5i)}{16+25} = \dfrac{8+10i}{41} = \dfrac{8}{41} + \dfrac{10}{41}i$

54. $\dfrac{13}{1-i} \cdot \dfrac{(1+i)}{(1+i)} = \dfrac{13+13i}{1-i^2} = \dfrac{13+13i}{2} = \dfrac{13}{2} + \dfrac{13}{2}i$

55. $\dfrac{5+i}{5-i} \cdot \dfrac{(5+i)}{(5+i)} = \dfrac{25+10i+i^2}{25-i^2}$
$= \dfrac{24+10i}{26} = \dfrac{12}{13} + \dfrac{5}{13}i$

56. $\dfrac{6-7i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{6+12i-7i-14i^2}{1-4i^2}$
$= \dfrac{20+5i}{5} = 4 + i$

57. $\dfrac{9-4i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-9i+4i^2}{-i^2}=-4-9i$

58. $\dfrac{8+16i}{2i}\cdot\dfrac{-2i}{-2i}=\dfrac{-16i-32i^2}{-4i^2}=8-4i$

59. $$\begin{aligned}\frac{3i}{(4-5i)^2}&=\frac{3i}{16-40i+25i^2}=\frac{3i}{-9-40i}\cdot\frac{-9+40i}{-9+40i}\\&=\frac{-27i+120i^2}{81+1600}=\frac{-120-27i}{1681}\\&=-\frac{120}{1681}-\frac{27}{1681}i\end{aligned}$$

60. $$\begin{aligned}\frac{5i}{(2+3i)^2}&=\frac{5i}{4+12i+9i^2}\\&=\frac{5i}{-5+12i}\cdot\frac{-5-12i}{-5-12i}\\&=\frac{-25i-60i^2}{25-144i^2}\\&=\frac{60-25i}{169}=\frac{60}{169}-\frac{25}{169}i\end{aligned}$$

61. $$\begin{aligned}\frac{2}{1+i}-\frac{3}{1-i}&=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}\\&=\frac{2-2i-3-3i}{1+1}\\&=\frac{-1-5i}{2}\\&=-\frac{1}{2}-\frac{5}{2}i\end{aligned}$$

62. $$\begin{aligned}\frac{2i}{2+i}+\frac{5}{2-i}&=\frac{2i(2-i)+5(2+i)}{(2+i)(2-i)}\\&=\frac{4i-2i^2+10+5i}{4-i^2}\\&=\frac{12+9i}{5}\\&=\frac{12}{5}+\frac{9}{5}i\end{aligned}$$

63. $$\begin{aligned}\frac{i}{3-2i}+\frac{2i}{3+8i}&=\frac{i(3+8i)+2i(3-2i)}{(3-2i)(3+8i)}\\&=\frac{3i+8i^2+6i-4i^2}{9+24i-6i-16i^2}\\&=\frac{4i^2+9i}{9+18i+16}\\&=\frac{-4+9i}{25+18i}\cdot\frac{25-18i}{25-18i}\\&=\frac{-100+72i+225i-162i^2}{625+324}\\&=\frac{62+297i}{949}=\frac{62}{949}+\frac{297}{949}i\end{aligned}$$

64. $$\begin{aligned}\frac{1+i}{i}-\frac{3}{4-i}&=\frac{(1+i)(4-i)-3i}{i(4-i)}\\&=\frac{4-i+4i-i^2-3i}{4i-i^2}\\&=\frac{5}{1+4i}\cdot\frac{1-4i}{1-4i}\\&=\frac{5-20i}{1-16i^2}\\&=\frac{5}{17}-\frac{20}{17}i\end{aligned}$$

65. $$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2}&=\left(\sqrt{6}i\right)\left(\sqrt{2}i\right)=\sqrt{12}i^2=\left(2\sqrt{3}\right)(-1)\\&=-2\sqrt{3}\end{aligned}$$

66. $$\begin{aligned}\sqrt{-5}\cdot\sqrt{-10}&=\left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\&=\sqrt{50}i^2=5\sqrt{2}(-1)=-5\sqrt{2}\end{aligned}$$

67. $\left(\sqrt{-15}\right)^2=\left(\sqrt{15}i\right)^2=15i^2=-15$

68. $\left(\sqrt{-75}\right)^2=\left(\sqrt{75}i\right)^2=75i^2=-75$

69. $$\begin{aligned}\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right)&=\left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)\\&=21-3\sqrt{10}i+7\sqrt{5}i-\sqrt{50}i^2\\&=\left(21+\sqrt{50}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\\&=\left(21+5\sqrt{2}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\end{aligned}$$

70. $$\left(2 - \sqrt{-6}\right)^2 = \left(2 - \sqrt{6}i\right)\left(2 - \sqrt{6}i\right)$$
$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$$
$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$$
$$= 4 - 6 - 4\sqrt{6}i$$
$$= -2 - 4\sqrt{6}i$$

71. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-4}}{2}$$
$$= \frac{2 \pm 2i}{2}$$
$$= 1 \pm i$$

72. $x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{-4}}{2}$$
$$= \frac{-6 + 2i}{2}$$
$$= -3 \pm i$$

73. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{-16}}{8}$$
$$= \frac{-16 \pm 4i}{8}$$
$$= -2 \pm \frac{1}{2}i$$

74. $9x^2 - 6x + 37 = 0;\ a = 9,\ b = -6,\ c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}$$
$$= \frac{6 \pm \sqrt{-1296}}{18}$$
$$= \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

75. $4x^2 + 16x + 15 = 0;\ a = 4,\ b = 16,\ c = 15$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$
$$x = -\frac{12}{8} = -\frac{3}{2} \text{ or } x = -\frac{20}{8} = -\frac{5}{2}$$

76. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$
$$= \frac{4 \pm \sqrt{-176}}{32}$$
$$= \frac{4 \pm 4\sqrt{11}i}{32}$$
$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

77. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{-72}}{6}$$
$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

78. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$
$$= \frac{12 \pm \sqrt{-136}}{28}$$
$$= \frac{12 \pm 2\sqrt{34}i}{28}$$
$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

79. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0;\ a = 7,\ b = -10,\ c = -50$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)} = \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14} = \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

80. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)} = \frac{3 \pm \sqrt{-207}}{9} = \frac{3 \pm 3\sqrt{23}i}{9} = \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

81. $-6i^3 + i^2 = -6i^2i + i^2$

$= -6(-1)i + (-1)$

$= 6i - 1$

$= -1 + 6i$

82. $4i^2 - 2i^3 = 4i^2 - 2i^2i = 4(-1) - 2(-1)i = -4 + 2i$

83. $-14i^5 = -14i^2i^2i = -14(-1)(-1)(i) = -14i$

84. $(-i)^3 = (-1)(i^3) = (-1)i^2i = (-1)(-1)i = i$

85. $\left(\sqrt{-72}\right)^3 = \left(6\sqrt{2}i\right)^3$

$= 6^3\left(\sqrt{2}\right)^3 i^3$

$= 216\left(2\sqrt{2}\right)i^2i$

$= 432\sqrt{2}(-1)i$

$= -432\sqrt{2}i$

86. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6$

$= 8i^6 = 8i^2i^2i^2$

$= 8(-1)(-1)(-1)$

$= -8$

87. $\dfrac{1}{i^3} = \dfrac{1}{i^2i} = \dfrac{1}{-i} = \dfrac{1}{-i}\cdot\dfrac{i}{i} = \dfrac{i}{-i^2} = i$

88. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{8i^2i} = \dfrac{1}{-8i} = \dfrac{1}{-8i}\cdot\dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

89. $(3i)^4 = 81i^4 = 81i^2i^2 = 81(-1)(-1) = 81$

90. $(-i)^6 = i^6 = i^2i^2i^2 = (-1)(-1)(-1) = -1$

91. (a) $z_1 = 9 + 16i,\ z_2 = 20 - 10i$

(b) $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{9 + 16i} + \dfrac{1}{20 - 10i} = \dfrac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \dfrac{29 + 6i}{340 + 230i}$

$z = \left(\dfrac{340 + 230i}{29 + 6i}\right)\left(\dfrac{29 - 6i}{29 - 6i}\right) = \dfrac{11{,}240 + 4630i}{877} = \dfrac{11{,}240}{877} + \dfrac{4630}{877}i$

92. (a) $\left(-1 + \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^2i$

$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$

$= 8$

(b) $\left(-1 - \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^2i$

$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$

$= 8$

93. False.

If $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

94. True.

$$\begin{aligned} x^4 - x^2 + 14 &= 56 \\ \left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 &\stackrel{?}{=} 56 \\ 36 + 6 + 14 &\stackrel{?}{=} 56 \\ 56 &= 56 \end{aligned}$$

95. False.

$$\begin{aligned} i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= \left(i^2\right)^{22} + \left(i^2\right)^{75} - \left(i^2\right)^{37} - \left(i^2\right)^{54} i + \left(i^2\right)^{30} i \\ &= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54} i + (-1)^{30} i \\ &= 1 - 1 + 1 - i + i = 1 \end{aligned}$$

96. False.

Sample answer: $4i + (3 + 2i) = 3 + 6i$, which is not a real number.

97.

$$\begin{aligned} i &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \\ i^5 &= i^4 i = i \\ i^6 &= i^4 i^2 = -1 \\ i^7 &= i^4 i^3 = -i \\ i^8 &= i^4 i^4 = 1 \\ i^9 &= i^4 i^4 i = i \\ i^{10} &= i^4 i^4 i^2 = -1 \\ i^{11} &= i^4 i^4 i^3 = -i \\ i^{12} &= i^4 i^4 i^4 = 1 \end{aligned}$$

The pattern $i, -1, -i, 1$ repeats. Divide the exponent by 4.

If the remainder is 1, the result is i.

If the remainder is 2, the result is -1.

If the remainder is 3, the result is $-i$.

If the remainder is 0, the result is 1.

98. (i) D

(ii) F

(iii) B

(iv) E

(v) A

(vi) C

99. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

100.

$$\begin{aligned} (a_1 + b_1 i)(a_2 + b_2 i) &= a_1 a_2 + a_1 b_2 i + a_2 b_1 i + b_1 b_2 i^2 \\ &= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i \end{aligned}$$

The complex conjugate of this product is $(a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i$.

The product of the complex conjugates is

$$\begin{aligned} (a_1 - b_1 i)(a_2 - b_2 i) &= a_1 a_2 - a_1 b_2 i - a_2 b_1 i - b_1 b_2 i^2 \\ &= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i. \end{aligned}$$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

101. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2) i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2) i$.

The sum of the complex conjugates is $(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2) i$.

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

Section 1.6 Other Types of Equations

1. polynomial

2. extraneous

3. rational

4. absolute value

5.
$$\begin{aligned} 6x^4 - 14x^2 &= 0 \\ 2x^2(3x^2 - 7) &= 0 \\ 2x^2 = 0 &\Rightarrow x = 0 \\ 3x^2 - 7 = 0 &\Rightarrow x = \pm\frac{\sqrt{21}}{3} \end{aligned}$$

6.
$$\begin{aligned} 36x^3 - 100x &= 0 \\ 4x(9x^2 - 25) &= 0 \\ 4x(3x + 5)(3x - 5) &= 0 \\ 4x = 0 &\Rightarrow x = 0 \\ 3x + 5 = 0 &\Rightarrow x = -\tfrac{5}{3} \\ 3x - 5 = 0 &\Rightarrow x = \tfrac{5}{3} \end{aligned}$$

7.
$$\begin{aligned} x^4 - 81 &= 0 \\ (x^2 + 9)(x + 3)(x - 3) &= 0 \\ x^2 + 9 = 0 &\Rightarrow x = \pm 3i \\ x + 3 = 0 &\Rightarrow x = -3 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

8.
$$\begin{aligned} x^6 - 64 &= 0 \\ (x^3 - 8)(x^3 + 8) &= 0 \\ (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4) &= 0 \\ x - 2 = 0 &\Rightarrow x = 2 \\ x^2 + 2x + 4 = 0 &\Rightarrow x = -1 \pm \sqrt{3}i \\ x + 2 = 0 &\Rightarrow x = -2 \\ x^2 - 2x + 4 = 0 &\Rightarrow x = 1 \pm \sqrt{3}i \end{aligned}$$

9.
$$\begin{aligned} x^3 + 512 &= 0 \\ x^3 + 8^3 &= 0 \\ (x + 8)(x^2 - 8x + 64) &= 0 \\ x + 8 = 0 &\Rightarrow x = -8 \\ x^2 - 8x + 64 = 0 &\Rightarrow x = 4 \pm 4\sqrt{3}i \end{aligned}$$

10.
$$\begin{aligned} 27x^3 - 343 &= 0 \\ (3x)^3 - 7^3 &= 0 \\ (3x - 7)(9x^2 + 21x + 49) &= 0 \\ 3x - 7 = 0 &\Rightarrow x = \frac{7}{3} \\ 9x^2 + 21x + 49 = 0 &\Rightarrow -\frac{7}{6} \pm \frac{7\sqrt{3}}{6}i \end{aligned}$$

11.
$$\begin{aligned} 5x^3 + 3 - x^2 + 45x &= 0 \\ 5x(x^2 + 6x + 9) &= 0 \\ 5x(x + 3)^2 &= 0 \\ 5x = 0 &\Rightarrow x = 0 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

12.
$$\begin{aligned} 9x^4 - 24x^3 + 16x^2 &= 0 \\ x^2(9x^2 - 24x + 16) &= 0 \\ x^2(3x - 4)^2 &= 0 \\ x^2 = 0 &\Rightarrow x = 0 \\ 3x - 4 = 0 &\Rightarrow x = \tfrac{4}{3} \end{aligned}$$

13.
$$\begin{aligned} x^3 - 3x^2 - x + 3 &= 0 \\ x^2(x - 3) - (x - 3) &= 0 \\ (x - 3)(x^2 - 1) &= 0 \\ (x - 3)(x + 3)(x - 1) &= 0 \\ x - 3 = 0 &\Rightarrow x = 3 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 1 = 0 &\Rightarrow x = 1 \end{aligned}$$

14.
$$\begin{aligned} x^3 + 2x^2 + 3x + 6 &= 0 \\ x^2(x + 2) + 3(x + 2) &= 0 \\ (x + 2)(x^2 + 3) &= 0 \\ x + 2 = 0 &\Rightarrow x = -2 \\ x^2 + 3 = 0 &\Rightarrow x = \pm\sqrt{3}i \end{aligned}$$

15.
$$\begin{aligned} x^4 - x^3 + x - 1 &= 0 \\ x^3(x - 1) + (x - 1) &= 0 \\ (x - 1)(x^3 + 1) &= 0 \\ (x - 1)(x + 1)(x^2 - x + 1) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x^2 - x + 1 = 0 &\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i \end{aligned}$$

16.
$$\begin{aligned} x^4 + 2x^3 - 8x - 16 &= 0 \\ x^3(x + 2) - 8(x + 2) &= 0 \\ (x^3 - 8)(x + 2) &= 0 \\ (x - 2)(x^2 + 2x + 4)(x + 2) &= 0 \\ x - 2 = 0 &\Rightarrow x = 2 \\ x^2 + 2x + 4 = 0 &\Rightarrow x = -1 \pm \sqrt{3}i \\ x + 2 = 0 &\Rightarrow x = -2 \end{aligned}$$

17.
$$\begin{aligned} x^4 - 4x^2 + 3 &= 0 \\ (x^2 - 3)(x^2 - 1) &= 0 \\ (x + \sqrt{3})(x - \sqrt{3})(x + 1)(x - 1) &= 0 \\ x + \sqrt{3} = 0 &\Rightarrow x = -\sqrt{3} \\ x - \sqrt{3} = 0 &\Rightarrow x = \sqrt{3} \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 1 = 0 &\Rightarrow x = 1 \end{aligned}$$

18.
$$\begin{aligned} x^4 + 5x^2 - 36 &= 0 \\ (x^2 + 9)(x^2 - 4) &= 0 \\ (x^2 + 9)(x + 2)(x - 2) &= 0 \\ x^2 + 9 = 0 &\Rightarrow x = \pm 3i \\ x + 2 = 0 &\Rightarrow x = -2 \\ x - 2 = 0 &\Rightarrow x = 2 \end{aligned}$$

19.
$$\begin{aligned} 4x^4 - 65x^2 + 16 &= 0 \\ (4x^2 - 1)(x^2 - 16) &= 0 \\ (2x + 1)(2x - 1)(x + 4)(x - 4) &= 0 \\ 2x + 1 = 0 &\Rightarrow x = -\tfrac{1}{2} \\ 2x - 1 = 0 &\Rightarrow x = \tfrac{1}{2} \\ x + 4x = 0 &\Rightarrow x = -4 \\ x - 4 = 0 &\Rightarrow x = 4 \end{aligned}$$

20.
$$\begin{aligned} 36t^4 + 29t^2 - 7 &= 0 \\ (36t^2 - 7)(t^2 + 1) &= 0 \\ (6t + \sqrt{7})(6t - \sqrt{7})(t^2 + 1) &= 0 \\ 6t + \sqrt{7} = 0 &\Rightarrow t = -\frac{\sqrt{7}}{6} \\ 6t - \sqrt{7} = 0 &\Rightarrow t = \frac{\sqrt{7}}{6} \\ t^2 + 1 = 0 &\Rightarrow t = \pm i \end{aligned}$$

21.
$$\begin{aligned} x^6 + 7x^3 - 8 &= 0 \\ (x^3 + 8)(x^3 - 1) &= 0 \\ (x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1) &= 0 \\ x + 2 = 0 &\Rightarrow x = -2 \\ x^2 - 2x + 4 = 0 &\Rightarrow x = 1 \pm \sqrt{3}i \\ x - 1 = 0 &\Rightarrow x = 1 \\ x^2 + x + 1 = 0 &\Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i \end{aligned}$$

22.
$$x^6 + 3x^3 + 2 = 0$$
$$(x^3 + 2)(x^3 + 1) = 0$$
$$\left(x + \sqrt[3]{2}\right)\left[x^2 - \sqrt[3]{2x} + \left(\sqrt[3]{2}\right)^2\right](x + 1)(x^2 - x + 1) = 0$$
$$x + \sqrt[3]{2x} + \left(\sqrt[3]{2}\right)^2 = 0 \Rightarrow x = -\sqrt[3]{2}$$
$$x^2 - \sqrt[3]{2x} + \left(\sqrt[3]{2}\right)^2 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2^{2/3}}$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{3}i}{2}$$

23.
$$\frac{1}{x^2} + \frac{8}{x} + 15 = 0$$
$$1 + 8x + 15x^2 = 0$$
$$(1 + 3x)(1 + 5x) = 0$$
$$1 + 3x = 0 \Rightarrow x = -\frac{1}{3}$$
$$1 + 5x = 0 \Rightarrow x = -\frac{1}{5}$$

24.
$$1 + \frac{3}{x} = \frac{2}{x^2}$$
$$x^2 + 3x = 2$$
$$x^2 + 3x - 2 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{17}}{2}$$

25.
$$2\left(\frac{x}{x+2}\right)^2 - 3\left(\frac{x}{x+2}\right) - 2 = 0$$
$$2x^2 - 3x(x + 2) - 2(x + 2)^2 = 0$$
$$2x^2 - 3x^2 - 6x - 2x^2 - 8x - 8 = 0$$
$$-3x^2 - 14x - 8 = 0$$
$$3x^2 + 14x + 8 = 0$$
$$(3x + 2)(x + 4) = 0$$
$$3x + 2 = 0 \Rightarrow x -\frac{2}{3}$$
$$x + 4 = 0 \Rightarrow x = -4$$

26. $6\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) - 6 = 0$

Let $u = x/(x + 1)$.
$$6u^2 + 5u - 6 = 0$$
$$(3u - 2)(2u + 3) = 0$$
$$3u - 2 = 0 \Rightarrow u = \frac{2}{3}$$
$$2u + 3 = 0 \Rightarrow u = -\frac{3}{2}$$
$$\frac{x}{x+1} = \frac{2}{3} \Rightarrow x = 2$$
$$\frac{x}{x+1} = \frac{-3}{2} \Rightarrow x = -\frac{3}{5}$$

27.
$$2x + 9\sqrt{x} = 5$$
$$2x + 9\sqrt{x} - 5 = 0$$
$$2\left(\sqrt{x}\right)^2 + 9\sqrt{x} - 5 = 0$$
$$\left(2\sqrt{x} - 1\right)\left(\sqrt{x} + 5\right) = 0$$
$$\sqrt{x} = \tfrac{1}{2} \Rightarrow x = \tfrac{1}{4}$$

$\left(\sqrt{x} = -5\right.$ is not a solution.$\left.\right)$

28. $6x - 7\sqrt{x} - 3 = 0$

Let $u = \sqrt{x}$.
$$6u^2 - 7u - 3 = 0$$
$$(3u + 1)(2u - 2) = 0$$
$$3u + 1 = 0 \Rightarrow u = -\tfrac{1}{3}$$
$$2u - 3 = 0 \Rightarrow u = \tfrac{3}{2}$$

$\sqrt{x} = -\frac{1}{3}$ is not a solution.

$\sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$

29.
$$\begin{aligned} 3x^{1/3} + 2x^{2/3} &= 5 \\ 2x^{2/3} + 3x^{1/3} - 5 &= 0 \\ 2\left(x^{1/3}\right)^2 + 3x^{1/3} - 5 &= 0 \\ \left(2x^{1/3} + 5\right)\left(x^{1/3} - 1\right) &= 0 \end{aligned}$$
$$2x^{1/3} + 5 = 0 \Rightarrow x^{1/3} = -\tfrac{5}{2} \Rightarrow x = \left(-\tfrac{5}{2}\right)^3 = -\tfrac{125}{8}$$
$$x^{1/3} - 1 = 0 \Rightarrow x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$$

30.
$$\begin{aligned} 9t^{2/3} + 24^{1/3} + 16 &= 0 \\ \left(3t^{1/3} + 4\right)^2 &= 0 \\ 3t^{1/3} + 4 = 0 \Rightarrow t^{1/3} &= -\tfrac{4}{3} \\ t &= -\tfrac{64}{27} \end{aligned}$$

31.
$$\begin{aligned} \sqrt{3x} - 12 &= 0 \\ \sqrt{3x} &= 12 \\ 3x &= 144 \\ x &= 48 \end{aligned}$$

32.
$$\begin{aligned} 7\sqrt{x} - 4 &= 0 \\ 7\sqrt{x} &= 4 \\ \sqrt{x} &= \tfrac{4}{7} \\ x &= \tfrac{16}{49} \end{aligned}$$

33.
$$\begin{aligned} \sqrt{x - 10} - 4 &= 0 \\ \sqrt{x - 10} &= 0 \\ x - 10 &= 16 \\ x &= 26 \end{aligned}$$

34.
$$\begin{aligned} \sqrt{5 - x} - 3 &= 0 \\ \sqrt{5 - x} &= 3 \\ 5 - x &= 9 \\ x &= -4 \end{aligned}$$

35.
$$\begin{aligned} \sqrt[3]{2x + 5} + 3 &= 0 \\ \sqrt[3]{2x + 5} &= -3 \\ 2x + 5 &= -27 \\ 2x &= -32 \\ x &= -16 \end{aligned}$$

36.
$$\begin{aligned} \sqrt[3]{3x + 1} - 5 &= 0 \\ \sqrt[3]{3x + 1} &= 5 \\ 3x + 1 &= 125 \\ 3x &= 124 \\ x &= \tfrac{124}{3} \end{aligned}$$

37.
$$\begin{aligned} -\sqrt{26 - 11x} + 4 &= x \\ 4 - x &= \sqrt{26 - 11x} \\ 16 - 8x + x^2 &= 26 - 11x \\ x^2 + 3x - 10 &= 0 \\ (x + 5)(x - 2) &= 0 \\ x + 5 = 0 \Rightarrow x &= -5 \\ x - 2 = 0 \Rightarrow x &= 2 \end{aligned}$$

38.
$$\begin{aligned} x + \sqrt{31 - 9x} &= 5 \\ \sqrt{31 - 9x} &= 5 - x \\ 31 - 9x &= 25 - 10x + x^2 \\ 0 &= x^2 - x - 6 \\ 0 &= (x - 3)(x + 2) \\ 0 &= x - 3 \Rightarrow x = 3 \\ x &= x - 2 \Rightarrow x = -2 \end{aligned}$$

39.
$$\begin{aligned} \sqrt{x} - \sqrt{x - 5} &= 1 \\ \sqrt{x} &= 1 + \sqrt{x - 5} \\ \left(\sqrt{x}\right)^2 &= \left(1 + \sqrt{x - 5}\right)^2 \\ x &= 1 + 2\sqrt{x - 5} + x - 5 \\ 4 &= 2\sqrt{x - 5} \\ 2 &= \sqrt{x - 5} \\ 4 &= x - 5 \\ 9 &= x \end{aligned}$$

40.
$$\begin{aligned} \sqrt{x} + \sqrt{x - 20} &= 10 \\ \sqrt{x} &= 10 - \sqrt{x - 20} \\ \left(\sqrt{x}\right)^2 &= \left(10 - \sqrt{x - 20}\right)^2 \\ x &= 100 - 20\sqrt{x - 20} + x - 20 \\ -80 &= -20\sqrt{x - 20} \\ 16 &= x - 20 \\ 36 &= x \end{aligned}$$

41.
$$\begin{aligned} \sqrt{x+5} + \sqrt{x-5} &= 10 \\ \sqrt{x+5} &= 10 - \sqrt{x-5} \\ \left(\sqrt{x+5}\right)^2 &= \left(10 - \sqrt{x-5}\right)^2 \\ x + 5 &= 100 - 20\sqrt{x-5} + x - 5 \\ -90 &= -20\sqrt{x-5} \\ 9 &= 2\sqrt{x-5} \\ 81 &= 4(x-5) \\ 81 &= 4x - 20 \\ 101 &= 4x \\ \tfrac{101}{4} &= x \end{aligned}$$

42.
$$\begin{aligned} 2\sqrt{x+1} - \sqrt{2x+3} &= 1 \\ 2\sqrt{x+1} &= 1 + \sqrt{2x+3} \\ \left(2\sqrt{x+1}\right)^2 &= \left(1 + \sqrt{2x+3}\right)^2 \\ 4(x+1) &= 1 + 2\sqrt{2x+3} + 2x + 3 \\ 2x &= 2\sqrt{2x+3} \\ x &= \sqrt{2x+3} \\ x^2 &= 2x + 3 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ x - 3 &= 0 \Rightarrow x = 3 \\ x + 1 &= 0 \Rightarrow x = -1, \text{ extraneous} \end{aligned}$$

43.
$$\begin{aligned} \sqrt{4\sqrt{4x+9}} &= \sqrt{8x+2} \\ \left(\sqrt{4\sqrt{4x+9}}\right)^2 &= \left(\sqrt{8x+2}\right)^2 \\ 4\sqrt{4x+9} &= 8x + 2 \\ 2\sqrt{4x+9} &= 4x + 1 \\ \left(2\sqrt{4x+9}\right)^2 &= (4x+1)^2 \\ 4(4x+9) &= 16x^2 + 8x + 1 \\ 16x + 36 &= 16x^2 + 8x + 1 \\ 0 &= 16x^2 - 8x - 35 \\ 0 &= (4x+5)(4x-7) \\ 4x + 5 &= 0 \Rightarrow x = -\frac{5}{4}, \text{ extraneous} \\ 4x - 7 &= 0 \Rightarrow x = \frac{7}{4} \end{aligned}$$

44.
$$\begin{aligned} \sqrt{16 + 9\sqrt{x}} &= 4 + \sqrt{x} \\ \left(\sqrt{16 + 9\sqrt{x}}\right)^2 &= \left(4 + \sqrt{x}\right)^2 \\ 16 + 9\sqrt{x} &= 16 + 8\sqrt{x} + x \\ \sqrt{x} &= x \\ \left(\sqrt{x}\right)^2 &= (x)^2 \\ x &= x^2 \\ x^2 - x &= 0 \\ x(x-1) &= 0 \\ x &= 0 \\ x - 1 &= 0 \Rightarrow x = 1 \end{aligned}$$

45.
$$\begin{aligned} (x-5)^{3/2} &= 8 \\ (x-5)^3 &= 8^2 \\ x - 5 &= \sqrt[3]{64} \\ x &= 5 + 4 = 9 \end{aligned}$$

46.
$$\begin{aligned} (x+2)^{2/3} &= 9 \\ (x+2)^2 &= 9^3 \\ x + 2 &= \pm\sqrt{729} \\ x &= -2 \pm 27 = -29, 25 \end{aligned}$$

47.
$$\begin{aligned} (x^2 - 5)^{3/2} &= 27 \\ (x^2 - 5)^3 &= 27^2 \\ x^2 - 5 &= \sqrt[3]{27^2} \\ x^2 &= 5 + 9 \\ x^2 &= 14 \\ x &= \pm\sqrt{14} \end{aligned}$$

48. $\left(x^2 - x - 22\right)^{3/2} = 27$

$$x^2 - x - 22 = 27^{2/3}$$
$$x^2 - x - 22 = 9$$
$$x^2 - x - 31 = 0$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-31)}}{2(1)} = \frac{1 \pm \sqrt{125}}{2} = \frac{1 \pm 5\sqrt{5}}{2}$$

49. $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

$$(x-1)^{1/2}\left[3x + 2(x-1)\right] = 0$$
$$(x-1)^{1/2}(5x-2) = 0$$
$$(x-1)^{1/2} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$$
$$5x - 2 = 0 \Rightarrow x = \tfrac{2}{5}, \text{ extraneous}$$

50. $4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} = 0$

$$2x\left[2x(x-1)^{1/3} + 3(x-1)^{4/3}\right] = 0$$
$$2x(x-1)^{1/3}\left[2x + 3(x-1)\right] = 0$$
$$2x(x-1)^{1/3}(5x-3) = 0$$
$$2x = 0 \Rightarrow x = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$5x - 3 = 0 \Rightarrow x = \tfrac{3}{5}$$

51. $x = \frac{3}{x} + \frac{1}{2}$

$$(2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right)$$
$$2x^2 = 6 + x$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$$
$$x - 2 = 0 \Rightarrow x = 2$$

52. $\frac{4}{x} - \frac{5}{3} = \frac{x}{6}$

$$(6x)\frac{4}{x} - (6x)\frac{5}{3} = (6x)\frac{x}{6}$$
$$24 - 10x = x^2$$
$$x^2 + 10x - 24 = 0$$
$$(x+12)(x-2) = 0$$
$$x + 12 = 0 \Rightarrow x = -12$$
$$x - 2 = 0 \Rightarrow x = 2$$

53. $\frac{1}{x} - \frac{1}{x+1} = 3$

$$x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} = x(x+1)(3)$$
$$x + 1 - x = 3x(x+1)$$
$$1 = 3x^2 + 3x$$
$$0 = 3x^2 + 3x - 1$$
$$a = 3, b = 3, c = -1$$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$$

54. $\frac{4}{x+1} - \frac{3}{x+2} = 1$

$$4(x+2) - 3(x+1) = (x+1)(x+2)$$
$$4x + 8 - 3x = x^2 + 3x + 2$$
$$x^2 + 2x - 3 = 0$$
$$(x-1)(x+3) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x + 3 = 0 \Rightarrow x = -3$$

55. $\frac{30 - x}{x} = x$

$$30 - x = x^2$$
$$0 = x^2 + x - 30$$
$$0 = (x+6)(x-5)$$
$$x + 6 = 0 \Rightarrow x = -6$$
$$x - 5 = 0 \Rightarrow x = 5$$

56.
$$\begin{aligned}
4x + 1 &= \frac{3}{x}\\
(x)4x + (x)1 &= (x)\frac{3}{x}\\
4x^2 + x &= 3\\
4x^2 + x - 3 &= 0\\
(4x - 3)(x + 1) &= 0\\
4x - 3 = 0 &\Rightarrow x = \frac{3}{4}\\
x + 1 = 0 &\Rightarrow x = -1
\end{aligned}$$

57.
$$\begin{aligned}
\frac{x}{x^2 - 4} + \frac{1}{x + 2} &= 3\\
(x + 2)(x - 2)\frac{x}{x^2 - 4} + (x + 2)(x - 2)\frac{1}{x + 2} &= 3(x + 2)(x - 2)\\
x + x - 2 &= 3x^2 - 12\\
3x^2 - 2x - 10 &= 0
\end{aligned}$$

$a = 3, b = -2, c = -10$

$$\begin{aligned}
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)}\\
&= \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3}
\end{aligned}$$

58.
$$\begin{aligned}
\frac{x + 1}{3} - \frac{x + 1}{x + 2} &= 0\\
3(x + 2)\frac{x + 1}{3} - 3(x + 2)\frac{x + 1}{x + 2} &= 0\\
(x + 2)(x + 1) - 3(x + 1) &= 0\\
x^2 + 3x + 2 - 3x - 3 &= 0\\
x^2 - 1 &= 0\\
(x + 1)(x - 1) &= 0\\
x + 1 = 0 &\Rightarrow x = -1\\
x - 1 = 0 &\Rightarrow x = 1
\end{aligned}$$

59. $|2x - 5| = 11$
$$\begin{aligned}
2x - 5 = 11 &\Rightarrow x = 8\\
-(2x - 5) = 11 &\Rightarrow x = -3
\end{aligned}$$

60. $|3x + 2| = 7$
$$\begin{aligned}
3x + 2 = 7 &\Rightarrow x = \tfrac{5}{3}\\
-(3x + 2) &= 7\\
-3x - 2 = 7 &\Rightarrow x = -3
\end{aligned}$$

61. $|x| = x^2 + x - 24$

First equation:
$$\begin{aligned}
x &= x^2 + x - 24\\
x^2 - 24 &= 0\\
x^2 &= 24\\
x &= \pm 2\sqrt{6}
\end{aligned}$$

Second equation:
$$\begin{aligned}
-x &= x^2 + x - 24\\
x^2 + 2x - 24 &= 0\\
(x + 6)(x - 4) &= 0\\
x + 6 = 0 &\Rightarrow x = -6\\
x - 4 = 0 &\Rightarrow x = 4
\end{aligned}$$

Only $x = 2\sqrt{6}$ and $x = -6$ are solutions of the original equation. $x = -2\sqrt{6}$ and $x = 4$ are extraneous.

62. $\left|x^2 + 6x\right| = 3x + 18$

First equation:

$$\begin{aligned} x^2 + 6x &= 3x + 18 \\ x^2 + 3x - 18 &= 0 \\ (x - 3)(x + 6) &= 0 \\ x - 3 = 0 &\Rightarrow x = 3 \\ x + 6 = 0 &\Rightarrow x = -6 \end{aligned}$$

Second equation:

$$\begin{aligned} -\left(x^2 + 6x\right) &= 3x + 18 \\ 0 &= x^2 + 9x + 18 \\ 0 &= (x + 3)(x + 6) \\ 0 &= x + 3 \Rightarrow x = -3 \\ x &= x + 6 \Rightarrow x = -6 \end{aligned}$$

The solutions of the original equation are $x = \pm 3$ and $x = -6$.

63. $\left|x + 1\right| = x^2 - 5$

First equation:

$$\begin{aligned} x + 1 &= x^2 - 5 \\ x^2 - x - 6 &= 0 \\ (x - 3)(x + 2) &= 0 \\ x - 3 = 0 &\Rightarrow x = 3 \\ x + 2 = 0 &\Rightarrow x = -2 \end{aligned}$$

Second equation:

$$\begin{aligned} -(x + 1) &= x^2 - 5 \\ -x - 1 &= x^2 - 5 \\ x^2 + x - 4 &= 0 \\ x &= \frac{-1 + \sqrt{17}}{2} \end{aligned}$$

Only $x = 3$ and $x = \dfrac{-1 - \sqrt{17}}{2}$ are solutions of the original equation. $x = -2$ and $x = \dfrac{-1 + \sqrt{17}}{2}$ are extraneous.

64. $\left|x - 5\right| = x^2 - 15x$

First equation:

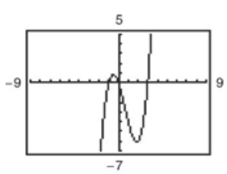
$$\begin{aligned} x - 15 &= x^2 - 15x \\ x^2 - 16x + 15 &= 0 \\ (x - 1)(x - 15) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x - 15 = 0 &\Rightarrow x = 15 \end{aligned}$$

Second equation:

$$\begin{aligned} -(x - 15) &= x^2 - 15x \\ x^2 - 14x - 15 &= 0 \\ (x + 1)(x - 15) &= 0 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 15 = 0 &\Rightarrow x = 15 \end{aligned}$$

Only $x = 15$ and $x = -1$ are solutions of the original equation. $x = 1$ is extraneous

65. (a)

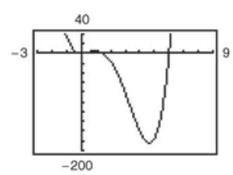


(b) x-intercepts: $(-1, 0), (0, 0), (3, 0)$

(c)
$$\begin{aligned} 0 &= x^3 - 2x^2 - 3x \\ 0 &= x(x + 1)(x - 3) \\ x &= 0 \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

66. (a)

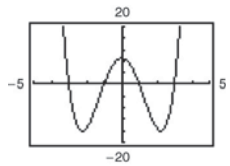


(b) x-intercepts: $(0, 0), \left(\frac{3}{2}, 0\right), (6, 0)$

(c)
$$\begin{aligned} y &= 2x^4 - 15x^3 + 18x^2 \\ 0 &= 2x^4 - 15x^3 + 18x^2 \\ &= x^2\left(2x^2 - 15x + 18\right) \\ &= x^2(2x - 3)(x - 6) \\ 0 &= x^2 \qquad \Rightarrow x = 0 \\ 0 &= 2x - 3 \Rightarrow x = \tfrac{3}{2} \\ 0 &= x - 6 \quad \Rightarrow x = 6 \end{aligned}$$

(d) The x-intercepts and the solutions are the same.

67. (a)

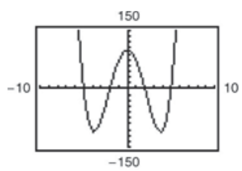


(b) x-intercepts: $(\pm 1, 0), (\pm 3, 0)$

(c)
$$\begin{aligned} 0 &= x^4 - 10x^2 + 9 \\ 0 &= (x^2 - 1)(x^2 - 9) \\ 0 &= (x + 1)(x - 1)(x + 3)(x - 3) \end{aligned}$$
$$\begin{aligned} x + 1 = 0 &\Rightarrow x = -1 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x + 3 = 0 &\Rightarrow x = -3 \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

68. (a)

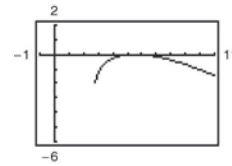


(b) x-intercepts: $(-2, 0), (2, 0), (-5, 0), (5, 0)$

(c)
$$\begin{aligned} y &= x^4 = 29x^2 + 100 \\ 0 &= x^4 - 29x^2 + 100 \\ &= (x^2 - 4)(x^2 - 25) \\ &= (x + 2)(x - 2)(x + 5)(x - 5) \\ 0 &= x + 2 \Rightarrow x = -2 \\ 0 &= x - 2 \Rightarrow x = 2 \\ 0 &= x + 5 \Rightarrow x = -5 \\ 0 &= x - 5 \Rightarrow x = 5 \\ x &= \pm 5, \ \pm 2 \end{aligned}$$

(d) The x-intercepts and the solutions are the same.

69. (a)

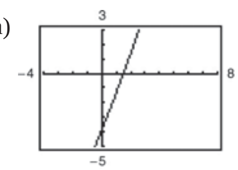


(b) x-intercepts: $(5, 0), (6, 0)$

(c)
$$\begin{aligned} 0 &= \sqrt{11x - 30} - x \\ x &= \sqrt{11x - 30} \\ x^2 &= 11x - 30 \\ x^2 - 11x + 30 &= 0 \\ (x - 5)(x - 6) &= 0 \\ x - 5 = 0 &\Rightarrow x = 5 \\ x - 6 = 0 &\Rightarrow x = 6 \end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

70. (a)

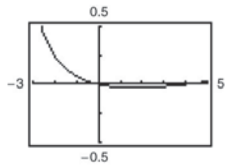


(b) x-intercept: $\left(\frac{3}{2}, 0\right)$

(c)
$$\begin{aligned} y &= 2x - \sqrt{15 - 4x} \\ 0 &= 2x - \sqrt{15 - 4x} \\ \sqrt{15 - 4x} &= 2x \\ 15 - 4x &= 4x^2 \\ 0 &= 4x^2 + 4x - 15 \\ 0 &= (2x + 5)(2x - 3) \\ 0 &= 2x + 5 \Rightarrow x = -\tfrac{5}{2}, \text{ extraneous} \\ 0 &= 2x - 3 \Rightarrow x = \tfrac{3}{2} \\ x &= \tfrac{3}{2} \end{aligned}$$

(d) The x-intercept and the solution are the same.

71. (a)

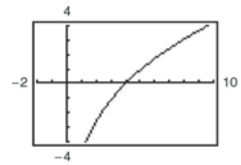


(b) x-intercepts: $(0, 0)$, $(4, 0)$

(c)
$$\begin{aligned}
0 &= \sqrt{7x+26} - \sqrt{5x+16} - 2 \\
-\sqrt{7x+36} &= -\sqrt{5x+16} - 2 \\
\sqrt{7x+36} &= 2 + \sqrt{5x+16} \\
\left(\sqrt{7x+36}\right)^2 &= \left(2+\sqrt{5x+16}\right)^2 \\
7x + 36 &= 4 + 4\sqrt{5x+16} + 5x + 16 \\
7x + 36 &= 5x + 20 + 4\sqrt{5x+16} \\
2x + 16 &= 4\sqrt{5x+16} \\
x + 8 &= 2\sqrt{5x+16} \\
(x+8)^2 &= \left(2\sqrt{5x+16}\right)^2 \\
x^2 + 16x + 64 &= 4(5x+16) \\
x^2 + 16x + 64 &= 20x + 64 \\
x^2 - 4x &= 0 \\
x(x-4) &= 0 \\
x &= 0 \\
x - 4 &= 0 \Rightarrow x = 4
\end{aligned}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

72. (a)

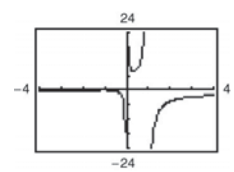


(b) x-intercept: $(4, 0)$

(c)
$$\begin{aligned}
y &= 3\sqrt{x} - \frac{4}{\sqrt{x}} - 4 \\
0 &= 3\sqrt{x} - \frac{4}{\sqrt{x}} - 4 \\
0 &= \sqrt{x}\left(3\sqrt{x} - \frac{4}{\sqrt{x}} - 4\right) \\
0 &= 3x - 4 - 4\sqrt{x} \\
0 &= \left(3\sqrt{x}+2\right)\left(\sqrt{x}-2\right) \\
0 &= 3\sqrt{x} + 2 \Rightarrow x = \frac{4}{9}, \text{ extraneous} \\
0 &= \sqrt{x} - 2 \Rightarrow x = 4 \\
x &= 4
\end{aligned}$$

(d) The x-intercept and the solution are the same.

73. (a)

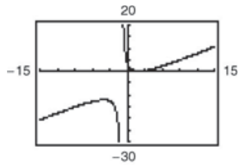


(b) x-intercept: $(-1, 0)$

(c)
$$\begin{aligned}
0 &= \frac{1}{x} - \frac{4}{x-1} - 1 \\
0 &= (x-1) - 4x - x(x-1) \\
0 &= x - 1 - 4x - x^2 + x \\
0 &= -x^2 - 2x - 1 \\
0 &= x^2 + 2x + 1 \\
0 &= (x+1)^2 \\
x + 1 = 0 &\Rightarrow x = -1
\end{aligned}$$

(d) The x-intercept of the graph is the same as the solution of the equation.

74. (a)

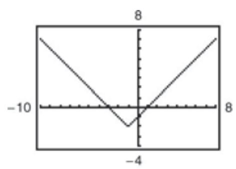


(b) x-intercept: $(2, 0)$

(c)
$$\begin{aligned}
0 &= x + \frac{9}{x+1} - 5 \\
0 &= x + \frac{9}{x+1} - 5 \\
0 &= x(x+1) + (x+1)\frac{9}{x+1} - 5(x+1) \\
0 &= x^2 + x + 9 - 5x - 5 \\
0 &= x^2 - 4x + 4 \\
0 &= (x-2)(x-2) \\
0 &= x - 2 \Rightarrow x = 2 \\
x &= 2
\end{aligned}$$

(d) The x-intercept and the solution are the same.

75. (a)

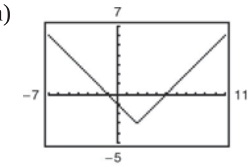


(b) x-intercepts: $(1, 0), (-3, 0)$

(c)
$$\begin{aligned}
0 &= |x+1| - 2 \\
2 &= |x+1|
\end{aligned}$$
$$\begin{array}{lll}
x + 1 = 2 & \text{or} & -(x+1) = 2 \\
x = 1 & \text{or} & -x - 1 = 2 \\
 & & -x = 3 \\
 & & x = -3
\end{array}$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

76. (a)



(b) x-intercepts: $(5, 0), (-1, 0)$

(c)
$$\begin{aligned}
0 &= |x-2| - 3 \\
3 &= |x-2|
\end{aligned}$$
$$x - 2 = 3 \Rightarrow x = 5 \text{ or } -(x-2) = 3$$
$$-x + 2 = 3 \Rightarrow x = -1$$
$$x = 5, -1$$

(d) The x-intercepts and the solutions are the same.

77. $3.2x^4 - 1.5x^2 - 2.1 = 0$

$$x^2 = \frac{1.5 \pm \sqrt{1.5^2 - 4(3.2)(-2.1)}}{2(3.2)}$$

Using the positive value for x^2, we have

$$x = \pm\sqrt{\frac{1.5 + \sqrt{29.13}}{6.4}} \approx \pm 1.038.$$

78. $0.1x^4 - 2.4x^2 - 3.6 = 0$

$$x^2 = \frac{2.4 \pm \sqrt{(-2.4)^2 - 4(0.1)(-3.6)}}{2(0.1)} = \frac{2.4 \pm 7.2}{0.2}$$

Using the positive values for x^2,

$$x = \pm\sqrt{\frac{2.4 + \sqrt{7.2}}{0.2}} \approx \pm 5.041.$$

79. $7.08x^6 + 4.15x^3 - 9.6 = 0$

$a = 7.8, b = 4.15, c = -9.6$

$$\begin{aligned}
x^3 &= \frac{-4.15 \pm \sqrt{(4.15)^2 - 4(7.08)(-9.6)}}{2(7.08)} \\
&= \frac{-4.15 \pm \sqrt{2.89.0945}}{14.16}
\end{aligned}$$

$$x = \sqrt[3]{\frac{-4.15 + \sqrt{289.0945}}{14.16}} \approx 0.968$$

$$x = \sqrt[3]{\frac{-4.15 - \sqrt{289.0945}}{14.16}} \approx -1.143$$

80. $5.25x^6 - 0.2x^3 - 1.55 = 0$

$$x^3 = \frac{0.2 \pm \sqrt{(-0.2)^2 - 4(5.25)(-1.55)}}{2(5.25)}$$

$$= \frac{0.2 \pm \sqrt{32.59}}{10.5}$$

$$x = \sqrt[3]{\frac{0.2 + \sqrt{32.59}}{10.5}} \approx 0.826$$

$$x = \sqrt[3]{\frac{0.2 - \sqrt{32.59}}{10.5}} \approx -0.807$$

81. $1.8x - 6\sqrt{x} - 5.6 = 0$ Given equation

$1.8(\sqrt{x})^2 - 6\sqrt{x} - 5.6 = 0$

Use the Quadratic Formula with $a = 1.8$, $b = -6$, and $c = -5.6$.

$$\sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} \approx \frac{6 \pm 8.7361}{3.6}$$

Considering only the positive value for $\sqrt{x}$, we have:

$\sqrt{x} \approx 4.0934$

$x \approx 16.756.$

82. $2.4x - 12.4\sqrt{x} + 0.28 = 0$

$$\sqrt{x} = \frac{12.4 \pm \sqrt{(-12.4)^2 - 4(2.4)(0.28)}}{2(2.4)}$$

$$= \frac{12.4 \pm \sqrt{151.072}}{4.8}$$

Using the positive values for $\sqrt{x}$,

$$x = \left(\frac{12.4 + \sqrt{151.072}}{4.8}\right)^2 \approx 26.461$$

$$x = \left(\frac{12.4 - \sqrt{151.072}}{4.8}\right)^2 \approx 0.001.$$

83. $4x^{2/3} + 8x^{1/3} + 3.6 = 0$

$a = 4, b = 8, c = 3.6$

$$x^{1/3} = \frac{-8 \pm \sqrt{8^2 - 4(4)(3.6)}}{2(4)}$$

$$x = \left[\frac{-8 + \sqrt{6.4}}{8}\right]^3 \approx -0.320$$

$$x = \left[\frac{-8 - \sqrt{6.4}}{8}\right]^3 \approx -2.280$$

84. $8.4x^{2/3} - 1.2x^{1/3} - 24 = 0$

$$x^{1/3} = \frac{1.2 \pm \sqrt{(-1.2)^2 - 4(8.4)(-24)}}{2(8.4)}$$

$$= \frac{1.2 \pm \sqrt{807.84}}{16.8}$$

$$x = \left(\frac{1.2 + \sqrt{807.84}}{16.8}\right)^3 \approx 5.482$$

$$x = \left(\frac{1.2 - \sqrt{807.84}}{16.8}\right)^3 \approx -4.255$$

85. $-4, 7$

Sample answer: $(x - (-4))(x - 7) = 0$

$(x + 4)(x - 7) = 0$

$x^2 - 3x - 28 = 0$

86. $0, 2, 9$

Sample answer: $(x - 0)(x - 2)(x - 9) = 0$

$x(x - 2)(x - 9) = 0$

$x(x^2 - 11x + 18) = 0$

$x^3 - 11x^2 + 18x = 0$

87. $-\frac{7}{3}, \frac{6}{7}$

One possible equation is:

$x = -\frac{7}{3} \Rightarrow 3x = -7 \Rightarrow 3x + 7$ is a factor.

$x = \frac{6}{7} \Rightarrow 7x = 6 \Rightarrow 7x - 6$ is a factor.

$(3x + 7)(7x - 6) = 0$

$21x^2 + 31x - 42 = 0$

Any non-zero multiple of this equation would also have these solutions.

88. $-\frac{1}{8}, -\frac{4}{5}$

$\left(x - \left(-\frac{1}{8}\right)\right)\left(x - \left(-\frac{4}{5}\right)\right) = 0$

$\left(x + \frac{1}{8}\right)\left(x + \frac{4}{5}\right) = 0$

$x^2 + \frac{4}{5}x + \frac{1}{8}x + \frac{4}{40} = 0$

$40x^2 + 32x + 5x + 4 = 0$

$40x^2 + 37x + 4 = 0$

Any non-zero multiple of this equation would also have these solutions.

89. $\sqrt{3}, -\sqrt{3}$, and 4

One possible equation is:

$$\begin{aligned}\left(x - \sqrt{3}\right)\left(x - \left(-\sqrt{3}\right)\right)(x - 4) &= 0\\ \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x - 4) &= 0\\ \left(x^2 - 3\right)(x - 4) &= 0\\ x^3 - 4x^2 - 3x + 12 &= 0\end{aligned}$$

Any non-zero multiple of this equation would also have these solutions.

90. $2\sqrt{7}, -\sqrt{7}$

$$\begin{aligned}\left(x - 2\sqrt{7}\right)\left(x + \sqrt{7}\right) &= 0\\ x^2 + x\sqrt{7} - 2x\sqrt{7} - 2(7) &= 0\\ x^2 - x\sqrt{7} - 14 &= 0\end{aligned}$$

Any non-zero multiple of this equation would also have these solutions.

91. $i, -i$

Sample answer: $(x - i)(x - (-i)) = 0$

$$\begin{aligned}(x - i)(x + i) &= 0\\ x^2 - i^2 &= 0\\ x^2 + 1 &= 0\end{aligned}$$

92. $2i, -2i$

Sample answer: $(x - 2i)(x - (-2i)) = 0$

$$\begin{aligned}(x - 2i)(x + 2i) &= 0\\ x^2 - 4i^2 &= 0\\ x^2 + 4 &= 0\end{aligned}$$

93. $-1, 1, i$, and $-i$

One possible equation is:

$$\begin{aligned}(x - (-1))(x - 1)(x - i)(x - (-i)) &= 0\\ (x + 1)(x - 1)(x - i)(x + i) &= 0\\ \left(x^2 - 1\right)\left(x^2 + 1\right) &= 0\\ x^4 - 1 &= 0\end{aligned}$$

Any non-zero multiple of this equation would also have these solutions.

94. $4i, -4i, 6, -6$

Sample answer: $(x - 4i)(x + 4i)(x - 6)(x + 6) = 0$

$$\begin{aligned}\left(x^2 + 16\right)\left(x^2 - 36\right) &= 0\\ x^4 - 20x^2 - 576 &= 0\end{aligned}$$

95. Let $x =$ the number of students in the original group. Then, $\dfrac{1700}{x} =$ the original cost per student.

When six more students join the group, the cost per student becomes $\dfrac{1700}{x} - 7.50$.

Model: (Cost per student) $\cdot$ (Number of students) $=$ (Total cost)

$$\begin{aligned}\left(\frac{1700}{x} - 7.5\right)(x + 6) &= 1700\\ (3400 - 15x)(x + 6) &= 3400x \quad \text{Multiply both sides by } 2x \text{ to clear fraction.}\\ -15x^2 - 90x + 20{,}400 &= 0\end{aligned}$$

$$x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20{,}400)}}{2(-15)} = \frac{90 \pm 1110}{-30}$$

Using the positive value for x we conclude that the original number was $x = 34$ students.

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96. *Model:* $\begin{pmatrix}\text{Cost per}\\ \text{student}\end{pmatrix}\cdot\begin{pmatrix}\text{Number of}\\ \text{students}\end{pmatrix} = \begin{pmatrix}\text{Monthly}\\ \text{rent}\end{pmatrix}$

Labels: Monthly rent $= x$

Number of students $= 4$

Original cost per student $= \dfrac{x}{3}$

Cost per student $= \dfrac{x}{3} - 75$

Equation:
$$\left(\frac{x}{3} - 75\right)(4) = x$$
$$\frac{4x}{3} - 300 = x$$
$$\frac{4x}{3} - x = 300$$
$$\frac{x}{3} = 300$$
$$x = 900$$

The monthly rent is \$900.

97. *Model:* $\text{Time} = \dfrac{\text{Distance}}{\text{Rate}}$

Labels: Let $x =$ average speed of the plane. Then we have a travel time of $t = 145/x$. If the average speed is increased by 40 mph, then

$$t - \frac{12}{60} = \frac{145}{x + 40}$$
$$t = \frac{145}{x + 40} + \frac{1}{5}.$$

Now, we equate these two equations and solve for x.

Equation:
$$\frac{145}{x} = \frac{145}{x + 40} + \frac{1}{5}$$
$$145(5)(x + 40) = 145(5)x + x(x + 40)$$
$$725x + 29{,}000 = 725x + x^2 + 40x$$
$$0 = x^2 + 40x - 29{,}000$$

Using the positive value for x found by the Quadratic Formula, we have $x \approx 151.5$ mph and $x + 40 = 191.5$ mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.

98. *Model:* $(\text{Rate})\cdot(\text{time}) = (\text{distance})$

Labels: Distance $= 1080$

Original time $= t$

Original rate $= \dfrac{1080}{t}$

Return time $= t + 2.5$

Return rate $= \dfrac{1080}{t} - 6$

Equation:
$$\left(\frac{1080}{t} - 6\right)(t + 2.5) = 1080$$
$$1080 + \frac{2700}{t} - 6t - 15 = 1080$$
$$\frac{2700}{t} - 6t - 15 = 0$$
$$270 - 0 - 6t^2 - 15t = 0$$
$$2t^2 + 5t - 900 = 0$$
$$(2t + 45)(t - 20) = 0$$
$$2t + 45 = 0 \Rightarrow t = -22.5$$
$$t - 20 = 0 \Rightarrow t = 20$$

The original rate was $\frac{1080}{20} = 54$ miles per hour.

99.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$3052.49 = 2500\left(1 + \frac{r}{12}\right)^{(12)(5)}$$
$$1.220996 = \left(1 + \frac{r}{12}\right)^{60}$$
$$(1.220996)^{1/60} = 1 + \frac{r}{12}$$
$$\left[(1.220996)^{1/60} - 1\right](12) = r$$
$$r \approx 0.04 = 4\%$$

100.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$4042.05 = 3000\left(1 + \frac{r}{4}\right)^{(4)(6)}$$
$$1.34735 = \left(1 + \frac{r}{4}\right)^{24}$$
$$(1.34735)^{1/24} = \left[\left(1 + \frac{r}{4}\right)^{24}\right]^{1/24}$$
$$1.0125 = 1 + \frac{r}{4}$$
$$0.0125 = \frac{r}{4}$$
$$4(0.0125) = r$$
$$0.05 = r$$

So, the annual interest rate is 5%.

101. When $C = 2.5$ we have:
$$2.5 = \sqrt{0.2x + 1}$$
$$6.25 = 0.2x + 1$$
$$5.25 = 0.2x$$
$$x = 26.25 = 26{,}250 \text{ passengers}$$

102. (a)
$$D = \sqrt{664{,}267.0 + 31{,}003.65t}$$
$$925 = \sqrt{664{,}267.0 + 31{,}003.65t}$$
$$(925)^2 = \left(\sqrt{664{,}267.0 + 31{,}003.65t}\right)^2$$
$$855{,}625 = 664{,}267.0 + 31{,}003.65t$$
$$855{,}625 = 31{,}003.65t$$
$$6.2 \approx t$$

So, the number of doctors reached 925,000 in the year 2006.

(b)
$$D = \sqrt{664{,}267.0 + 31{,}003.65t}$$
$$1{,}100 = \sqrt{664{,}267.0 + 31{,}003.65t}$$
$$(1{,}100)^2 = \left(\sqrt{664{,}267.0 + 31{,}003.65t}\right)^2$$
$$1{,}210{,}000 = 664{,}267.0 + 31{,}003.65t$$
$$545{,}733 = 31{,}003.65t$$
$$17.6 \approx t$$

Using the model, the number of doctors will reach 1,100,000 in the year 2017. Answers will vary. *Sample answer:* Yes, it is reasonable that the number of medical doctors continues to increase at a rate in which the number reaches 1,100,000 in the year 2017.

103. $T = 75.82 - 2.11x + 43.51\sqrt{x}, 5 \le x \le 40$

(a)
$$212 = 75.82 - 2.11x + 43.51\sqrt{x}$$
$$0 = -2.11x + 43.51\sqrt{x} - 136.18$$

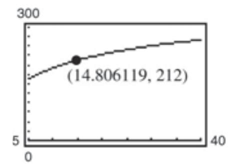
By the Quadratic Formula, we have

$\sqrt{x} \approx 16.77928 \Rightarrow x \approx 281.333$

$\sqrt{x} \approx 3.84787 \Rightarrow x \approx 14.806.$

Since x is restricted to $5 \le x \le 40$, let $x = 14.806$ pounds per square inch.

(b)



104. (a)
$$P = \frac{181.34 + 0.788t}{1 - 0.07t}$$
$$\frac{181.34 + 0.788t}{1 - 0.007t} = 210$$
$$181.34 + 0.788t = 210(1 - 0.007t)$$
$$181.34 + 0.788t = 210 - 1.47t$$
$$2.258t = 28.66$$
$$t = \frac{28.66}{2.258}$$
$$t \approx 12.69 \rightarrow \text{during 2002}$$

(b) To find the year when the total voting age population is expected to reach 280 million, let $P = 280$ and solve for t.
$$P = \frac{181.34 + 0.788t}{1 - 0.07t}$$
$$\frac{181.34 + 0.788t}{1 - 0.007t} = 280$$
$$181.34 + 0.788t = 280(1 - 0.007t)$$
$$181.34 + 0.788t = 280 - 1.96t$$
$$2.748t = 98.66$$
$$t = \frac{98.66}{2.748}$$
$$t \approx 35.9 \rightarrow \text{during 2025}$$

This prediction is reasonable if population of the United States in general continues to increase at a similar rate as it had during the years 1990 through 2010. You could assume that the voting age population would increase at a rate similar to that for which the model was created.

105.
$$\begin{aligned}
37.55 &= 40 - \sqrt{0.01x + 1}\\
\sqrt{0.01x + 1} &= 2.45\\
0.01x + 1 &= 6.0025\\
0.01x &= 5.0025\\
x &= 500.25
\end{aligned}$$

Rounding x to the nearest whole unit yields $x \approx 500$ units.

106. *Verbal Model:* Total cost = Cost underwater · Distance underwater + Cost overland · Distance overland

Labels: Total cost : \$1,098,662.40

Cost overland: \$24 per foot

Distance overland in feet: $5280(8 - x)$

Cost underwater: \$30 per foot

Distance underwater in feet: $5280\sqrt{x^2 + (3/4)^2} = 5280\sqrt{\dfrac{16x^2 + 9}{16}} = 1320\sqrt{16x^2 + 9}$

Equation:
$$\begin{aligned}
1{,}098{,}662.40 &= 30\left(1320\sqrt{16x^2 + 9}\right) + 24\left[5280(8 - x)\right]\\
1{,}098{,}662.40 &= 39{,}600\sqrt{16x^2 + 9} + 126{,}720(8 - x)\\
1{,}098{,}662.40 &= 7920\left[5\sqrt{16x^2 + 9} + 16(8 - x)\right]\\
138.72 &= 5\sqrt{16x^2 + 9} + 16(8 - x)\\
138.72 &= 5\sqrt{16x^2 + 9} + 128 - 16x\\
16x + 10.72 &= 5\sqrt{16x^2 + 9}\\
(16x + 10.72)^2 &= \left(5\sqrt{16x^2 + 9}\right)^2\\
256x^2 + 343.04x + 114.9184 &= 25\left(16x^2 + 9\right)\\
256x^2 + 343.04x + 114.9184 &= 400x^2 + 225\\
0 &= 144x^2 - 343.04x + 110.0816
\end{aligned}$$

By the Quadratic Formula, $x \approx 2$ or $x \approx 0.382$.

So, the length of x could either be 0.382 mile or 2 miles.

107.
$$\begin{aligned}
\frac{1}{t} + \frac{1}{t + 3} &= \frac{1}{y}\\
\frac{1}{t} + \frac{1}{t + 3} &= \frac{1}{2}\\
2t(t + 3)\frac{1}{t} + 2t(t + 3)\frac{1}{t + 3} &= 2t(t + 3)\frac{1}{2}\\
2(t + 3) + 2t &= t(t + 3)\\
2t + 6 + 2t &= t^2 + 3t\\
0 &= t^2 - t - 6\\
0 &= (t - 3)(t + 2)\\
t - 3 = 0 &\Rightarrow t = 3\\
t + 2 = 0 &\Rightarrow t = -2
\end{aligned}$$

Since t represents time, $t = 3$ is the only solution. It takes 3 hours for you working alone to tile the floor.

108.

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{y}2$$

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{3}$$

$$3t(t+2)\left(\frac{1}{t}\right) + 3t(t+2)\left(\frac{1}{t+2}\right) = 3t(t+2)\left(\frac{1}{3}\right)$$

$$3(t+2) + 3t = t(t+2)$$

$$3t + 6 + 3t = t^2 + 2t$$

$$0 = t^2 - 4t - 6$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$$= \frac{2\left(2 \pm \sqrt{10}\right)}{2}$$

$$= 2 \pm \sqrt{10}$$

$$t \approx 5.2 \text{ or } t \approx -1.2$$

Since t represents time, $t \approx 5.2$ is the only solution. It takes approximately 5.2 hours for you working alone to paint the fence.

109.

$$v = \sqrt{\frac{gR}{\mu s}}$$

$$v^2 = \frac{gR}{\mu s}$$

$$v^2\mu s = gR$$

$$\frac{v^2\mu s}{R} = g$$

110.

$$i = \pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q}$$

$$i^2 = \left(\pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q}\right)^2$$

$$i^2 = \frac{1}{LC}\left(Q^2 - q\right)$$

$$LCi^2 = Q^2 - q$$

$$LCi^2 + q = Q^2$$

$$\pm\sqrt{LCi^2 + q} = Q$$

111. False—See Example 7 on page 125.

112. True. There is no value to satisfy this equation.

$$\sqrt{x+10} - \sqrt{x-10} = 0$$

$$\sqrt{x+10} = \sqrt{x-10}$$

$$x + 10 = x - 10$$

$$10 \neq -10$$

113. The distance between $(1, 2)$ and $(x, -10)$ is 13.

$$\sqrt{(x-1)^2 + (-10-2)^2} = 13$$

$$(x-1)^2 + (-12)^2 = 13^2$$

$$x^2 - 2x + 1 + 144 = 169$$

$$x^2 - 2x - 24 = 0$$

$$(x+4)(x-6) = 0$$

$$x + 4 = 0 \Rightarrow x = -4$$

$$x - 6 = 0 \Rightarrow x = 6$$

Both $(-4, -10)$ and $(6, -10)$ are a distance of 13 from $(1, 2)$.

114. The distance between $(-8, 0)$ and $(x, 5)$ is 13.

$$\begin{aligned}
\sqrt{(x+8)^2 + (5-0)^2} &= 13 \\
(x+8)^2 + 5^2 &= 13^2 \\
x^2 + 16x + 64 + 25 &= 169 \\
x^2 + 16x - 80 &= 0 \\
(x+20)(x-4) &= 0 \\
x + 20 = 0 &\Rightarrow x = -20 \\
x - 4 = 0 &\Rightarrow x = 4
\end{aligned}$$

Both $(-20, 5)$ and $(4, 5)$ are a distance of 13 from $(-8, 0)$.

115. The distance between $(0, 0)$ and $(8, y)$ is 17.

$$\begin{aligned}
(8-0)^2 + (y-0)^2 &= 17 \\
(8)^2 + (y)^2 &= 17^2 \\
64 + y^2 &= 289 \\
y^2 &= 225 \\
y &= \pm\sqrt{225} \\
&= \pm 15
\end{aligned}$$

Both $(8, 15)$ and $(8, -15)$ are a distance of 17 from $(0, 0)$.

116. The distance between $(-8, 4)$ and $(7, y)$ is 17.

$$\begin{aligned}
\sqrt{(7-(-8))^2 + (y-4)^2} &= 17 \\
(15)^2 + (y-4)^2 &= 17^2 \\
225 + (y-4)^2 &= 289 \\
(y-4)^2 &= 64 \\
y - 4 &= \pm 8 \\
y &= 4 \pm 8 = -4, 12
\end{aligned}$$

Both $(7, -4)$ and $(7, 12)$ are a distance of 17 from $(-8, 4)$.

117. $9 + |9 - a| = b$

$$|9 - a| = b - 9$$

$$\begin{aligned}
9 - a &= b - 9 & \text{or} \quad 9 - a &= -(b - 9) \\
-a &= b - 18 & 9 - a &= -b + 9 \\
a &= 18 - b & -a &= -b \\
& & a &= b
\end{aligned}$$

Thus, $a = 18 - b$ *or* $a = b$. From the original equation we know that $b \geq 9$.

Some possibilities are: $b = 9$, $a = 9$
$b = 10$, $a = 8$ *or* $a = 10$
$b = 11$, $a = 7$ *or* $a = 11$
$b = 12$, $a = 6$ *or* $a = 12$
$b = 13$, $a = 5$ *or* $a = 13$
$b = 14$, $a = 4$ *or* $a = 14$

118. Isolate the absolute value by subtracting x from both sides of the equations. The expression inside the absolute value signs can be positive or negative, so two separate equations must be solved. Each solution must be checked since extraneous solutions may be included.

119.

$$\begin{aligned}
20 + \sqrt{20 - 1} &= b \\
\sqrt{20 - a} &= b - 20 \\
20 - a &= b^2 - 40b + 400 \\
-a &= b^2 - 40b + 380 \\
a &= -b^2 + 40b - 380
\end{aligned}$$

This formula gives the relationship between a and b. From the original equation we know that $a \leq 20$ and $b \geq 20$. Choose a b value, where $b \geq 20$ and then solve for a, keeping in mind that $a \leq 20$.

Some possibilities are: $b = 20$, $a = 20$
$b = 21$, $a = 19$
$b = 22$, $a = 16$
$b = 23$, $a = 11$
$b = 24$, $a = 4$
$b = 25$, $a = -5$

120. First isolate the radical by subtracting x from both sides of the equation. Then, square both sides and solve the resulting equation using the Quadratic Formula. Each solution must be checked since extraneous solutions may be included.

121. The quadratic equation was not written in general form before the values of a, b, and c were substituted in the Quadratic Formula.

122. (a) The formula for volume of the glass cube is V = Length × Width × Height.
The volume of water in the cube is the length × width × height of the water.
So, the volume is $x \cdot x \cdot (x - 3) = x^2(x - 3)$.

(b) Given the equation $x^2(x - 3) = 320$. The dimensions of the glass cube can be found by solving for x.
So, the capacity of the cube is equal to $V = x^3$.

Section 1.7 Linear Inequalities in One Variable

1. solution set

2. graph

3. double

4. union

5. Interval: $[0, 9)$

(a) Inequality: $0 \le x \le 9$

(b) The interval is bounded.

6. Interval: $(-7, 4)$

(a) Inequality: $-7 \le x \le 4$

(b) The interval is bounded.

7. Interval: $[-1, 5]$

(a) Inequality: $-1 \le x \le 5$

(b) The interval is bounded.

8. Interval: $(2, 10]$

(a) Inequality: $2 < x \le 10$

(b) The interval is bounded.

9. Interval: $(11, \infty)$

(a) Inequality: $x > 11$

(b) The interval is unbounded.

10. Interval: $[-5, \infty)$

(a) Inequality: $-5 \le x < \infty$ or $x \ge -5$

(b) The interval is unbounded.

11. Interval: $(-\infty, -2)$

(a) Inequality: $x < -2$

(b) The interval is unbounded.

12. Interval: $(-\infty, 7]$

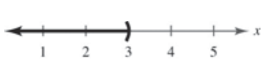
(a) Inequality: $-\infty < x \le 7$ or $x \le -7$

(b) The interval is unbounded.

13. $4x < 12$

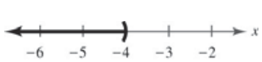
$\frac{1}{4}(4x) < \frac{1}{4}(12)$

$x < 3$



14. $10x < -40$

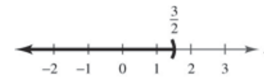
$x < -4$



15. $-2x > -3$

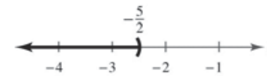
$-\frac{1}{2}(-2x) < \left(-\frac{1}{2}\right)(-3)$

$x < \frac{3}{2}$



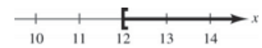
16 $-6x > 15$

$x < -\frac{15}{6}$ or $x < -\frac{5}{2}$



17. $x - 5 \ge 7$

$x \ge 12$



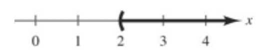
18. $x + 7 \le 12$

$x \le 5$

19. $2x + 7 < 3 + 4x$

$-2x < -4$

$x > 2$



20. $3x + 1 \ge 2 + x$

$2x \ge 1$

$x \ge \frac{1}{2}$

21. $2x - 1 \ge 1 - 5x$

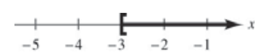
$7x \ge 2$

$x \ge \frac{2}{7}$

22. $6x - 4 \le 2 + 8x$

$-2x \le 6$

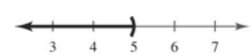
$x \ge -3$



23. $4 - 2x < 3(3 - x)$

$4 - 2x < 9 - 3x$

$x < 5$

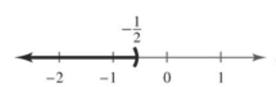


24. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

$x < -\frac{1}{2}$



25. $\frac{3}{4}x - 6 \le x - 7$

$-\frac{1}{4}x \le -1$

$x \ge 4$

26. $3 + \frac{2}{7}x > x - 2$

$21 + 2x > 7x - 14$

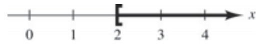
$-5x > -35$

$x < 7$

27. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$

$4x + \frac{1}{2} \geq 3x + \frac{5}{2}$

$x \geq 2$

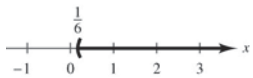


28. $9x - 1 < \frac{3}{4}(16x - 2)$

$36x - 4 < 48x - 6$

$-12x < -2$

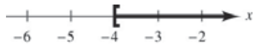
$x > \frac{1}{6}$



29. $3.6x + 11 \geq -3.4$

$3.6x \geq 14.4$

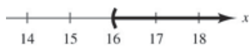
$x \geq -4$



30. $15.6 - 1.3x < -5.2$

$-1.3x < -20.8$

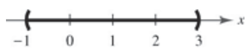
$x > 16$



31. $1 < 2x + 3 < 9$

$-2 < 2x < 6$

$-1 < x < 3$

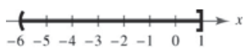


32. $-9 \leq -2x - 7 < 5$

$-2 \leq -2x < 12$

$1 \geq x > -6$

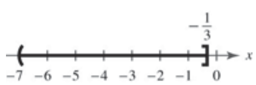
$-6 < x \leq 1$



33. $0 < 3(x + 7) \leq 20$

$0 < x + 7 \leq \dfrac{20}{3}$

$-7 < x \leq -\dfrac{1}{3}$

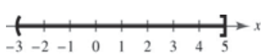


34. $-1 \leq -(x - 4) < 7$

$1 \geq x - 4 > -7$

$5 \geq x > -3$

$-3 < x \leq 5$

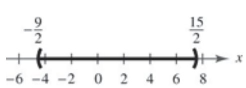


35. $-4 < \dfrac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

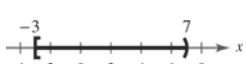
$-\dfrac{9}{2} < x < \dfrac{15}{2}$



36. $0 \leq \dfrac{x + 3}{2} < 5$

$0 \leq x + 3 < 10$

$-3 \leq x < 7$



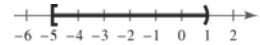
37. $-1 < \dfrac{-x - 2}{3} \leq 1$

$-3 < -x - 2 \leq 3$

$-1 < -x \leq 5$

$1 > x \geq -5$

$-5 \leq x < 1$



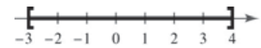
38. $-1 \leq \dfrac{-3x + 5}{7} \leq 2$

$-7 \leq -3x + 5 \leq 14$

$-12 \leq -3x \leq 9$

$4 \geq x \geq -3$

$-3 \leq x \leq 4$



39. $\frac{3}{4} > x + 1 > \frac{1}{4}$

$-\frac{1}{4} > x > -\frac{3}{4}$

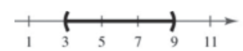
$-\frac{3}{4} < x < -\frac{1}{4}$

40. $-1 < 2 - \dfrac{x}{3} < 1$

$-3 < 6 - x < 3$

$-9 < -x < -3$

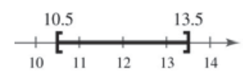
$3 < x < 9$



41. $3.2 \leq 0.4x - 1 \leq 4.4$

$4.2 \leq 0.4x \leq 5.4$

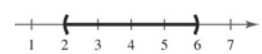
$10.5 \leq x \leq 13.5$



42. $1.6 < 0.3x + 1 < 2.8$

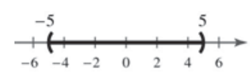
$0.6 < 0.3x < 1.8$

$2 < x < 6$



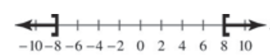
43. $|x| < 5$

$-5 < x < 5$



44. $|x| \geq 8$

$x \geq 8$ or $x \leq -8$



45. $\left|\dfrac{x}{2}\right| > 1$

$\dfrac{x}{2} < -1$ or $\dfrac{x}{2} > 1$

$x < -2 \qquad x > 2$

46. $\left|\dfrac{x}{5}\right| > 3$

$\dfrac{x}{5} < -3$ or $\dfrac{x}{5} > 3$

$x < -15 \qquad x > 15$

47. $|x - 5| < -1$

No solution. The absolute value of a number cannot be less than a negative number.

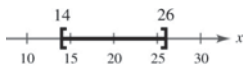
48. $|x - 7| < -5$

No solution. The absolute value of a number cannot be less than a negative number.

49. $|x - 20| \le 6$

$-6 \le x - 20 \le 6$

$14 \le x \le 26$



50. $|x - 8| \ge 0$

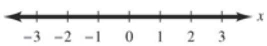
$x - 8 \ge 0$ or $-(x - 8) \ge 0$

$x \ge 8$ $\quad$ $-x + 8 \ge 0$

$-x \ge -8$

$x \le 8$

All real numbers x



51. $|3 - 4x| \ge 9$

$3 - 4x \le -9$ or $3 - 4x \ge 9$

$-4x \le -12$ $\quad$ $-4x \ge 6$

$x \ge 3$ $\quad$ $x \le -\frac{3}{2}$

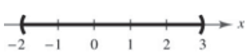
52. $|1 - 2x| < 5$

$-5 < 1 - 2x < 5$

$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$

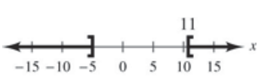


53. $\left|\dfrac{x - 3}{2}\right| \ge 4$

$\dfrac{x - 3}{2} \le -4$ or $\dfrac{x - 3}{2} \ge 4$

$x - 3 \le -8$ $\quad$ $x - 3 \ge 8$

$x \le -5$ $\quad$ $x \ge 11$



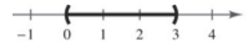
54. $\left|1 - \dfrac{2x}{3}\right| < 1$

$-1 < 1 - \dfrac{2x}{3} < 1$

$-2 < -\dfrac{2x}{3} < 0$

$3 > x > 0$

$0 < x < 3$



55. $|9 - 2x| - 2 < -1$

$|9 - 2x| < 1$

$-1 < 9 - 2x < 1$

$-10 < -2x < -8$

$5 > x > 4$

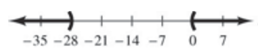
$4 < x < 5$

56. $|x + 14| + 3 > 17$

$|x + 14| > 14$

$x + 14 < -14$ or $x + 14 > 14$

$x < -28$ $\quad$ $x > 0$

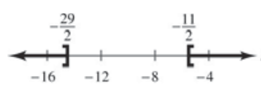


57. $2|x + 10| \ge 9$

$|x + 10| \ge \frac{9}{2}$

$x + 10 \le -\frac{9}{2}$ or $x + 10 \ge \frac{9}{2}$

$x \le -\frac{29}{2}$ $\quad$ $x \ge -\frac{11}{2}$



58. $3|4 - 5x| \le 9$

$|4 - 5x| \le 3$

$-3 \le 4 - 5x \le 3$

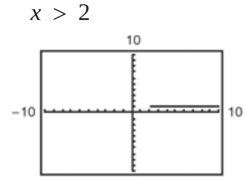
$-7 \le -5x \le -1$

$\frac{7}{5} \ge x \ge \frac{1}{5}$

$\frac{1}{5} \le x \le \frac{7}{5}$

59. $6x > 12$

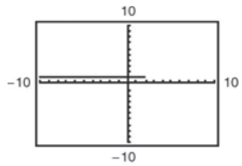
$x > 2$



60. $3x - 1 \le 5$

$3x \le 6$

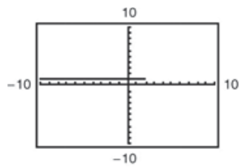
$x \le 2$



61. $5 - 2x \ge 1$

$-2x \ge -4$

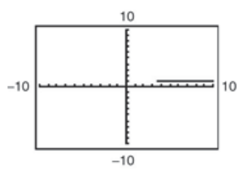
$x \le 2$



62. $20 < 6x - 1$

$21 < 6x$

$\frac{7}{2} < x$

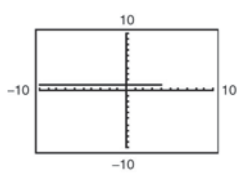


63. $4(x - 3) \le 8 - x$

$4x - 12 \le 8 - x$

$5x \le 20$

$x \le 4$

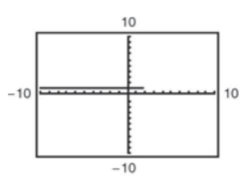


64. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

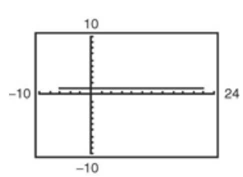
$x < 2$



65. $|x - 8| \le 14$

$-14 \le x - 8 \le 14$

$-6 \le x \le 22$

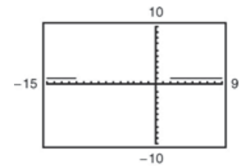


66. $|2x + 9| > 13$

$2x + 9 < -13$ or $2x + 9 > 13$

$2x < -22 \qquad 2x > 4$

$x < -11 \qquad x > 2$

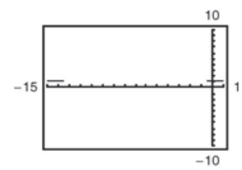


67. $2|x + 7| \ge 13$

$|x + 7| \ge \frac{13}{2}$

$x + 7 \le -\frac{13}{2}$ or $x + 7 \ge \frac{13}{2}$

$x \le -\frac{27}{2} \qquad x \ge -\frac{1}{2}$

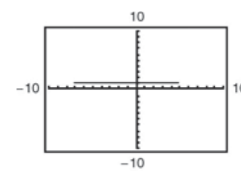


68. $\frac{1}{2}|x + 1| \le 3$

$|x + 1| \le 6$

$-6 \le x + 1 \le 6$

$-7 \le x \le 5$



69. $y = 2x - 3$

(a) $y \ge 1$

$2x - 3 \ge 1$

$2x \ge 4$

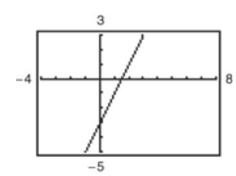
$x \ge 2$

(b) $y \le 0$

$2x - 3 \le 0$

$2x \le 3$

$x \le \frac{3}{2}$



70. $y = \frac{2}{3}x + 1$

(a) $y \le 5$

$\frac{2}{3}x + 1 \le 5$

$\frac{2}{3}x \le 4$

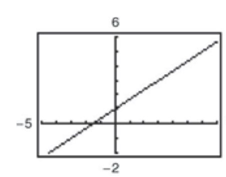
$x \le 6$

(b) $y \ge 0$

$\frac{2}{3}x + 1 \ge 0$

$\frac{2}{3}x \ge -1$

$x \ge -\frac{3}{2}$



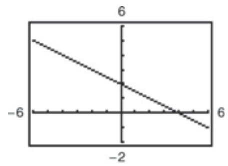
71. $y = -\frac{1}{2}x + 2$

(a) $0 \le y \le 3$

$0 \le -\frac{1}{2}x + 2 \le 3$

$-2 \le -\frac{1}{2}x \le 1$

$4 \ge x \ge -2$



(b) $y \ge 0$

$-\frac{1}{2}x + 2 \ge 0$

$-\frac{1}{2}x \ge -2$

$x \le 4$

72. $y = -3x + 8$

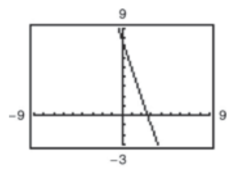
(a) $-1 \le y \le 3$

$-1 \le -3x + 8 \le 3$

$-9 \le -3x \le -5$

$3 \ge x \ge \frac{5}{3}$

$\frac{5}{3} \le x \le 3$



(b) $y \le 0$

$-3x + 8 \le 0$

$-3x \le -8$

$x \ge \frac{8}{3}$

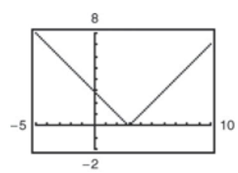
73. $y = |x - 3|$

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$



(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4$ or $x - 3 \ge 4$

$x \le -1$ or $x \ge 7$

74. $y = \left|\frac{1}{2}x + 1\right|$

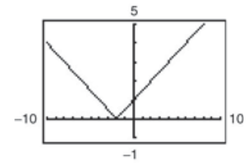
(a) $y \le 4$

$\left|\frac{1}{2}x + 1\right| \le 4$

$-4 \le \frac{1}{2}x + 1 \le 4$

$-5 \le \frac{1}{2}x \le 3$

$-10 \le x \le 6$



(b) $y \ge 1$

$\left|\frac{1}{2}x + 1\right| \ge 1$

$\frac{1}{2}x + 1 \le -1$ or $\frac{1}{2}x + 1 \ge 1$

$\frac{1}{2}x \le -2$ $\quad$ $\frac{1}{2}x \ge 0$

$x \le -4$ $\quad$ $x \ge 0$

75. $x - 5 \ge 0$

$x \ge 5$

$[5, \infty)$

76. $x - 10 \ge 0$

$x \ge 10$

$[10, \infty)$

77. $x + 3 \ge 0$

$x \ge -3$

$[-3, \infty)$

78. $3 - x \ge 0$

$3 \ge x$

$(-\infty, 3]$

79. $7 - 2x \ge 0$

$-2x \ge -7$

$x \le \frac{7}{2}$

$\left(-\infty, \frac{7}{2}\right]$

80. $6x + 15 \ge 0$

$6x \ge -15$

$x \ge -\frac{5}{2}$

$\left[-\frac{5}{2}, \infty\right)$

81. All real numbers less than 8 units from 10.

82. $|x - 8| > 4$

All real numbers more than 4 units from 8

83. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \leq 3$

$|x| \leq 3$

84. The graph shows all real numbers more than 3 units from 0.

$|x - 0| > 3$

$|x| > 3$

85. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \geq 3$

86. The graph shows all real numbers no more than 4 units from -1.

$|x + 1| \leq 4$

87. All real numbers at least 10 units from 12

$|x - 12| \geq 10$

88. All real numbers at least 5 units from 8

$|x - 8| \geq 5$

89. All real numbers more than 4 units from -3

$|x - (-3)| > 4$

$|x + 3| > 4$

90. All real numbers no more than 7 units from -6

$|x + 6| \leq 7$

91. $\$4.10 \leq E \leq \$4.25$

92. $2{,}000{,}000 < p < 2{,}400{,}000$

93. $r \leq 0.08$

94. $I \geq \$239{,}000{,}000$

95. $r = 220 - A = 220 - 20 = 200$ beats per minute

$0.50(200) \leq r \leq 0.85(200)$

$100 \leq r \leq 170$

The target heart rate is at least 100 beats per minute and at most 170 beats per minute.

96. $r = 220 - A = 220 - 40 = 180$ beats per minute

$0.50(180) \leq r \leq 0.85(180)$

$90 \leq r \leq 153$

The target heartrate is at least 90 beats per minute and at most 153 beats per minute.

97. $9.00 + 0.75x > 13.50$

$0.75x > 4.50$

$x > 6$

You must produce at least 6 units each hour in order to yield a greater hourly wage at the second job.

98. Let $x =$ gross sales per month.

$1000 + 0.04x > 3000$

$0.04x > 2000$

$x > \$50{,}000$

You must earn at least \$50,000 each month in order to earn a greater monthly wage at the second job.

99. $1000(1 + r(2)) > 1062.50$

$1 + 2r > 1.0625$

$2r > 0.0625$

$r > 0.03125$

$r > 3.125\%$

100. $825 < 750(1 + r(2))$

$825 < 750(1 + 2r)$

$825 < 750 + 1500r$

$75 < 1500r$

$0.05 < r$

The rate must be more than 5%.

101. $R > C$

$115.95x > 95x + 750$

$20.95x > 750$

$x \geq 35.7995$

$x \geq 36$ units

102. $24.55x > 15.4x + 150{,}000$

$9.15 > 150{,}000$

$x > 16{,}393.44262$

Because the number of units x must be an integer, the product will return a profit when at least 16,394 units are sold.

103. Let x = number of dozen doughnuts sold per day.

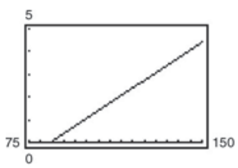
$$\begin{aligned} \textit{Revenue}: R &= 7.95x \\ \textit{Cost}: C &= 1.45x + 165 \\ P &= R - C \\ &= 7.95x - (1.45x + 165) \\ &= 6.50x - 165 \\ 400 \le P &\le 1200 \\ 400 \le 6.50x - 165 &\le 1200 \\ 565 \le 6.50x &\le 1365 \\ 86.9 \le x &\le 210 \end{aligned}$$

The daily sales vary between 87 and 210 dozen doughnuts per day.

104. The goal is to lose $164 - 128 = 36$ pounds. At $1\frac{1}{2}$ pounds per week, it will take 24 weeks.

$$\begin{aligned} 36 \div 1\tfrac{1}{2} &= 36 \times \tfrac{2}{3} \\ &= 12 \times 2 \\ &= 24 \end{aligned}$$

105. (a)



(b) From the graph you see that $y \ge 3$ when $x \ge 129$.

(c) Algebraically:

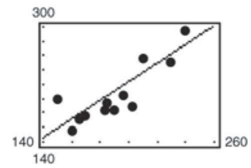
$$\begin{aligned} 3 &\le 0.067x - 5.638 \\ 8.638 &\le 0.067x \\ x &\ge 129 \end{aligned}$$

(d) IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitude.

106. (a)

x	165	184	150	210	196	240
y	170	185	200	255	205	295
$1.3x - 36$	179	203	159	237	219	276

x	202	170	185	190	230	160
y	190	175	195	185	250	155
$1.3x - 36$	227	185	205	211	263	172



(b) One estimate is $x \ge 182$ pounds.

(c)
$$\begin{aligned} 1.3x - 36 &\ge 200 \\ 1.3x &\ge 236 \\ x &\ge 181.5385 \approx 181.54 \text{ pounds} \end{aligned}$$

(d) An athlete's weight is not a particularly good indicator of the athlete's maximum bench-press weight. Other factors, such as muscle tone and exercise habits, influence maximum bench press weight.

107. (a)
$$\begin{aligned} S &= 1.36t + 41.1 \\ 45 \le 1.36t + 41.1 &\le 50 \\ 3.9 \le 1.36t &\le 8.9 \\ 2.9 \le t &\le 6.5 \end{aligned}$$

Between the years 2002 and 2006 the average salary was between \$45,000 and \$50,000.

(b)
$$\begin{aligned} 1.36t + 41.1 &\ge 62 \\ 1.36t &\ge 20.9 \\ t &\ge 15.4 \end{aligned}$$

The average salary will exceed \$62,000 sometime during the year 2015.

108. (a)
$$\begin{aligned} M &= 3.24t + 161.5 \\ 178 < 3.24t + 161.5 &\le 190 \\ 16.5 < 3.24t &\le 28.5 \\ 5.1 < t &\le 8.8 \end{aligned}$$

Milk production was between 178 billion pounds and 190 billion pounds between the years 2005 and 2008.

(b)
$$\begin{aligned} 3.24t + 161.5 &> 225 \\ 3.24t &> 63.5 \\ t &> 19.6 \end{aligned}$$

Milk production will exceed 225 billion pounds sometime during the year 2019.

109. $|s - 10.4| \le \frac{1}{16}$

$$-\tfrac{1}{16} \le s - 10.4 \le \tfrac{1}{16}$$
$$-0.0625 \le s - 10.4 \le 0.0625$$
$$10.3375 \le s \le 10.4625$$

Because $A = s^2$,

$$(10.3375)^2 \le \text{area} \le (10.4625)^2$$
$$106.864 \text{ in.}^2 \le \text{area} \le 109.464 \text{ in.}^2.$$

110. $24.2 - 0.25 \le s \le 24.2 + 0.25$

$$23.95 \le s \le 24.45$$

The interval containing the possible side lengths s in centimeters of the square is $[23.95, 24.45]$, so the interval containing the possible areas in square centimeters is $[23.95^2, 24.45^2]$, or $[573.6025, 597.8025]$.

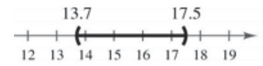
111. $\frac{1}{10}(3.61) \approx 0.361$

You might have been undercharged or overcharged by \$0.36.

112. $1 \text{ oz} = \frac{1}{16}$ lb, so $\frac{1}{2}$ oz $= \frac{1}{32}$ lb.

Because $8.99 \cdot \frac{1}{32} = 0.2809375$, you may be undercharged or overcharged by \$0.28.

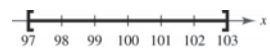
113. $\left|\frac{t - 15.6}{1.9}\right| < 1$



$$-1 < \frac{t - 15.6}{1.9} < 1$$
$$-1.9 < t - 15.6 < 1.9$$
$$13.7 < t < 17.5$$

Two-thirds of the workers could perform the task in the time interval between 13.7 minutes and 17.5 minutes.

114. Let x be the thickness of the given cover layers. So, the possible thickness of the cover layer is $|x - 100| \le 3$.



115. True. This is the Addition of a Constant Property of Inequality.

116. False. If c is negative, then $ac \ge bc$.

117. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

118. (a) Estimate from the graph: when the plate thickness is 2 millimeters, the frequency is approximately 330 vibrations per second.

(b) Estimate from the graph: when the frequency is 600, the plate thickness is approximately 3.6 millimeters.

(c) Estimate from the graph: when the frequency is between 200 and 400 vibrations per second, the plate thickness is between 1.2 and 2.4 millimeter.

(d) Estimate from the graph: when the plate thickness is less than 3 millimeters, the frequency is less than 500 vibrations per second.

Section 1.8 Other Types of Inequalities

1. positive; negative

2. key; test intervals

3. zeros; undefined values

4. $P = R - C$

5. $x^2 - 3 < 0$

(a) $x = 3$

$$(3)^2 - 3 \stackrel{?}{<} 0$$
$$6 \not< 0$$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$$(0)^2 - 3 \stackrel{?}{<} 0$$
$$-3 < 0$$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$$\left(\tfrac{3}{2}\right)^2 - 3 \stackrel{?}{<} 0$$
$$-\tfrac{3}{4} < 0$$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$$(-5)^2 - 3 \stackrel{?}{<} 0$$
$$22 \not< 0$$

No, $x = -5$ *is not* a solution.

6. $x^2 - x - 12 \geq 0$

(a) $x = 5$

$$(5)^2 - (5) - 12 \overset{?}{\geq} 0$$
$$8 \geq 0$$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$$(0)^2 - 0 - 12 \overset{?}{\geq} 0$$
$$-12 \ngeq 0$$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$$(-4)^2 - (-4) - 12 \overset{?}{\geq} 0$$
$$16 + 4 - 12 \overset{?}{\geq} 0$$
$$8 \geq 0$$

Yes, $x = -4$ *is* a solution.

(d) $x = -3$

$$(-3)^2 - (-3) - 12 \overset{?}{\geq} 0$$
$$9 + 3 - 12 \overset{?}{\geq} 0$$
$$0 \geq 0$$

Yes, $x = -3$ *is* a solution.

7. $\dfrac{x + 2}{x - 4} \geq 3$

(a) $x = 5$

$$\frac{5 + 2}{5 - 4} \overset{?}{\geq} 3$$
$$7 \geq 3$$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$$\frac{4 + 2}{4 - 4} \overset{?}{\geq} 3$$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$$\frac{-\frac{9}{2} + 2}{-\frac{9}{2} - 4} \overset{?}{\geq} 3$$
$$\frac{5}{17} \ngeq 3$$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$$\frac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \overset{?}{\geq} 3$$
$$13 \geq 3$$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

8. $\dfrac{3x^2}{x^2 + 4} < 1$

(a) $x = -2$

$$\frac{3(-2)^2}{(-2)^2 + 4} \overset{?}{<} 1$$
$$\frac{12}{8} \nless 1$$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$$\frac{3(-1)^2}{(-1)^2 + 4} \overset{?}{<} 1$$
$$\frac{3}{5} < 1$$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$$\frac{3(0)^2}{(0)^2 + 4} \overset{?}{<} 1$$
$$0 < 1$$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$$\frac{3(3)^2}{(3)^2 + 4} \overset{?}{<} 1$$
$$\frac{27}{13} \nless 1$$

No, $x = 3$ *is not* a solution.

9. $3x^2 - x - 2 = (3x + 2)(x - 1)$

$$3x + 2 = 0 \Rightarrow x = -\tfrac{2}{3}$$
$$x - 1 = 0 \Rightarrow x = 1$$

The key numbers are $-\frac{2}{3}$ and 1.

10. $9x^3 - 25x^2 = 0$

$$x^2(9x - 25) = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$9x - 25 = 0 \Rightarrow x = \tfrac{25}{9}$$

The key numbers are 0 and $\frac{25}{9}$.

11. $\dfrac{1}{x - 5} + 1 = \dfrac{1 + 1(x - 5)}{x - 5}$

$$= \frac{x - 4}{x - 5}$$
$$x - 4 = 0 \Rightarrow x = 4$$
$$x - 5 = 0 \Rightarrow x = 5$$

The key numbers are 4 and 5.

12. $\dfrac{x}{x+2} - \dfrac{2}{x-1} = \dfrac{x(x-1) - 2(x+2)}{(x+2)(x-1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x+2)(x-1)}$

$= \dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x + 1 = 0 \Rightarrow x = -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-2, -1, 1,$ and 4.

13. $x^2 < 9$

$x^2 - 9 < 0$

$(x+3)(x-3) < 0$

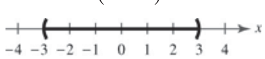
Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x+3)(x-3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	-4	7	Positive
$(-3, 3)$	0	-9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$



14. $x^2 \le 16$

$x^2 - 16 \le 0$

$(x+4)(x-4) \le 0$

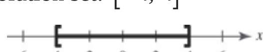
Key numbers: $x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 4), (4, \infty)$

Test: Is $(x+4)(x-4) \le 0$?

Interval	x-Value	Value of $x^2 - 16$	Conclusion
$(-\infty, -4)$	-5	9	Positive
$(-4, 4)$	0	-16	Negative
$(4, \infty)$	5	9	Positive

Solution set: $[-4, 4]$



15. $(x+2)^2 \le 25$

$x^2 + 4x + 4 \le 25$

$x^2 + 4x - 21 \le 0$

$(x+7)(x-3) \le 0$

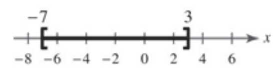
Key numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty, -7)$	-8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



16. $(x-3)^2 \ge 1$

$x^2 - 6x + 8 \ge 0$

$(x-2)(x-4) \ge 0$

Key numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$

17. $x^2 + 4x + 4 \ge 9$

$x^2 + 4x - 5 \ge 0$

$(x+5)(x-1) \ge 0$

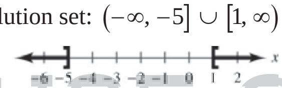
Key numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x+5)(x-1) \ge 0$?

Interval	x-Value	Value of $(x+5)(x-1)$	Conclusion
$(-\infty, -5)$	-6	$(-1)(-7) = 7$	Positive
$(-5, 1)$	0	$(5)(-1) = -5$	Negative
$(1, \infty)$	2	$(7)(1) = 7$	Positive

Solution set: $(-\infty, -5] \cup [1, \infty)$



18. $x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

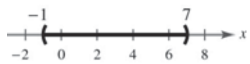
Key numbers: $x = -1, x = 7$

Test intervals: $(-\infty, -1) \Rightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Rightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 1)(x - 7) > 0$

Solution set: $(-1, 7)$



19. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x + 3)(x - 2) < 0$

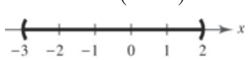
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x + 3)(x - 2) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



20. $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x + 3)(x - 1) > 0$

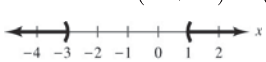
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



21. $x^2 + 2x - 3 < 0$

$(x + 3)(x - 1) < 0$

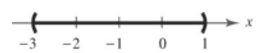
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



22. $x^2 > 2x + 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

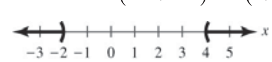
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



23. $3x^2 - 11x > 20$

$3x^2 - 11x - 20 > 0$

$(3x + 4)(x - 5) > 0$

Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$

24. $-2x^2 + 6x + 15 \le 0$

$2x^2 - 6x - 15 \ge 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{156}}{4}$$

$$= \frac{6 \pm 2\sqrt{39}}{4}$$

$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

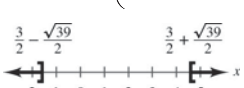
Test intervals:

$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$

$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$

$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



25. $x^2 - 3x - 18 > 0$

$(x + 3)(x - 6) > 0$

Key numbers: $x = -3, x = 6$

Test intervals: $(-\infty, -3), (-3, 6), (6, \infty)$

Test: Is $(x + 3)(x - 6) > 0$?

Interval	x-Value	Value of $(x + 3)(x - 6)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-10) = 10$	Positive
$(-3, 6)$	0	$(3)(-6) = -18$	Negative
$(6, \infty)$	7	$(10)(1) = 10$	Positive

Solution set: $(-\infty, -3) \cup (6, \infty)$

26. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

$(x + 2)^2(x - 2) \le 0$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$

27. $x^3 - 3x^2 - x > -3$

$x^3 - 3x^2 - x + 3 > 0$

$x^2(x - 3) - (x - 3) > 0$

$(x - 3)(x^2 - 1) > 0$

$(x - 3)(x + 1)(x - 1) > 0$

Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty, -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0$?

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$

28. $2x^3 + 13x^2 - 8x - 46 \ge 6$

$2x^3 + 13x^2 - 8x - 52 \ge 0$

$x^2(2x + 13) - 4(2x + 13) \ge 0$

$(2x + 13)(x^2 - 4) \ge 0$

$(2x + 13)(x + 2)(x - 2) \ge 0$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

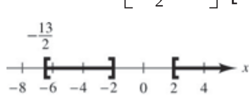
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



29. $4x^3 - 6x^2 < 0$

$2x^2(2x - 3) < 0$

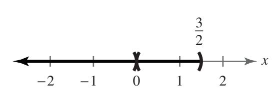
Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$



30. $4x^3 - 12x^2 > 0$

$4x^2(x - 3) > 0$

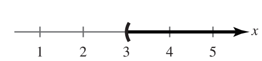
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow 4x^2(x - 3) < 0$

$(0, 3) \Rightarrow 4x^2(x - 3) < 0$

$(3, \infty) \Rightarrow 4x^2(x - 3) > 0$

Solution set: $(3, \infty)$



31. $x^3 - 4x \geq 0$

$x(x + 2)(x - 2) \geq 0$

Key numbers: $x = 0, x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow x(x + 2)(x - 2) < 0$

$(-2, 0) \Rightarrow x(x + 2)(x - 2) > 0$

$(0, 2) \Rightarrow x(x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow x(x + 2)(x - 2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$

32. $2x^3 - x^4 \leq 0$

$x^3(2 - x) \leq 0$

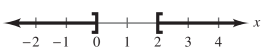
Key numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \Rightarrow x^3(2 - x) < 0$

$(0, 2) \Rightarrow x^3(2 - x) > 0$

$(2, \infty) \Rightarrow x^3(2 - x) < 0$

Solution set: $(-\infty, 0] \cup [2, \infty)$



33. $(x - 1)^2(x + 2)^3 \geq 0$

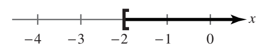
Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x - 1)^2(x + 2)^3 < 0$

$(-2, 1) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

$(1, \infty) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

Solution set: $[-2, \infty)$



34. $x^4(x - 3) \leq 0$

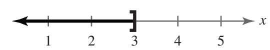
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x - 3) < 0$

$(0, 3) \Rightarrow x^4(x - 3) < 0$

$(3, \infty) \Rightarrow x^4(x - 3) > 0$

Solution set: $(-\infty, 3]$



35. $4x^2 - 4x + 1 \leq 0$

$(2x - 1)^2 \leq 0$

Key number: $x = \frac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$\left(-\infty, \frac{1}{2}\right]$	$x = 0$	$[2(0) - 1]^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	$x = 1$	$[2(1) - 1]^2 = 1$	Positive

The solution set consists of the single real number $\frac{1}{2}$.

36. $x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are $x = -\frac{3}{2} \pm \frac{\sqrt{23}}{2}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

37. $x^2 - 6x + 12 \le 0$

Using the Quadratic Formula, you can determine that the key numbers are $x = 3 \pm \sqrt{3}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

38. $x^2 - 8x + 16 > 0$

$(x - 4)^2 > 0$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0 - 4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5 - 4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

39. $\dfrac{4x - 1}{x} > 0$

Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x - 1}{x} > 0$?

Interval	x-Value	Value of $\dfrac{4x - 1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\dfrac{-5}{-1} = 5$	Positive
$\left(0, \dfrac{1}{4}\right)$	$\dfrac{1}{8}$	$\dfrac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\dfrac{1}{4}, \infty\right)$	1	$\dfrac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$

40. $\dfrac{x^2 - 1}{x} < 0$

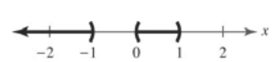
$\dfrac{(x - 1)(x + 1)}{x} < 0$

Key numbers: $x = -1, x = 0, x = 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Interval	x-Value	Value of $\dfrac{(x - 1)(x + 1)}{x}$	Conclusion
$(-\infty, -1)$	-2	$\dfrac{(-3)(-1)}{-2} = -\dfrac{3}{2}$	Negative
$(-1, 0)$	$-\dfrac{1}{2}$	$\dfrac{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \dfrac{3}{2}$	Positive
$(0, 1)$	$\dfrac{1}{2}$	$\dfrac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{1}{2}} = -\dfrac{3}{2}$	Negative
$(1, \infty)$	2	$\dfrac{(1)(3)}{2} = \dfrac{3}{2}$	Positive

Solution set: $(-\infty, -1) \cup (0, 1)$



41. $\dfrac{3x-5}{x-5} \ge 0$

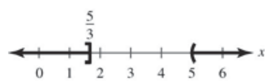
Key numbers: $x = \dfrac{5}{3}, x = 5$

Test intervals: $\left(-\infty, \dfrac{5}{3}\right), \left(\dfrac{5}{3}, 5\right), (5, \infty)$

Test: Is $\dfrac{3x-5}{x-5} \ge 0$?

Interval	x-Value	Value of $\dfrac{3x-5}{x-5}$	Conclusion
$\left(-\infty, \dfrac{5}{3}\right)$	0	$\dfrac{-5}{-5} = 1$	Positive
$\left(\dfrac{5}{3}, 5\right)$	2	$\dfrac{6-5}{2-5} = -\dfrac{1}{3}$	Negative
$(5, \infty)$	6	$\dfrac{18-5}{6-5} = 13$	Positive

Solution set: $\left(-\infty, \dfrac{5}{3}\right] \cup (5, \infty)$



42. $\dfrac{5+7x}{1+2x} \le 4$

$\dfrac{5+7x-4(1+2x)}{1+2x} \le 0$

$\dfrac{1-x}{1+2x} \le 0$

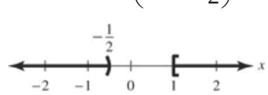
Key numbers: $x = -\dfrac{1}{2}, x = 1$

Test intervals: $\left(-\infty, -\dfrac{1}{2}\right), \left(-\dfrac{1}{2}, 1\right), (1, \infty)$

Test: Is $\dfrac{1-x}{1+2x} \le 0$?

Interval	x-Value	Value of $\dfrac{1-x}{1+2x}$	Conclusion
$\left(-\infty, -\dfrac{1}{2}\right)$	-1	$\dfrac{2}{-1} = -2$	Negative
$\left(-\dfrac{1}{2}, 1\right)$	0	$\dfrac{1}{1} = 1$	Positive
$(1, \infty)$	2	$\dfrac{-1}{5} = -\dfrac{1}{5}$	Negative

Solution set: $\left(-\infty, -\dfrac{1}{2}\right) \cup [1, \infty)$



43. $\dfrac{x+6}{x+1} - 2 < 0$

$\dfrac{x+6-2(x+1)}{x+1} < 0$

$\dfrac{4-x}{x+1} < 0$

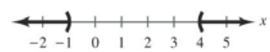
Key numbers: $x = -1, x = 4$

Test intervals: $(-\infty, -1) \Rightarrow \dfrac{4-x}{x+1} < 0$

$(-1, 4) \Rightarrow \dfrac{4-x}{x+1} > 0$

$(4, \infty) \Rightarrow \dfrac{4-x}{x+1} < 0$

Solution set: $(-\infty, -1) \cup (4, \infty)$



44. $\dfrac{x+12}{x+2} - 3 \ge 0$

$\dfrac{x+12-3(x+2)}{x+2} \ge 0$

$\dfrac{6-2x}{x+2} \ge 0$

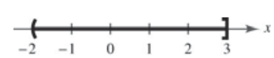
Key numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2) \Rightarrow \dfrac{6-2x}{x+2} < 0$

$(-2, 3) \Rightarrow \dfrac{6-2x}{x+2} > 0$

$(3, \infty) \Rightarrow \dfrac{6-2x}{x+2} < 0$

Solution interval: $(-2, 3]$



45.
$$\frac{2}{x+5} > \frac{1}{x-3}$$
$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$
$$\frac{2(x-3) - 1(x+5)}{(x+5)(x-3)} > 0$$
$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5, x = 3, x = 11$

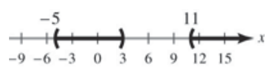
Test intervals: $(-\infty, -5) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$



46.
$$\frac{5}{x-6} > \frac{3}{x+2}$$
$$\frac{5(x+2) - 3(x-6)}{(x-6)(x+2)} > 0$$
$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Key numbers: $x = -14, x = -2, x = 6$

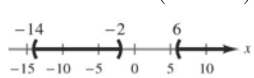
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



47.
$$\frac{1}{x-3} \le \frac{9}{4x+3}$$
$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$
$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$
$$\frac{30-5x}{(x-3)(4x+3)} \le 0$$

Key numbers: $x = 3, x = -\frac{3}{4}, x = 6$

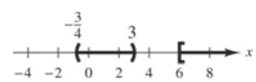
Test intervals: $\left(-\infty, -\frac{3}{4}\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$\left(-\frac{3}{4}, 3\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

$(3, 6) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$(6, \infty) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

Solution set: $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$



48.
$$\frac{1}{x} \ge \frac{1}{x+3}$$
$$\frac{1(x+3) - 1(x)}{x(x+3)} \ge 0$$
$$\frac{3}{x(x+3)} \ge 0$$

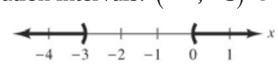
Key numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



49.
$$\frac{x^2 + 2x}{x^2 - 9} \le 0$$
$$\frac{x(x + 2)}{(x + 3)(x - 3)} \le 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \frac{x(x + 2)}{(x + 3)(x - 3)} > 0$

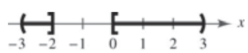
$(-3, -2) \Rightarrow \frac{x(x + 2)}{(x + 3)(x - 3)} < 0$

$(-2, 0) \Rightarrow \frac{x(x + 2)}{(x + 3)(x - 3)} > 0$

$(0, 3) \Rightarrow \frac{x(x + 2)}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \frac{x(x + 2)}{(x + 3)(x - 3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



50.
$$\frac{x^2 + x - 6}{x} \ge 0$$
$$\frac{(x + 3)(x - 2)}{x} \ge 0$$

Key numbers: $x = -3, x = 0, x = 2$

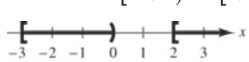
Test intervals: $(-\infty, -3) \Rightarrow \frac{(x + 3)(x - 2)}{x} < 0$

$(-3, 0) \Rightarrow \frac{(x + 3)(x - 2)}{x} > 0$

$(0, 2) \Rightarrow \frac{(x + 3)(x - 2)}{x} < 0$

$(2, \infty) \Rightarrow \frac{(x + 3)(x - 2)}{x} > 0$

Solution set: $[-3, 0) \cup [2, \infty)$



51.
$$\frac{3}{x - 1} + \frac{2x}{x + 1} > -1$$
$$\frac{3(x + 1) + 2x(x - 1) + 1(x + 1)(x - 1)}{(x - 1)(x + 1)} > 0$$
$$\frac{3x^2 + x + 2}{(x - 1)(x + 1)} > 0$$

Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \frac{3x^2 + x + 2}{(x - 1)(x + 1)} > 0$

$(-1, 1) \Rightarrow \frac{3x^2 + x + 2}{(x - 1)(x + 1)} < 0$

$(1, \infty) \Rightarrow \frac{3x^2 + x + 2}{(x - 1)(x + 1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$

52.
$$\frac{3x}{x - 1} \le \frac{x}{x + 4} + 3$$
$$\frac{3x(x + 4) - x(x - 1) - 3(x + 4)(x - 1)}{(x - 1)(x + 4)} \le 0$$
$$\frac{-x^2 + 4x + 12}{(x - 1)(x + 4)} \le 0$$
$$\frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

$(-4, -2) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0$

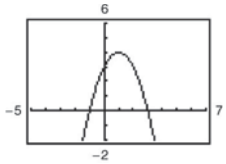
$(-2, 1) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

$(1, 6) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} > 0$

$(6, \infty) \Rightarrow \frac{-(x - 6)(x + 2)}{(x - 1)(x + 4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$

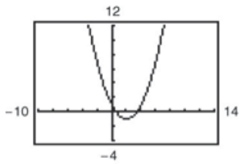
53. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

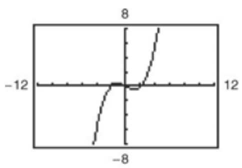
54. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$ when $x \le -2$ or $x \ge 6$.

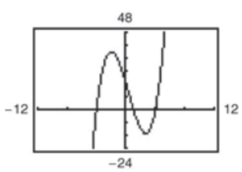
55. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0$ or $2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

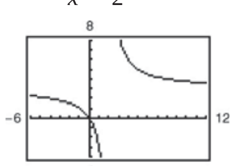
56. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$ when $-\infty < x \le -4$ or $1 \le x \le 4$.

(b) $y \ge 36$ when $x = -2$ or $5 \le x < \infty$.

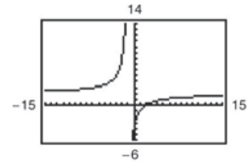
57. $y = \dfrac{3x}{x - 2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

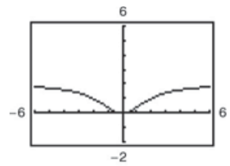
58. $y = \dfrac{2(x - 2)}{x + 1}$



(a) $y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$ when $-2 \le x < -1$.

59. $y = \dfrac{2x^2}{x^2 + 4}$



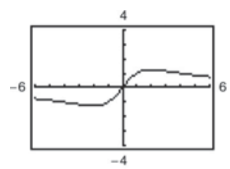
(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

60. $y = \dfrac{5x}{x^2 + 4}$



(a) $y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$ when $-\infty < x \le 0$.

61. $4 - x^2 \ge 0$

$(2 + x)(2 - x) \ge 0$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

62. $x^2 - 4 \ge 0$

$(x + 2)(x - 2) \ge 0$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow (x + 2)(x - 2) > 0$

$(-2, 2) \Rightarrow (x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow (x + 2)(x - 2) > 0$

Domain: $(-\infty, -2] \cup [2, \infty)$

63. $x^2 - 9x + 20 \geq 0$

$(x - 4)(x - 5) \geq 0$

Key numbers: $x = 4$, $x = 5$

Test intervals: $(-\infty, 4)$, $(4, 5)$, $(5, \infty)$

Interval	x-Value	Value of $(x - 4)(x - 5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

64. $81 - 4x^2 \geq 0$

$(9 - 2x)(9 + 2x) \geq 0$

Key numbers: $x = \pm\dfrac{9}{2}$

Test intervals: $\left(-\infty, -\dfrac{9}{2}\right), \left(-\dfrac{9}{2}, \dfrac{9}{2}\right), \left(\dfrac{9}{2}, \infty\right)$

Interval	x-Value	Value of $(9 - 2x)(9 + 2x)$	Conclusion
$\left(-\infty, -\frac{9}{2}\right)$	-5	$(19)(-1) = -19$	Negative
$\left(-\frac{9}{2}, \frac{9}{2}\right)$	0	$(9)(9) = 81$	Positive
$\left(\frac{9}{2}, \infty\right)$	5	$(-1)(19) = -19$	Negative

Domain: $\left[-\dfrac{9}{2}, \dfrac{9}{2}\right]$

65. $\dfrac{x}{x^2 - 2x - 35} \geq 0$

$\dfrac{x}{(x + 5)(x - 7)} \geq 0$

Key numbers: $x = 0$, $x = -5$, $x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

66. $\dfrac{x}{x^2 - 9} \geq 0$

$\dfrac{x}{(x + 3)(x - 3)} \geq 0$

Key numbers: $x = -3$, $x = 0$, $x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

67. $0.4x^2 + 5.26 < 10.2$

$0.4x^2 - 4.94 < 0$

$0.4(x^2 - 12.35) < 0$

Key numbers: $x \approx \pm 3.51$

Test intervals: $(-\infty, -3.51)$, $(-3.51, 3.51)$, $(3.51, \infty)$

Solution set: $(-3.51, 3.51)$

68. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13)$, $(-1.13, 1.13)$, $(1.13, \infty)$

Solution set: $(-1.13, 1.13)$

69. $-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13$, $x \approx 25.13$

Test intervals: $(-\infty, -0.13)$, $(-0.13, 25.13)$, $(25.13, \infty)$

Solution set: $(-0.13, 25.13)$

70. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42$, $x \approx 0.42$

Test intervals: $(-\infty, -4.42)$, $(-4.42, 0.42)$, $(0.42, \infty)$

Solution set: $(-4.42, 0.42)$

71. $\dfrac{1}{2.3x - 5.2} > 3.4$

$\dfrac{1}{2.3x - 5.2} - 3.4 > 0$

$\dfrac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$

$\dfrac{-7.82x + 18.68}{2.3x - 5.2} > 0$

Key numbers: $x \approx 2.39,\ x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

72. $\dfrac{2}{3.1x - 3.7} > 5.8$

$\dfrac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$

$\dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

Key numbers: $x \approx 1.19,\ x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

73. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0,\ t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4,\ t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

74. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2},\ t = 4 + 2\sqrt{2}$

Test intervals:
$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}),$
$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\le t < 4 - 2\sqrt{2}$ seconds
and
$4 + 2\sqrt{2}$ seconds $< t \le$ 8 seconds

75. $2L + 2W = 100 \Rightarrow W = 50 - L$

$LW \ge 500$

$L(50 - L) \ge 500$

$-L^2 + 50L - 500 \ge 0$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$
13.8 meters $\le L \le$ 36.2 meters

76. $2L + 2W = 440 \Rightarrow W = 220 - L$

$LW \ge 8000$

$L(220 - L) \ge 8000$

$-L^2 + 220L - 8000 \ge 0$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \ge 0$?

Solution set: $110 - 10\sqrt{41} \le L \le 110 + 10\sqrt{41}$
45.97 feet $\le L \le$ 174.03 feet

77. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$$\begin{aligned} P &= R - C \\ &= (75x - 0.0005x^2) - (30x + 250{,}000) \\ &= -0.0005x^2 + 45x - 250{,}000 \end{aligned}$$

$$P \geq 750{,}000$$

$$-0.0005x^2 + 45x - 250{,}000 \geq 750{,}000$$

$$-0.0005x^2 + 45x - 1{,}000{,}000 \geq 0$$

Key numbers: $x = 40{,}000$, $x = 50{,}000$

(These were obtained by using the Quadratic Formula.)

Test intervals:
$(0, 40{,}000)$, $(40{,}000, 50{,}000)$, $(50{,}000, \infty)$

The solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \leq x \leq 50{,}000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. So, for $40{,}000 \leq x \leq 50{,}000$, $\$50.00 \leq p \leq \$55.00$.

78. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$$\begin{aligned} P &= R - C \\ &= (50x - 0.0002x^2) - (12x + 150{,}000) \\ &= -0.0002x^2 + 38x - 150{,}000 \end{aligned}$$

$$P \geq 1{,}650{,}000$$

$$-0.0002x^2 + 38x - 150{,}000 \geq 1{,}650{,}000$$

$$-0.0002x^2 + 38x - 1{,}800{,}000 \geq 0$$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

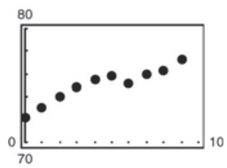
Test intervals:
$(0, 90{,}000)$, $(90{,}000, 100{,}000)$, $(100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \leq x \leq 100{,}000$. The price per unit is

$$p = \frac{R}{x} = 50 - 0.0002x.$$

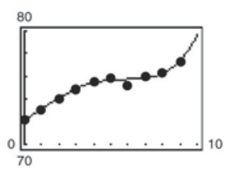
For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \leq x \leq 100{,}000$, $\$30 \leq p \leq \$32$.

79. (a)



(b) $N = 0.00406t^4 - 0.0564t^3 + 0.147t^2 + 0.86t + 72.2$

(c)



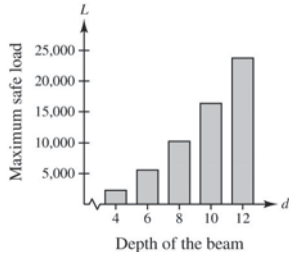
The model fits the data well.

(d) Using the zoom and trace features, the number of students enrolled in schools exceeded 74 million in the year 2001.

(e) No. The model can be used to predict enrollments for years close to those in its domain but when you project too far into the future, the numbers predicted by the model increase too rapidly to be considered reasonable.

80. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b)
$$\begin{aligned} 2000 &\le 168.5d^2 - 472.1 \\ 2472.1 &\le 168.5d^2 \\ 14.67 &\le d^2 \\ 3.83 &\le d \end{aligned}$$

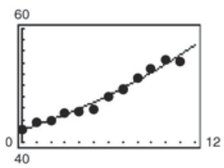
The minimum depth is 3.83 inches.

81.
$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{2} \\ 2R_1 &= 2R + RR_1 \\ 2R_1 &= R(2 + R_1) \\ \frac{2R_1}{2 + R_1} &= R \end{aligned}$$

Because $R \ge 1$,
$$\begin{aligned} \frac{2R_1}{2 + R_1} &\ge 1 \\ \frac{2R_1}{2 + R_1} - 1 &\ge 0 \\ \frac{R_1 - 2}{2 + R_1} &\ge 0. \end{aligned}$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \ge 2$ ohms.

82. (a)



(b) The model fits the data well.

(c)
$$\begin{aligned} S &= \frac{42.16 - 0.236t}{1 - 0.026t} \\ 60 &\le \frac{42.16 - 0.236t}{1 - 0.026t} \\ 0 &\le \frac{42.16 - 0.236t}{1 - 0.026t} - 60 \\ 0 &\le \frac{-17.84 + 1.324t}{1 - 0.026t} \end{aligned}$$

Key numbers: $t \approx 38.5$ and $t \approx 13.5$

Test Intervals	t-Value	Expression Value	Conclusion
$(0, 13.5)$	$t = 1$	-17.0	Negative
$(13.5, 38.5)$	$t = 20$	18.0	Positive
$(38.5, \infty)$	$t = 40$	-878.0	Negative

So, the mean salary for classroom teachers will exceed \$60,000 during the year 2013.

(d) No. The model yields negative values for values of $t \ge 38.5$. The graph also has a vertical asymptote at

$$t = \frac{500}{13} \approx 38.5.$$

After testing the intervals, you can see that the inequality is satisfied on the open interval (13.5, 38.5).

83. True.

$$x^3 - 2x^2 - 11x + 12 = (x + 3)(x - 1)(x - 4)$$

The test intervals are $(-\infty, -3)$, $(-3, 1)$, $(1, 4)$, and $(4, \infty)$.

84. True.

The y-values are greater than zero for all values of x.

85. $x^2 + bx + 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \ge 0$.

$b^2 - 4(1)(4) \ge 0$

$b^2 - 16 \ge 0$

Key numbers: $b = -4, b = 4$

Test intervals: $(-\infty, -4) \Rightarrow b^2 - 16 > 0$

$(-4, 4) \Rightarrow b^2 - 16 < 0$

$(4, \infty) \Rightarrow b^2 - 16 > 0$

Solution set: $(-\infty, -4] \cup [4, \infty]$

(b) $b^2 - 4ac \ge 0$

Key numbers: $b = -2\sqrt{ac}, b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$,

$b \le -2\sqrt{ac}$ or $b \ge 2ac$.

86. $x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \ge 0$.

$b^2 - 4(1)(-4) \ge 0$

$b^2 + 16 \ge 0$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \ge 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

87. $3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \ge 0$.

$b^2 - 4(3)(10) \ge 0$

$b^2 - 120 \ge 0$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $(-\infty, -2\sqrt{30}) \Rightarrow b^2 - 120 > 0$

$(-2\sqrt{30}, 2\sqrt{30}) \Rightarrow b^2 - 120 < 0$

$(2\sqrt{30}, \infty) \Rightarrow b^2 - 120 > 0$

Solution set: $(-\infty, -2\sqrt{30}] \cup [2\sqrt{30}, \infty]$

(b) $b^2 - 4ac \ge 0$

Similar to part (a), if $a > 0$ and $c > 0$,

$b \le -2\sqrt{ac}$ or $b \ge 2ac$.

88. $2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \ge 0$.

$b^2 - 4(2)(5) \ge 0$

$b^2 - 40 \ge 0$

Key numbers: $b = -2\sqrt{10}, b = 2\sqrt{10}$

Test intervals: $(-\infty, -2\sqrt{10}) \Rightarrow b^2 - 40 > 0$

$(-2\sqrt{10}, 2\sqrt{10}) \Rightarrow b^2 - 40 < 0$

$(2\sqrt{10}, \infty) \Rightarrow b^2 - 40 > 0$

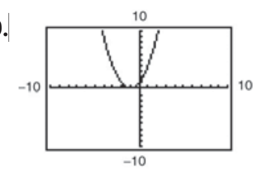
Solution set: $(-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)$

(b) $b^2 - 4ac \ge 0$

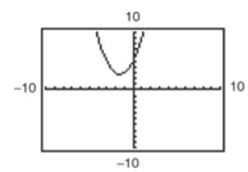
Similar to part (a), if $a > 0$ and $c > 0$,

$b \le -2\sqrt{ac}$ or $b \ge 2ac$.

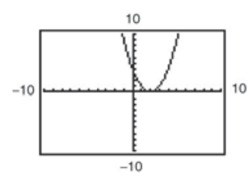
89.



For part (b), the y-values that are less than or equal to 0 occur only at $x = -1$.



For part (c), there are no y-values that are less than 0.



For part (d), the y-values that are greater than 0 occur for all values of x except 2

90. (a) $x = a, x = b$

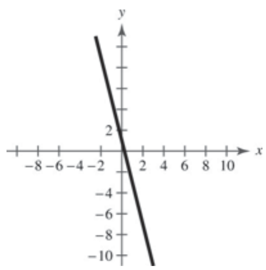
(b)

(c) The real zeros of the polynomial

Review Exercises for Chapter 1

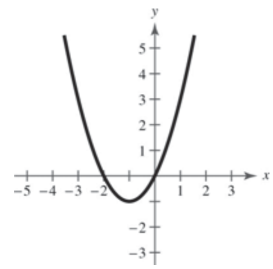
1. $y = -4x + 1$

x	-2	-1	0	1	2
y	9	5	1	-3	-7



2. $y = x^2 + 2x$

x	-3	-2	-1	0	1
y	3	0	-1	0	3



3. x-intercepts: $(1, 0), (5, 0)$

y-intercept: $(0, 5)$

4. x-intercepts: $(-4, 0), (2, 0)$

y-intercept: $(0, -2)$

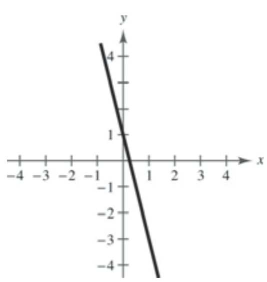
5. $y = -4x + 1$

Intercepts: $\left(\frac{1}{4}, 0\right), (0, 1)$

$y = -4(-x) + 1 \Rightarrow y = 4x + 1 \Rightarrow$ No y-axis symmetry

$-y = -4x + 1 \Rightarrow y = 4x - 1 \Rightarrow$ No x-axis symmetry

$-y = -4(-x) + 1 \Rightarrow y = -4x - 1 \Rightarrow$ No origin symmetry



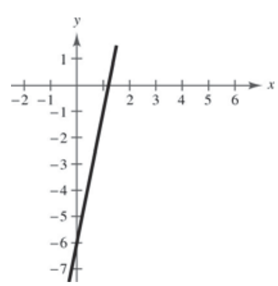
6. $y = 5x - 6$

Intercepts: $\left(\frac{6}{5}, 0\right), (0, -6)$

$y = 5(-x) - 6 \Rightarrow y = -5x - 6 \Rightarrow$ No y-axis symmetry

$-y = 5x - 6 \Rightarrow y = -5x + 6 \Rightarrow$ No x-axis symmetry

$-y = 5(-x) - 6 \Rightarrow y = 5x + 6 \Rightarrow$ No origin symmetry



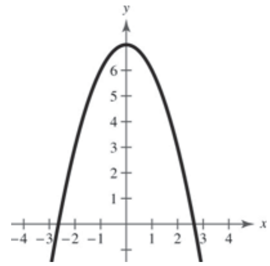
7. $y = 7 - x^2$

Intercepts: $\left(\pm\sqrt{7}, 0\right), (0, 7)$

$y = 7 - (-x)^2 \Rightarrow y = 7 - x^2 \Rightarrow$ y-axis symmetry

$y = 7 - x^2 \Rightarrow y = -7 + x^2 \Rightarrow$ No x-axis symmetry

$-y = 7 - (-x)^2 \Rightarrow y = -7 + x^2 \Rightarrow$ No origin symmetry



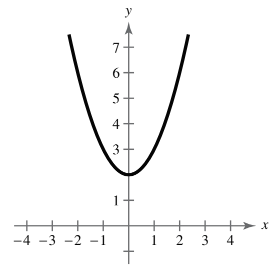
8. $y = x^2 + 2$

Intercept: $(0, 2)$

$y = (-x)^2 + 2 \Rightarrow y = x^2 + 2 \Rightarrow$ y-axis symmetry

$y = x^2 + 2 \Rightarrow y = -x^2 - 2 \Rightarrow$ No x-axis symmetry

$-y = (-x)^2 + 2 \Rightarrow y = x^2 - 2 \Rightarrow$ No origin symmetry



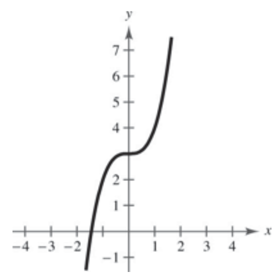
9. $y = x^3 + 3$

Intercepts: $\left(-\sqrt[3]{3}, 0\right), (0, 3)$

$y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow$ No y-axis symmetry

$-y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow$ No origin symmetry



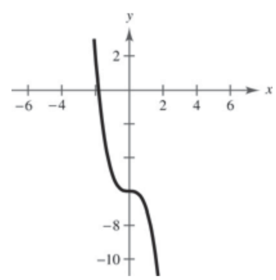
10. $y = -6 - x^3$

Intercepts: $\left(\sqrt[3]{-6}, 0\right), (0, -6)$

$y = -6 - (-x)^3 \Rightarrow y = -6 + x \Rightarrow$ No y-axis symmetry

$-y = -6 - x^3 \Rightarrow y = 6 + x^3 \Rightarrow$ No x-axis symmetry

$-y = -6 - (-x)^3 \Rightarrow y = 6 - x^3 \Rightarrow$ No origin symmetry



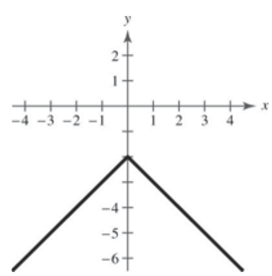
11. $y = -|x| - 2$

Intercept: $(0, -2)$

$y = -|-x| - 2 \Rightarrow y = -|x| - 2 \Rightarrow$ y-axis symmetry

$-y = -|x| - 2 \Rightarrow y = |x| + 2 \Rightarrow$ No x-axis symmetry

$-y = -|-x| - 2 \Rightarrow y = |x| + 2 \Rightarrow$ No origin symmetry



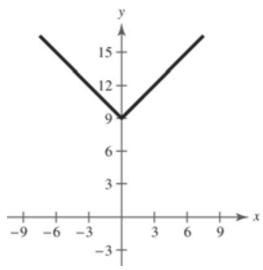
12. $y = |x| + 9$

Intercept: $(0, 9)$

$y = |-x| + 9 \Rightarrow y = |x| + 9 \Rightarrow$ y-axis symmetry

$-y = |x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow$ No x-axis symmetry

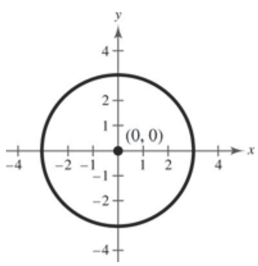
$-y = |-x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow$ No origin symmetry



13. $x^2 + y^2 = 9$

Center: $(0, 0)$

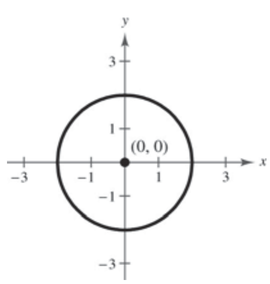
Radius: 3



14. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

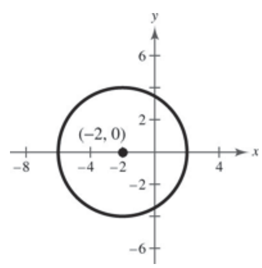


15. $(x + 2)^2 + y^2 = 16$

$(x - (-2))^2 + (y - 0)^2 = 4^2$

Center: $(-2, 0)$

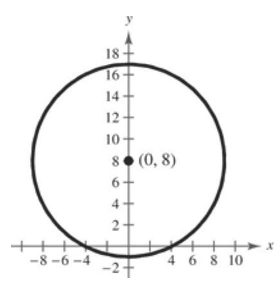
Radius: 4



16. $x^2 + (y - 8)^2 = 81$

Center: $(0, 8)$

Radius: 9



17. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\dfrac{0 + 4}{2}, \dfrac{0 + (-6)}{2}\right) = (2, -3)$

Radius: $r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}$

Standard form: $(x - 2)^2 + (y - (-3))^2 = (\sqrt{13})^2$

$$(x - 2)^2 + (y + 3)^2 = 13$$

18. Endpoints of a diameter: $(-2, -3)$ and $(4, -10)$

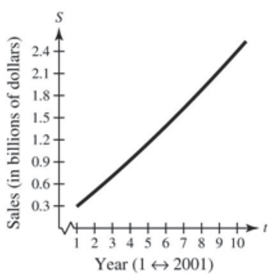
Center: $\left(\dfrac{-2 + 4}{2}, \dfrac{-3 + (-10)}{2}\right) = \left(1, -\dfrac{13}{2}\right)$

Radius: $r = \sqrt{(1 - (-2))^2 + \left(-\dfrac{13}{2} - (-3)\right)^2} = \sqrt{9 + \dfrac{49}{4}} = \sqrt{\dfrac{85}{4}}$

Standard form: $(x - 1)^2 + \left(y - \left(-\dfrac{13}{2}\right)\right)^2 = \left(\sqrt{\dfrac{85}{4}}\right)^2$

$$(x - 1)^2 + \left(y + \frac{13}{2}\right)^2 = \frac{85}{4}$$

19. (a)



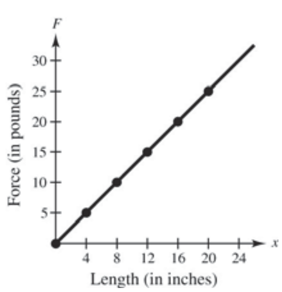
(b) Using the zoom and trace features, sales were $1.2 million during the year 2005.

20. $F = \frac{5}{4}x, 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)



(c) When $x = 10, F = \frac{50}{4} = 12.5$ pounds.

21. $2(x - 2) = 2x - 4$

$2x - 4 = 2x - 4$

$0 = 0$ Identity

All real numbers are solutions.

22. $3(x - 2) + 2x = 2(x + 3)$

$3x - 6 + 2x = 2x + 6$

$3x = 12$

$x = 4$

Conditional equation

23. $2(x + 3) = 2x - 2$

$2x + 6 = 2x - 2$

$6 = -2$ Contradiction

No solution

24. $5(x - 1) - 2x = 3x - 5$

$5x - 5 - 2x = 3x - 5$

$3x - 5 = 3x - 5$

$0 = 0$ Identity

All real numbers are solutions.

25. $8x - 5 = 3x + 20$

$5x = 25$

$x = 5$

26. $7x + 3 = 3x - 17$
$$4x = -20$$
$$x = -5$$

27. $2(x + 5) - 7 = 3(x - 2)$
$$2x + 10 - 7 = 3x - 6$$
$$2x + 3 = 3x - 6$$
$$-x = -9$$
$$x = 9$$

28. $3(x + 3) = 5(1 - x) - 1$
$$3x + 9 = 5 - 5x - 1$$
$$3x + 9 = 4 - 5x$$
$$8x = -5$$
$$x = -\tfrac{5}{8}$$

29. $\dfrac{x}{5} - 3 = \dfrac{2x}{2} + 1$
$$5\left(\frac{x}{5} - 3\right) = (x + 1)5$$
$$x - 15 = 5x + 5$$
$$-4x = 20$$
$$x = -5$$

30. $\dfrac{4x - 3}{6} + \dfrac{x}{4} = x - 2$
$$2(4x - 3) + 3x = 12x - 24$$
$$8x - 6 + 3x = 12x - 24$$
$$-x = -18$$
$$x = 18$$

31. $y = 3x - 1$

x-intercept: $0 = 3x - 1 \Rightarrow x = \tfrac{1}{3}$

y-intercept: $y = 3(0) - 1 \Rightarrow y = -1$

The x-intercept is $\left(\tfrac{1}{3}, 0\right)$ and the y-intercept is $(0, -1)$.

32.
$$y = -5x + 6 \qquad y = -5x + 6$$
$$0 = -5x + 6 \qquad y = -5(0) + 6$$
$$-6 = -5x \qquad y = 6$$
$$\tfrac{6}{5} = x$$

The x-intercept is $\left(\tfrac{6}{5}, 0\right)$ and the y-intercept is $(0, 6)$.

33. $y = 2(x - 4)$

x-intercept: $0 = 2(x - 4) \Rightarrow x = 4$

y-intercept: $y = 2(0 - 4) \Rightarrow y = -8$

The x-intercept is $(4, 0)$ and the y-intercept is $(0, -8)$.

34.
$$y = 4(7x + 1) \qquad y = 4(7x + 1)$$
$$0 = 4(7x + 1) \qquad y = 4[7(0) + 1]$$
$$0 = 28x + 4 \qquad y = 4$$
$$-4 = 28x$$
$$-\tfrac{1}{7} = x$$

The x-intercept is $\left(-\tfrac{1}{7}, 0\right)$ and the y-intercept is $(0, 4)$.

35. $y = -\dfrac{1}{2}x + \dfrac{2}{3}$

x-intercept: $0 = -\dfrac{1}{2}x + \dfrac{2}{3} \Rightarrow x = \dfrac{2/3}{1/2} = \dfrac{4}{3}$

y-intercept: $y = -\dfrac{1}{2}(0) + \dfrac{2}{3} \Rightarrow y = \dfrac{2}{3}$

The x-intercept is $\left(\dfrac{4}{3}, 0\right)$ and the y-intercept is $\left(0, \dfrac{2}{3}\right)$.

36.
$$y = \tfrac{3}{4}x - \tfrac{1}{4} \qquad y = \tfrac{3}{4}x - \tfrac{1}{4}$$
$$0 = \tfrac{3}{4}x - \tfrac{1}{4} \qquad y = \tfrac{3}{4}(0) - \tfrac{1}{4}$$
$$\tfrac{4}{3} \cdot \tfrac{1}{4} = \tfrac{4}{3} \cdot \tfrac{3}{4}x \qquad y = -\tfrac{1}{4}$$
$$\tfrac{1}{3} = x$$

The x-intercept is $\left(\tfrac{1}{3}, 0\right)$ and the y-intercept is $\left(0, -\tfrac{1}{4}\right)$.

37. $244.92 = 2(3.14)(3)^2 + 2(3.14)(3)h$
$$244.92 = 56.52 + 18.84h$$
$$188.40 = 18.84h$$
$$10 = h$$

The height is 10 inches.

38.
$$C = \tfrac{5}{9}F - \tfrac{160}{9}$$
$$\tfrac{5}{9}F = C + \tfrac{160}{9}$$
$$F = \tfrac{9}{5}\left(C + \tfrac{160}{9}\right)$$

For $C = 100°$, $F = \tfrac{9}{5}\left(100 + \tfrac{160}{9}\right) = 212°\text{F}$.

39. *Verbal Model:* September's profit + October's profit = 689,000

Labels: Let x = September's profit. Then $x + 0.12x$ = October's profit.

Equation:
$$\begin{aligned} x + (x + 0.12x) &= 689{,}000 \\ 2.12x &= 689{,}000 \\ x &= 325{,}000 \\ x + 0.12x &= 364{,}000 \end{aligned}$$

So, September profit was \$325,000 and October profit was $325{,}000 + 0.12(325{,}000) = \$364{,}000$.

40. *Model:* (Original price)(1 − discount rate) = (sale price)

Labels: Original price $= 340 + 85 = 425$

Discount rate $= x$

Sale price $= 340$

Equation:
$$\begin{aligned} 425(1 - x) &= 340 \\ 1 - x &= 0.8 \\ x &= 0.2 \end{aligned}$$

The percent discount is 20%.

41. Let x = the number of original investors.

Each person's share is $\dfrac{90{,}000}{x}$. If three more people invest, each person's share is $\dfrac{90{,}000}{x + 3}$.

Since this is \$2500 less than the original cost, we have:

$$\begin{aligned} \frac{90{,}000}{x} - 2500 &= \frac{90{,}000}{x + 3} \\ 90{,}000(x + 3) - 2500x(x + 3) &= 90{,}000x \\ 90{,}000x + 270{,}000 - 2500x^2 - 7500x &= 90{,}000x \\ -2500x^2 - 7500x + 270{,}000 &= 0 \\ -2500(x^2 + 3x - 108) &= 0 \\ -2500(x + 12)(x - 9) &= 0 \end{aligned}$$

$x = -12$, extraneous or $x = 9$

There are currently nine investors.

42.

	Rate	Time	Distance
To work	r	$\dfrac{56}{r}$	56
From work	$r + 8$	$\dfrac{56}{r+8}$	56

$$\text{Time} = \frac{\text{distance}}{\text{rate}}$$

Time to work = time from work + 10 minutes

$$\frac{56}{r} = \frac{56}{r+8} + \frac{1}{6} \quad \text{Convert minutes to portion of an hour.}$$

$$6(r+8)(56) = 6r(56) + r(r+8)$$

$$336r + 2688 = 336r + r^2 + 8r$$

$$0 = r^2 + 8r - 2688$$

$$0 = (r-48)(r+56)$$

Using the positive value for r, we have $r = 48$ miles per hour. The average speed on the trip home was $r + 8 = 56$ miles per hour.

43. Let $x =$ the number of liters of pure antifreeze.

$$30\% \text{ of } (10 - x) + 100\% \text{ of } x = 50\% \text{ of } 10$$

$$0.30(10 - x) + 1.00x = 0.50(10)$$

$$3 - 0.30x + 1.00x = 5$$

$$0.70x = 2$$

$$x = \frac{2}{0.70} = \frac{20}{7} = 2\frac{6}{7} \text{ liters}$$

44. *Model:* $\left(\text{Interest from } 4\tfrac{1}{2}\%\right) + \left(\text{Interest from } 5\tfrac{1}{2}\%\right) = (\text{total interest})$

Labels: Amount invested at $4\frac{1}{2}\% = x$, amount invested at $5\frac{1}{2}\% = 6000 - x$

Interest from $4\frac{1}{2}\% = x(0.045)(1)$, interest from $5\frac{1}{2}\% = (6000 - x)(0.055)(1)$, total interest $= \$3.05$

Equation: $0.045x + 0.055(6000 - x) = 305$

$$0.045x + 330 - 0.055x = 305$$

$$-0.01x = -25$$

$$x = 2500$$

The amount invested at $4\frac{1}{2}\%$ was \$2500 and the amount invested at $5\frac{1}{2}\%$ was $6000 - 2500 = \$3500$.

45.

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi r^2} = h$$

46.

$$E = \frac{1}{2}mv^2$$

$$mv^2 = 2E$$

$$m = \frac{2E}{v^2}$$

47. $15 + x - 2x^2 = 0$

$$0 = 2x^2 - x - 15$$

$$0 = (2x + 5)(x - 3)$$

$$2x + 5 = 0 \Rightarrow x = -\tfrac{5}{2}$$

$$x - 3 = 0 \Rightarrow x = 3$$

48. $2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$

$x - 4 = 0 \Rightarrow x = 4$

49. $6 = 3x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

50. $16x^2 = 25$

$16x^2 - 25 = 0$

$(4x - 5)(4x + 5) = 0$

$4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$

51. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = -13 \pm 5$

$x = -18$ or $x = -8$

52. $(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

53. $x^2 + 12x + 250 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-12 \pm 2\sqrt{11}}{2}$$

$$= -6 \pm \sqrt{11}$$

54. $9x^2 - 12x = 14$

$9x^2 - 12x - 14 = 0$

$$x = \frac{-12 \pm \sqrt{(-12)^2 - 4(9)(-14)}}{2(9)}$$

$$= \frac{-12 \pm 18\sqrt{2}}{18}$$

$$= \frac{2}{3} \pm \sqrt{2}$$

55. $-2x^2 - 5x + 27 = 0$

$2x^2 + 5x - 27 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-27)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{241}}{4}$$

56. $-20 - 3x + 3x^2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

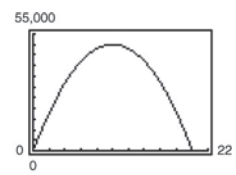
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6}$$

57. $M = 500x(20 - x)$

(a) $500x(20 - x) = 0$ when $x = 0$ feet and $x = 20$ feet.

(b)



(c) The bending moment is greatest when $x = 10$ feet.

58. (a) $h(t) = -16t^2 + 30t + 5.8$

(b) $h(1) = -16 \cdot 1^2 + 30 \cdot 1 + 5.8 = 19.8$ feet

(c) $-16t^2 + 30t + 5.8 = 0$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(5.8)}}{2(-16)}$$

$$= \frac{-30 \pm \sqrt{1271.2}}{-32}$$

$$\approx 2.052 \text{ or } -0.1767$$

The ball will hit the ground in about two seconds.

59. $4 + \sqrt{-9} = 4 + 3i$

60. $3 + \sqrt{-16} = 3 + 4i$

61. $i^2 + 3i = -1 + 3i$

62. $-5i + i^2 = -1 - 5i$

63. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i$

64. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = -2\left(\frac{\sqrt{2}}{2}i\right) = -\sqrt{2}i$

65. $6i(5 - 2i) = 30i - 12i^2 = 12 + 30i$

66.
$$\begin{aligned}(1 + 6i)(5 - 2i) &= 5 - 2i + 30i - 12i^2\\ &= 5 + 28i + 12\\ &= 17 + 28i\end{aligned}$$

67.
$$\begin{aligned}\frac{6 - 5i}{i} &= \frac{6 - 5i}{i} \cdot \frac{-i}{-i}\\ &= \frac{-6i + 5i^2}{-i^2}\\ &= -5 - 6i\end{aligned}$$

68.
$$\begin{aligned}\frac{3 + 2i}{5 + i} &= \frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i}\\ &= \frac{15 - 3i + 10i - 2i^2}{25 - i^2}\\ &= \frac{17 + 7i}{26}\\ &= \frac{17}{26} + \frac{7i}{26}\end{aligned}$$

69.
$$\begin{aligned}\frac{4}{2 - 3i} + \frac{2}{1 + i} &= \frac{4}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} + \frac{2}{1 + i} \cdot \frac{1 - i}{1 - i}\\ &= \frac{8 + 12i}{4 + 9} + \frac{2 - 2i}{1 + 1}\\ &= \frac{8}{13} + \frac{12}{13}i + 1 - i\\ &= \left(\frac{8}{13} + 1\right) + \left(\frac{12}{13}i - i\right)\\ &= \frac{21}{13} - \frac{1}{13}i\end{aligned}$$

70.
$$\begin{aligned}\frac{1}{2 + i} - \frac{5}{1 + 4i} &= \frac{(1 + 4i) - 5(2 + i)}{(2 + i)(1 + 4i)}\\ &= \frac{1 + 4i - 10 - 5i}{2 + 81 + i + 4i^2}\\ &= \frac{-9 - i}{-2 + 9i} \cdot \frac{(-2 - 9i)}{(-2 - 9i)}\\ &= \frac{18 + 81i + 2i + 9i^2}{4 - 81i^2}\\ &= \frac{9 + 83i}{85} = \frac{9}{85} + \frac{83i}{85}\end{aligned}$$

71. $x^2 - 2x + 10 = 0$
$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}\\ &= \frac{2 \pm \sqrt{-36}}{2}\\ &= \frac{2 \pm 6i}{2}\\ &= 1 \pm 3i\end{aligned}$$

72. $x^2 + 6x + 34 = 0$
$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(34)}}{2(1)}\\ &= \frac{-6 \pm \sqrt{-100}}{2}\\ &= \frac{-6 \pm 10i}{2}\\ &= -3 \pm 5i\end{aligned}$$

73. $4x^2 + 4x + 7 = 0$
$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(7)}}{2(4)}\\ &= \frac{-4 \pm \sqrt{-96}}{8}\\ &= \frac{-4 \pm 4\sqrt{6}i}{8}\\ &= -\frac{1}{2} \pm \frac{\sqrt{6}}{2}i\end{aligned}$$

74. $6x^2 + 3x + 27 = 0$
$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-3 \pm \sqrt{3^2 - 4(6)(27)}}{2(6)}\\ &= \frac{-3 \pm \sqrt{-639}}{12}\\ &= \frac{-3 \pm 3i\sqrt{71}}{12} = -\frac{1}{4} \pm \frac{\sqrt{71}}{4}i\end{aligned}$$

75. $5x^4 - 12x^3 = 0$

$x^3(5x - 12) = 0$

$x^3 = 0$ or $5x - 12 = 0$

$x = 0$ or $x = \frac{12}{5}$

76. $4x^3 - 6x^2 = 0$

$x^2(4x - 6) = 0$

$x^2 = 0 \Rightarrow x = 0$

$4x - 6 = 0 \Rightarrow x = \frac{3}{2}$

77. $x^4 - 5x^2 + 6 = 0$

$(x^2 - 2)(x^2 - 3) = 0$

$x^2 - 2 = 0$ or $x^2 - 3 = 0$

$x^2 = 2$ $\quad$ $x^2 = 3$

$x = \pm\sqrt{2}$ $\quad$ $x = \pm\sqrt{3}$

78. $9x^4 + 27x^3 - 4x^2 - 12x = 0$

$9x^3(x + 3) - 4x(x + 3) = 0$

$(x + 3)(9x^3 - 4x) = 0$

$(x + 3)(x)(9x^2 - 4) = 0$

$x + 3 = 0 \Rightarrow x = -3$

$x = 0$

$9x^2 - 4 = 0 \Rightarrow x = \pm\frac{2}{3}$

79. $\sqrt{2x + 3} + \sqrt{x - 2} = 2$

$\left(\sqrt{2x + 3}\right)^2 = \left(2 - \sqrt{x - 2}\right)^2$

$2x + 3 = 4 - 4\sqrt{x - 2} + x - 2$

$x + 1 = -4\sqrt{x - 2}$

$(x + 1)^2 = \left(-4\sqrt{x - 2}\right)^2$

$x^2 + 2x + 1 = 16(x - 2)$

$x^2 - 14x + 33 = 0$

$(x - 3)(x - 11) = 0$

$x = 3$, extraneous or $x = 11$, extraneous

No solution

80. $5\sqrt{x} - \sqrt{x - 1} = 6$

$5\sqrt{x} = 6 + \sqrt{x - 1}$

$25x = 36 + 12\sqrt{x - 1} + x - 1$

$24x - 35 = 12\sqrt{x - 1}$

$576x^2 - 1680x + 1225 = 144(x - 1)$

$576x^2 - 1824x + 1369 = 0$

$$x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)} = \frac{1824 \pm \sqrt{172{,}800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}$$

$$x = \frac{38 + 5\sqrt{3}}{24}$$

$x = \dfrac{38 - 5\sqrt{3}}{24}$, extraneous

81. $(x - 1)^{2/3} - 25 = 0$

$(x - 1)^{2/3} = 25$

$(x - 1)^2 = 25^3$

$x - 1 = \pm\sqrt{25^3}$

$x = 1 \pm 125$

$x = 126$ or $x = -124$

82. $(x + 2)^{3/4} = 27$

$x + 2 = 27^{4/3}$

$x + 2 = 81$

$x = 79$

83.
$$\frac{5}{x} = 1 + \frac{3}{x+2}$$
$$5(x+2) = 1(x)(x+2) + 3x$$
$$5x + 10 = x^2 + 2x + 3x$$
$$10 = x^2$$
$$\pm\sqrt{10} = x$$

84.
$$\frac{6}{x} + \frac{8}{x+5} = 3$$
$$x(x+5)\frac{6}{x} + x(x+5)\frac{8}{x+5} = 3x(x+5)$$
$$6(x+5) + 8x = 3x(x+5)$$
$$14x + 30 = 3x^2 + 15x$$
$$0 = 3x^2 + x - 30$$
$$0 = (3x+10)(x-3)$$
$$0 = 3x + 10 \Rightarrow x = -\frac{10}{3}$$
$$0 = x - 3 \Rightarrow x = 3$$

85. $|x - 5| = 10$

$x - 5 = -10$ or $x - 5 = 10$

$x = -5$ $\qquad$ $x = 15$

86. $|2x + 3| = 7$

$|2x + 3| = 7$ or $2x + 3 = -7$

$2x = 4$ $\qquad$ $2x = -10$

$x = 2$ $\qquad$ $x = -5$

87. $|x^2 - 3| = 2x$ or $x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$ $\qquad$ $x2 + 2x - 3 = 0$

$(x - 3)(x + 1) = 0$ $\qquad$ $(x + 3)(x - 1) = 0$

$x = 3$ or $x = -1$ $\qquad$ $x = -3$ or $x = 1$

The only solutions of the original equation are $x = 3$ or $x = 1$. $(x = 3$ and $x = -1$ are extraneous.$)$

88. $|x^2 - 6| = x$

$x^2 - 6 = x$ or $-(x^2 - 6) = x$

$x^2 - x - 6 = 0$ $\qquad$ $x^2 + x - 6 = 0$

$(x - 3)(x + 2) = 0$ $\qquad$ $(x + 3)(x - 2) = 0$

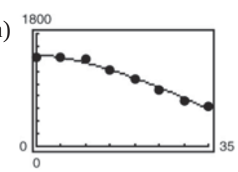
$x - 3 = 0 \Rightarrow x = 3$ $\qquad$ $x - 2 = 0 \Rightarrow x = 2$

$x + 2 = 0 \Rightarrow x = -2$, extraneous $\qquad$ $x + 3 = 0 \Rightarrow x = -3$, extraneous

89.
$$29.95 = 42 - \sqrt{0.001x + 2}$$
$$-12.05 = -\sqrt{0.001x + 2}$$
$$\sqrt{0.001x + 2} = 12.05$$
$$0.001x + 2 = 145.2025$$
$$0.001x = 143.2025$$
$$x = 143{,}202.5$$
$$\approx 143{,}203 \text{ units}$$

90. $N = 1465 - 4.2t^{3/2}$

(a)



The model fits the data well.

(b) There were 800 daily evening newspapers in 1999.

(c)
$$800 = 1465 - 4.2t^{3/2}$$
$$4.2t^{3/2} = 665$$
$$t^{3/2} = \frac{665}{4.2}$$
$$t = \left(\frac{665}{4.2}\right)^{2/3}$$
$$\approx 29.27 \Rightarrow 1999$$

91. Interval: $(-7, 2]$

Inequality: $-7 < x \le 2$

The interval is bounded.

92. Interval: $(4, \infty)$

Inequality: $4 < x < \infty$

The interval is unbounded.

93. Interval: $(-\infty, -10]$

Inequality: $x \le -10$

The interval is unbounded.

94. Interval: $[-2, 2]$

Inequality: $-2 \le x \le 2$

The interval is bounded.

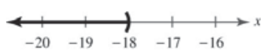
95. $3(x + 2) + 7 < 2x - 5$

$3x + 6 + 7 < 2x - 5$

$3x + 13 < 2x - 5$

$x < -18$

$(-\infty\ -18)$



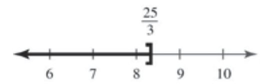
96. $2(x + 7) - 4 \ge 5(x - 3)$

$2x + 10 \ge 5x - 15$

$-3x \ge -25$

$x \le \frac{25}{3}$

$\left(-\infty, \frac{25}{3}\right]$



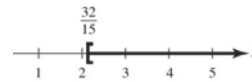
97. $4(5 - 2x) \le \frac{1}{2}(8 - x)$

$20 - 8x \le 4 - \frac{1}{2}x$

$-\frac{15}{2}x \le -16$

$x \ge \frac{32}{15}$

$\left[\frac{32}{15}, \infty\right)$



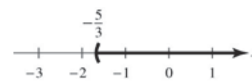
98. $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

$9 - 3x > 4 - 6x$

$3x > -5$

$x > -\frac{5}{3}$

$\left(-\frac{5}{3}, \infty\right)$



99. $|x - 3| > 4$

$x - 3 < -4$ or $x - 3 > 4$

$x < -1$ or $x > 7$

$(-\infty, -1) \cup (7, \infty)$

100. $\left|x - \frac{3}{2}\right| \ge \frac{3}{2}$

$x - \frac{3}{2} \le -\frac{3}{2}$ or $x - \frac{3}{2} \ge \frac{3}{2}$

$x \le 0$ or $x \ge 3$

$(-\infty, 0] \cup [3, \infty)$

101. If the side is 19.3 cm, then with the possible error of 0.5 cm we have:

$18.8 \le \text{side} \le 19.8$

$353.44\text{ cm}^2 \le \text{area} \le 392.04\text{ cm}^2$

102. $125.33x > 92x + 1200$

$33.33x > 1200$

$x > 36$ units

So, the smallest value of x for which the product returns a profit is 37 units.

103. $x^2 - 6x - 27 < 0$

$(x + 3)(x - 9) < 0$

Key numbers: $x = -3, x = 9$

Test intervals: $(-\infty, -3), (-3, 9), (9, \infty)$

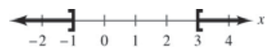
Test: Is $(x + 3)(x - 9) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-3, 9)$.

104. $x^2 - 2x \geq 3$

$x2 - 2x - 3 \geq 0$

$(x - 3)(x + 1) \geq 0$



Key numbers: $x = -1, x = 3$

Test intervals: $(-\infty, -1), (-1, 3), (3, \infty)$

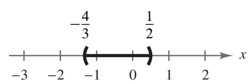
Test: Is $(x - 3)(x + 1) \geq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -1] \cup [3, \infty)$.

105. $6x^2 + 5x < 4$

$6x^2 + 5x - 4 < 0$

$(3x + 4)(2x - 1) < 0$



Key numbers: $x = -\frac{4}{3}, x = \frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

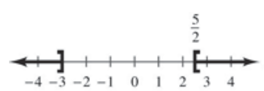
Test: Is $(3x + 4)(2x - 1) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $\left(-\frac{4}{3}, \frac{1}{2}\right)$.

106. $2x^2 + x \geq 15$

$2x^2 + x - 15 \geq 0$

$(2x - 5)(x + 3) \geq 0$



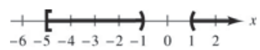
Key numbers: $x = \frac{5}{2}, x = -3$

Test intervals: $(-\infty, -3), \left(-3, \frac{5}{2}\right), \left(\frac{5}{2}, \infty\right)$

Test: Is $(2x - 5)(x + 3) \geq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -3] \cup \left[\frac{5}{2}, \infty\right)$.

107. $$\frac{2}{x+1} \le \frac{3}{x-1}$$



$$\frac{2(x-1) - 3(x+1)}{(x+1)(x-1)} \le 0$$

$$\frac{2x - 2 - 3x - 3}{(x+1)(x-1)} \le 0$$

$$\frac{-(x+5)}{(x+1)(x-1)} \le 0$$

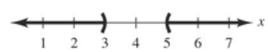
Key numbers: $x = -5, x = -1, x = 1$

Test intervals: $(-5, -1), (-1, 1), (1, \infty)$

Test: Is $\dfrac{-(x+5)}{(x+1)(x-1)} \le 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $[-5, -1) \cup (1, \infty)$.

108. $\dfrac{x-5}{3-x} < 0$



Key numbers: $x = 5, x = 3$

Test intervals: $(-\infty, 3), (3, 5), (5, \infty)$

Test: Is $\dfrac{x-5}{3-x} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, 3) \cup (5, \infty)$.

109.
$$\begin{aligned} 5000(1+r)^2 &> 5500 \\ (1+r)^2 &> 1.1 \\ 1 + r &> 1.0488 \\ r &> 0.0488 \\ r &> 4.9\% \end{aligned}$$

110.
$$\begin{aligned} P &= \frac{1000(1+3t)}{5+t} \\ 2000 &\le \frac{1000(1+3t)}{5+t} \\ 2000(5+t) &\le 1000(1+3t) \\ 10{,}000 + 2000t &\le 1000 + 3000t \\ -1000t &\le -9000 \\ t &\ge 9 \text{ days} \end{aligned}$$

111. False.

$$\sqrt{-18}\sqrt{-2} = \left(\sqrt{18}i\right)\left(\sqrt{2}i\right) = \sqrt{36}i^2 = -6$$

$$\sqrt{(-8)(-2)} = \sqrt{36} = 6$$

112. False. The equation has no real solution.

The solutions are

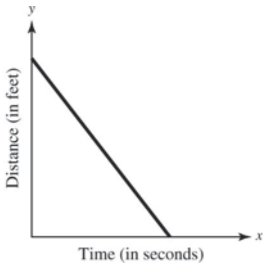
$$\frac{717}{650} \pm \frac{\sqrt{3311}i}{650}.$$

113. Rational equations, equations involving radicals, and absolute value equations, may have "solutions" that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

114. $\left[-\infty, -\frac{36}{11}\right]$ or $[2, \infty)$. *Sample answer:* The first equivalent inequality written is incorrect. It should be $11x + 4 \le -26$. This leads to the solution $\left(-\infty, -\frac{30}{11}\right]$ or $[2, \infty)$.

Problem Solving for Chapter 1

1.



2. (a) $1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

(b) $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$

When $n = 5$: $\frac{1}{2}(5)(6) = 15$

When $n = 8$: $\frac{1}{2}(8)(9) = 36$

When $n = 10$: $\frac{1}{2}(10)(11) = 55$

(c)
$$\frac{1}{2}n(n + 1) = 210$$
$$n(n + 1) = 420$$
$$n^2 + n - 420 = 0$$
$$(n + 21)(n - 20) = 0$$
$$n = -21 \text{ or } n = 20$$

Since n is a natural number, choose $n = 20$.

3. (a) $A = \pi ab$

$a + b = 20 \Rightarrow b = 20 - a$, thus:

$A = \pi a(20 - a)$

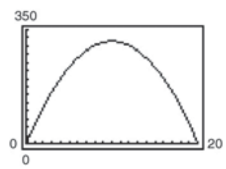
(b)

a	4	7	10	13	16
A	64π	91π	100π	91π	64π

(c)
$$300 = \pi a(20 - a)$$
$$300 = 20\pi a - \pi a^2$$
$$\pi a^2 - 20\pi a + 300 = 0$$
$$a = \frac{20\pi \pm \sqrt{(-20\pi)^2 - 4\pi(300)}}{2\pi}$$
$$= \frac{20\pi \pm \sqrt{400\pi^2 - 1200\pi}}{2\pi}$$
$$= \frac{20\pi \pm 20\sqrt{\pi(\pi - 3)}}{2\pi}$$
$$= 10 \pm \frac{10}{\pi}\sqrt{\pi(\pi - 3)}$$

$a \approx 12.123$ or $a \approx 7.877$

(d)



(e) The a-intercepts occur at $a = 0$ and $a = 20$. Both yield an area of 0. When $a = 0, b = 20$ and you have a vertical line of length 40. Likewise when $a = 20, b = 0$ and you have a horizontal line of length 40. They represent the minimum and maximum values of a.

(f) The maximum value of A is $100\pi \approx 314.159$. This occurs when $a = b = 10$ and the ellipse is actually a circle.

4. $P = 0.00256s^2$

(a) $$\begin{aligned} 0.00256s^2 &= 20 \\ s^2 &= 7812.5 \\ s &\approx 88.4 \text{ miles per hour} \end{aligned}$$

(b) $$\begin{aligned} 0.00256s^2 &= 40 \\ s^2 &= 15625 \\ s &= 125 \text{ miles per hour} \end{aligned}$$

No, actually it can survive wind blowing at $\sqrt{2}$ times the speed found in part (a).

(c) The wind speed in the formula is squared, so a small increase in wind speed could have potentially serious effects on a building.

5. $$h = \left(\sqrt{h_0} - \frac{2\pi d^2\sqrt{3}}{lw}t\right)^2$$

$l = 60''$, $w = 30''$, $h_0 = 25''$, $d = 2''$

$$h = \left(5 - \frac{8\pi\sqrt{3}}{1800}t\right)^2 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

(a) $$12.5 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$\sqrt{12.5} = 5 - \frac{\pi\sqrt{3}}{225}t$$

$$t = \frac{225}{\pi\sqrt{3}}\left(5 - \sqrt{12.5}\right) \approx 60.6 \text{ seconds}$$

(b) $$0 = \left(\sqrt{12.5} - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$t = \frac{225\sqrt{12.5}}{\pi\sqrt{3}} \approx 146.2 \text{ seconds}$$

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.

6. (a) If $x^2 + 9 = (x + m)(x + n)$ then $mn = 9$ and $m + n = 0$.

(b) $m + n = 0 \Rightarrow n = -m$

$m(-m) = 9 \Rightarrow -m^2 = 9 \Rightarrow m^2 = -9$

There is no **integer** m such that m^2 equals a negative number. $x^2 + 9$ cannot be factored over the integers.

7. (a) 5, 12, and 13; 8, 15, and 17

7, 24, and 25

(b) $5 \cdot 12 \cdot 13 = 780$ which is divisible by 3, 4, and 5.

$8 \cdot 15 \cdot 17 = 2040$ which is divisible by 3, 4, and 5.

$7 \cdot 24 \cdot 25 = 4200$ which is also divisible by 3, 4, and 5.

(c) Conjecture: If $a^2 + b^2 = c^2$ where a, b, and c are positive integers, then abc is divisible by 60.

8.

Equation	x_1, x_2	$x_1 + x_2$	$x_1 \cdot x_2$
(a) $x^2 - x - 6 = 0$ $(x + 2)(x - 3) = 0$	$-2, 3$	1	-6
(b) $2x^2 + 5x - 3 = 0$ $(2x - 1)(x + 3) = 0$	$\frac{1}{2}, -3$	$-\frac{5}{2}$	$-\frac{3}{2}$
(c) $4x^2 - 9 = 0$ $(2x + 3)(2x - 3) = 0$	$-\frac{3}{2}, \frac{3}{2}$	0	$-\frac{9}{4}$
(d) $x^2 - 10x + 34 = 0$ $x = 5 \pm 3i$	$5 + 3i, 5 - 3i$	10	34

9. (a) $$S = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2a}{2a} = -\frac{b}{a}$$

(b) $$P = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

10. (a) (i) $\left(\frac{-5 + 5\sqrt{3}i}{2}\right)^3 = 125$

(ii) $\left(\frac{-5 - 5\sqrt{3}i}{2}\right)^3 = 125$

(b) (i) $\left(\frac{-3 + 3\sqrt{3}i}{2}\right)^3 = 27$

(ii) $\left(\frac{-3 - 3\sqrt{3}i}{2}\right)^3 = 27$

(c) (i) The cube roots of 1 are: $1, \frac{-1 \pm \sqrt{3}i}{2}$

(ii) The cube roots of 8 are: $2, -1 \pm \sqrt{3}i$

(iii) The cube roots of 64 are: $4, -2 \pm 2\sqrt{3}i$

11. (a) $$z_m = \frac{1}{z} = \frac{1}{1 + i} = \frac{1}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{1 - i}{2} = \frac{1}{2} - \frac{1}{2}i$$

(b) $$z_m = \frac{1}{z} = \frac{1}{3 - i} = \frac{1}{3 - i} \cdot \frac{3 + i}{3 + i} = \frac{3 + i}{10} = \frac{3}{10} + \frac{1}{10}i$$

(c) $$z_m = \frac{1}{z} = \frac{1}{-2 + 8i} = \frac{1}{-2 + 8i} \cdot \frac{-2 - 8i}{-2 - 8i} = \frac{-2 - 8i}{68} = -\frac{1}{34} - \frac{2}{17}i$$

12. $(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

13. (a) $c = 1$

The terms are: $i, -1 + i, -i, -1 + i, -i, -1 + i, -i, -1 + i, -i, \ldots$

The sequence is bounded so $c = i$ *is* in the Mandelbrot Set.

(b) $c = -2$

The terms are: $1 + i, 1 + 3i, -7 + 7i, 1 - 97i, -9407 - 1931i, \ldots$

The sequence is unbounded so $c = 1 + i$ *is not* in the Mandelbrot Set.

(c) $c = -2$

The terms are: $-2, 2, 2, 2, 2, \ldots$

The sequence is bounded so $c = -2$ *is* in the Mandelbrot Set.

14.
$$4\sqrt{x} = 2x + k$$
$$2x - 4\sqrt{x} + k = 0 \qquad \text{Complete the square.}$$
$$x - 2\sqrt{x} = -\frac{k}{2}$$
$$x - 2\sqrt{x} + 1 = 1 - \frac{k}{2}$$
$$\left(\sqrt{x} - 1\right)^2 = 1 - \frac{k}{2}$$

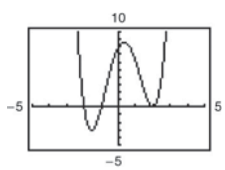
Number of solutions (real)	Some k-values
2	−1, 0, 1
1	2 only
0	3, 4, 5

This equation will have two solutions when $1 - \frac{k}{2} > 0$ or when $k < 2$.

This equation will have one solution when $1 - \frac{k}{2} = 0$ or when $k = 2$.

This equation will have no solutions when $1 - \frac{k}{2} < 0$ or when $k > 2$.

15. $y = x^4 - x^3 - 6x^2 + 4x + 8 = (x - 2)^2(x + 1)(x + 2)$



From the graph you see that $x^4 - x^3 - 6x^2 + 4x + 8 > 0$ on the intervals $(-\infty, -2) \cup (-1, 2) \cup (2, \infty)$.

Practice Test for Chapter 1

1. Graph $3x - 5y = 15$.

2. Graph $y = \sqrt{9 - x}$.

3. Solve $5x + 4 = 7x - 8$.

4. Solve $\frac{x}{3} - 5 = \frac{x}{5} + 1$.

5. Solve $\frac{3x + 1}{6x - 7} = \frac{2}{5}$.

6. Solve $(x - 3)^2 + 4 = (x + 1)^2$.

7. Solve $A = \frac{1}{2}(a + b)h$ for a.

8. 301 is what percent of 4300?

9. Cindy has $6.05 in quarter and nickels. How many of each coin does she have if there are 53 coins in all?

10. Ed has $15,000 invested in two fund paying $9\frac{1}{2}\%$ and 11% simple interest, respectively. How much is invested in each if the yearly interest is $1582.50?

11. Solve $28 + 5x - 3x^2 = 0$ by factoring.

12. Solve $(x - 2)^2 = 24$ by taking the square root of both sides.

13. Solve $x^2 - 4x - 9 = 0$ by completing the square.

14. Solve $x^2 + 5x - 1 = 0$ by the Quadratic Formula.

15. Solve $3x^2 - 2x + 4 = 0$ by the Quadratic Formula.

16. The perimeter of a rectangle is 1100 feet. Find the dimensions so that the enclosed area will be 60,000 square feet.

17. Find two consecutive even positive integers whose product is 624.

18. Solve $x^3 - 10x^2 + 24x = 0$ by factoring.

19. Solve $\sqrt[3]{6 - x} = 4$.

20. Solve $(x^2 - 8)^{2/5} = 4$.

21. Solve $x^4 - x^2 - 12 = 0$.

22. Solve $4 - 3x > 16$.

23. Solve $\left|\frac{x - 3}{2}\right| < 5$.

24. Solve $\frac{x + 1}{x - 3} < 2$.

25. Solve $|3x - 4| \geq 9$.