Complete Solutions Manual to Accompany

Numerical Analysis

TENTH EDITION

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Preface

This Instructor's Manual for the Tenth edition of Numerical Analysis by Burden, Faires, and Burden contains solutions to all the exercises in the book. Although the answers to the odd exercises are also in the back of the text, we have found that users of the book appreciate having all the solutions in one source. In addition, the results listed in this Instructor's Manual often go beyond those given in the back of the book. For example, we do not place the long solutions to theoretical and applied exercises in the book. You will find them here.

A Student Study Guide for the Tenth edition of Numerical Analysis is also available and the solutions given in the Guide are generally more detailed than those in the Instructor's Manual.

We have added a number of exercises to the text that can be implemented in any generic computer algebra system such as Maple, Matlab, Mathematica, Sage, and FreeMat. In our recent teaching of the course we found that students understood the concepts better when they worked through the algorithms step-by-step, but let the computer algebra system do the tedious computation.

It has been our practice to include in our Numerical Analysis book structured algorithms of all the techniques discussed in the text. The algorithms are given in a form that can be coded in any appropriate programming language, by students with even a minimal amount of programming expertise.

At our companion website for the book,

https://sites.google.com/site/numericalanalysis1burden/

you will find all the algorithms written in the programming languages FORTRAN, Pascal, C, Java, and in the Computer Algebra Systems, Maple, MATLAB, and Mathematica. For the Tenth edition, we have added new Maple programs to reflect the *NumericalAnalysis* package.

The companion website also contains additional information about the book and will be updated regularly to reflect any modifications that might be made. For example, we will place there any responses to questions from users of the book concerning interpretations of the exercises and appropriate applications of the techniques. We also have a set of PowerPoint files for many of the methods in the book. Many

of these files were created by Professor John Carroll of Dublin City University and several were developed by Dr. Annette M. Burden of Youngstown State University.

We hope our supplement package provides flexibility for instructors teaching Numerical Analysis. If you have any suggestions for improvements that can be incorporated into future editions of the book or the supplements, we would be most grateful to receive your comments. We can be most easily contacted by electronic mail at the addresses listed below.

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Mathematical Preliminaries

Exercise Set 1.1, page 14

1. For each part, $f \in C[a,b]$ on the given interval. Since $f(a)$ and $f(b)$ are of opposite sign, the Intermediate Value Theorem implies that a number c exists with $f(c) = 0$.

2. (a) $f(x) = \sqrt{(x)} - \cos x; f(0) = -1 < 0, f(1) = 1 - \cos 1 > 0.45 > 0$; Intermediate Value Theorem implies there is a c in $(0,1)$ such that $f(c) = 0$.
   (b) $f(x) = e^x - x^2 + 3x - 2; f(0) = -1 < 0, f(1) = e > 0$; Intermediate Value Theorem implies there is a c in $(0,1)$ such that $f(c) = 0$.
   (c) $f(x) = -3\tan(2x) + x; f(0) = 0$ so there is a c in $[0,1]$ such that $f(c) = 0$.
   (d) $f(x) = \ln x - x^2 + \frac{5}{2}x - 1; f(\frac{1}{2}) = -\ln 2 < 0, f(1) = \frac{1}{2} > 0$; Intermediate Value Theorem implies there is a c in $(\frac{1}{2}, 1)$ such that $f(c) = 0$.

3. For each part, $f \in C[a,b]$, f' exists on (a,b) and $f(a) = f(b) = 0$. Rolle's Theorem implies that a number c exists in (a,b) with $f'(c) = 0$. For part (d), we can use $[a,b] = [-1,0]$ or $[a,b] = [0,2]$.

4. (a) $[0,1]$
   (b) $[0,1]$, $[4,5]$, $[-1,0]$
   (c) $[-2,-2/3]$, $[0,1]$, $[2,4]$
   (d) $[-3,-2]$, $[-1,-0.5]$, and $[-0.5,0]$

5. The maximum value for $|f(x)|$ is given below.
   (a) 0.4620981
   (b) 0.8
   (c) 5.164000
   (d) 1.582572

6. (a) $f(x) = \frac{2x}{x^2+1}; 0 \leq x \leq 2; f(x) \geq 0$ on $[0,2], f'(1) = 0, f(0) = 0, f(1) = 1, f(2) = \frac{4}{5}, \max_{0 \leq x \leq 2} |f(x)| = 1$.
   (b) $f(x) = x^2\sqrt{4-x}; 0 \leq x \leq 4; f'(0) = 0, f'(3.2) = 0, f(0) = 0, f(3.2) = 9.158934436, f(4) = 0, \max_{0 \leq x \leq 4} |f(x)| = 9.158934436$.
   (c) $f(x) = x^3 - 4x + 2; 1 \leq x \leq 2; f'(\frac{2\sqrt{3}}{3}) = 0, f'(1) = -1, f(\frac{2\sqrt{3}}{3}) = -1.079201435, f(2) = 2, \max_{1 \leq x \leq 2} |f(x)| = 2$.

(d) $f(x) = x\sqrt{3-x^2}; 0 \le x \le 1; f'(\sqrt{\frac{3}{2}}) = 0, \sqrt{\frac{3}{2}}$ not in $[0,1], f(0) = 0, f(1) = \sqrt{2}, \max_{0\le x\le 1}|f(x)| = \sqrt{2}$.

7. For each part, $f \in C[a,b]$, f' exists on (a,b) and $f(a) = f(b) = 0$. Rolle's Theorem implies that a number c exists in (a,b) with $f'(c) = 0$. For part (d), we can use $[a,b] = [-1,0]$ or $[a,b] = [0,2]$.

8. Suppose p and q are in $[a,b]$ with $p \ne q$ and $f(p) = f(q) = 0$. By the Mean Value Theorem, there exists $\xi \in (a,b)$ with
$$f(p) - f(q) = f'(\xi)(p-q).$$
But, $f(p) - f(q) = 0$ and $p \ne q$. So $f'(\xi) = 0$, contradicting the hypothesis.

9. (a) $P_2(x) = 0$
   (b) $R_2(0.5) = 0.125$; actual error $= 0.125$
   (c) $P_2(x) = 1 + 3(x-1) + 3(x-1)^2$
   (d) $R_2(0.5) = -0.125$; actual error $= -0.125$

10. $P_3(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

x	0.5	0.75	1.25	1.5
$P_3(x)$	1.2265625	1.3310547	1.5517578	1.6796875
$\sqrt{x+1}$	1.2247449	1.3228757	1.5	1.5811388
$\lvert\sqrt{x+1} - P_3(x)\rvert$	0.0018176	0.0081790	0.0517578	0.0985487

11. Since
$$P_2(x) = 1 + x \quad \text{and} \quad R_2(x) = \frac{-2e^{\xi}(\sin\xi + \cos\xi)}{6}x^3$$
for some ξ between x and 0, we have the following:
   (a) $P_2(0.5) = 1.5$ and $|f(0.5) - P_2(0.5)| \le 0.0932$;
   (b) $|f(x) - P_2(x)| \le 1.252$;
   (c) $\int_0^1 f(x)\,dx \approx 1.5$;
   (d) $|\int_0^1 f(x)\,dx - \int_0^1 P_2(x)\,dx| \le \int_0^1 |R_2(x)|\,dx \le 0.313$, and the actual error is 0.122.

12. $P_2(x) = 1.461930 + 0.617884\left(x - \frac{\pi}{6}\right) - 0.844046\left(x - \frac{\pi}{6}\right)^2$ and $R_2(x) = -\frac{1}{3}e^{\xi}(\sin\xi + \cos\xi)\left(x - \frac{\pi}{6}\right)^3$ for some ξ between x and $\frac{\pi}{6}$.
   (a) $P_2(0.5) = 1.446879$ and $f(0.5) = 1.446889$. An error bound is 1.01×10^{-5}, and the actual error is 1.0×10^{-5}.
   (b) $|f(x) - P_2(x)| \le 0.135372$ on $[0,1]$
   (c) $\int_0^1 P_2(x)\,dx = 1.376542$ and $\int_0^1 f(x)\,dx = 1.378025$
   (d) An error bound is 7.403×10^{-3}, and the actual error is 1.483×10^{-3}.

13. $P_3(x) = (x-1)^2 - \frac{1}{2}(x-1)^3$
    (a) $P_3(0.5) = 0.312500$, $f(0.5) = 0.346574$. An error bound is $0.291\overline{6}$, and the actual error is 0.034074.
    (b) $|f(x) - P_3(x)| \le 0.291\overline{6}$ on $[0.5, 1.5]$
    (c) $\int_{0.5}^{1.5} P_3(x)\,dx = 0.08\overline{3}$, $\int_{0.5}^{1.5}(x-1)\ln x\,dx = 0.088020$
    (d) An error bound is $0.058\overline{3}$, and the actual error is 4.687×10^{-3}.

14. (a) $P_3(x) = -4 + 6x - x^2 - 4x^3$; $P_3(0.4) = -2.016$
    (b) $|R_3(0.4)| \le 0.05849$; $|f(0.4) - P_3(0.4)| = 0.013365367$
    (c) $P_4(x) = -4 + 6x - x^2 - 4x^3$; $P_4(0.4) = -2.016$
    (d) $|R_4(0.4)| \le 0.01366$; $|f(0.4) - P_4(0.4)| = 0.013365367$

15. $P_4(x) = x + x^3$
    (a) $|f(x) - P_4(x)| \le 0.012405$
    (b) $\int_0^{0.4} P_4(x)\,dx = 0.0864$, $\int_0^{0.4} xe^{x^2}\,dx = 0.086755$
    (c) 8.27×10^{-4}
    (d) $P_4'(0.2) = 1.12$, $f'(0.2) = 1.124076$. The actual error is 4.076×10^{-3}.

16. First we need to convert the degree measure for the sine function to radians. We have $180° = \pi$ radians, so $1° = \frac{\pi}{180}$ radians. Since,

$$f(x) = \sin x, \quad f'(x) = \cos x, \quad f''(x) = -\sin x, \quad \text{and} \quad f'''(x) = -\cos x,$$

we have $f(0) = 0$, $f'(0) = 1$, and $f''(0) = 0$.

The approximation $\sin x \approx x$ is given by

$$f(x) \approx P_2(x) = x, \quad \text{and} \quad R_2(x) = -\frac{\cos\xi}{3!}x^3.$$

If we use the bound $|\cos\xi| \le 1$, then

$$\left|\sin\frac{\pi}{180} - \frac{\pi}{180}\right| = \left|R_2\left(\frac{\pi}{180}\right)\right| = \left|\frac{-\cos\xi}{3!}\left(\frac{\pi}{180}\right)^3\right| \le 8.86 \times 10^{-7}.$$

17. Since $42° = 7\pi/30$ radians, use $x_0 = \pi/4$. Then

$$\left|R_n\left(\frac{7\pi}{30}\right)\right| \le \frac{\left(\frac{\pi}{4} - \frac{7\pi}{30}\right)^{n+1}}{(n+1)!} < \frac{(0.053)^{n+1}}{(n+1)!}.$$

For $|R_n(\frac{7\pi}{30})| < 10^{-6}$, it suffices to take $n = 3$. To 7 digits,

$$\cos 42° = 0.7431448 \quad \text{and} \quad P_3(42°) = P_3(\frac{7\pi}{30}) = 0.7431446,$$

so the actual error is 2×10^{-7}.

18. $P_n(x) = \sum_{k=0}^{n} x^k,\ n \geq 19$

19. $P_n(x) = \displaystyle\sum_{k=0}^{n} \frac{1}{k!} x^k,\ n \geq 7$

20. For n odd, $P_n(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots + \frac{1}{n}(-1)^{(n-1)/2}x^n$. For n even, $P_n(x) = P_{n-1}(x)$.

21. A bound for the maximum error is 0.0026.

22. For $x < 0$, $f(x) < 2x + k < 0$, provided that $x < -\frac{1}{2}k$. Similarly, for $x > 0$, $f(x) > 2x + k > 0$, provided that $x > -\frac{1}{2}k$. By Theorem 1.11, there exists a number c with $f(c) = 0$. If $f(c) = 0$ and $f(c') = 0$ for some $c' \neq c$, then by Theorem 1.7, there exists a number p between c and c' with $f'(p) = 0$. However, $f'(x) = 3x^2 + 2 > 0$ for all x.

23. Since $R_2(1) = \frac{1}{6}e^{\xi}$, for some ξ in $(0, 1)$, we have $|E - R_2(1)| = \frac{1}{6}|1 - e^{\xi}| \leq \frac{1}{6}(e - 1)$.

24. (a) Use the series

$$e^{-t^2} = \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{k!} \quad \text{to integrate} \quad \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt,$$

and obtain the result.

(b) We have

$$\begin{aligned}\frac{2}{\sqrt{\pi}} e^{-x^2} \sum_{k=0}^{\infty} \frac{2^k x^{2k+1}}{1 \cdot 3 \cdots (2k+1)} &= \frac{2}{\sqrt{\pi}} \left[1 - x^2 + \frac{1}{2}x^4 - \frac{1}{6}x^7 + \frac{1}{24}x^8 + \cdots \right] \\ &\quad \cdot \left[x + \frac{2}{3}x^3 + \frac{4}{15}x^5 + \frac{8}{105}x^7 + \frac{16}{945}x^9 + \cdots \right] \\ &= \frac{2}{\sqrt{\pi}} \left[x - \frac{1}{3}x^3 + \frac{1}{10}x^5 - \frac{1}{42}x^7 + \frac{1}{216}x^9 + \cdots \right] = \text{erf}\ (x)\end{aligned}$$

(c) 0.8427008

(d) 0.8427069

(e) The series in part (a) is alternating, so for any positive integer n and positive x we have the bound

$$\left| \text{erf}(x) - \frac{2}{\sqrt{\pi}} \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)k!} \right| < \frac{x^{2n+3}}{(2n+3)(n+1)!}\ .$$

We have no such bound for the positive term series in part (b).

25. (a) $P_n^{(k)}(x_0) = f^{(k)}(x_0)$ for $k = 0, 1, \ldots, n$. The shapes of P_n and f are the same at x_0.

(b) $P_2(x) = 3 + 4(x - 1) + 3(x - 1)^2$.

26. (a) The assumption is that $f(x_i) = 0$ for each $i = 0, 1, \ldots, n$. Applying Rolle's Theorem on each on the intervals $[x_i, x_{i+1}]$ implies that for each $i = 0, 1, \ldots, n-1$ there exists a number z_i with $f'(z_i) = 0$. In addition, we have

$$a \leq x_0 < z_0 < x_1 < z_1 < \cdots < z_{n-1} < x_n \leq b.$$

(b) Apply the logic in part (a) to the function $g(x) = f'(x)$ with the number of zeros of g in $[a, b]$ reduced by 1. This implies that numbers w_i, for $i = 0, 1, \ldots, n-2$ exist with

$$g'(w_i) = f''(w_i) = 0, \quad \text{and} \quad a < z_0 < w_0 < z_1 < w_1 < \cdots w_{n-2} < z_{n-1} < b.$$

(c) Continuing by induction following the logic in parts (a) and (b) provides $n+1-j$ distinct zeros of $f^{(j)}$ in $[a, b]$.

(d) The conclusion of the theorem follows from part (c) when $j = n$, for in this case there will be (at least) $(n+1) - n = 1$ zero in $[a, b]$.

27. First observe that for $f(x) = x - \sin x$ we have $f'(x) = 1 - \cos x \geq 0$, because $-1 \leq \cos x \leq 1$ for all values of x.

(a) The observation implies that $f(x)$ is non-decreasing for all values of x, and in particular that $f(x) > f(0) = 0$ when $x > 0$. Hence for $x \geq 0$, we have $x \geq \sin x$, and $|\sin x| = \sin x \leq x = |x|$.

(b) When $x < 0$, we have $-x > 0$. Since $\sin x$ is an odd function, the fact (from part (a)) that $\sin(-x) \leq (-x)$ implies that $|\sin x| = -\sin x \leq -x = |x|$.

As a consequence, for all real numbers x we have $|\sin x| \leq |x|$.

28. (a) Let x_0 be any number in $[a, b]$. Given $\epsilon > 0$, let $\delta = \epsilon/L$. If $|x - x_0| < \delta$ and $a \leq x \leq b$, then $|f(x) - f(x_0)| \leq L|x - x_0| < \epsilon$.

(b) Using the Mean Value Theorem, we have

$$|f(x_2) - f(x_1)| = |f'(\xi)||x_2 - x_1|,$$

for some ξ between x_1 and x_2, so

$$|f(x_2) - f(x_1)| \leq L|x_2 - x_1|.$$

(c) One example is $f(x) = x^{1/3}$ on $[0, 1]$.

29. (a) The number $\frac{1}{2}(f(x_1) + f(x_2))$ is the average of $f(x_1)$ and $f(x_2)$, so it lies between these two values of f. By the Intermediate Value Theorem 1.11 there exist a number ξ between x_1 and x_2 with

$$f(\xi) = \frac{1}{2}(f(x_1) + f(x_2)) = \frac{1}{2}f(x_1) + \frac{1}{2}f(x_2).$$

(b) Let $m = \min\{f(x_1), f(x_2)\}$ and $M = \max\{f(x_1), f(x_2)\}$. Then $m \leq f(x_1) \leq M$ and $m \leq f(x_2) \leq M$, so

$$c_1 m \leq c_1 f(x_1) \leq c_1 M \quad \text{and} \quad c_2 m \leq c_2 f(x_2) \leq c_2 M.$$

Thus

$$(c_1 + c_2)m \leq c_1 f(x_1) + c_2 f(x_2) \leq (c_1 + c_2)M$$

and

$$m \leq \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2} \leq M.$$

By the Intermediate Value Theorem 1.11 applied to the interval with endpoints x_1 and x_2, there exists a number ξ between x_1 and x_2 for which

$$f(\xi) = \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2}.$$

(c) Let $f(x) = x^2 + 1$, $x_1 = 0$, $x_2 = 1$, $c_1 = 2$, and $c_2 = -1$. Then for all values of x,

$$f(x) > 0 \quad \text{but} \quad \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2} = \frac{2(1) - 1(2)}{2 - 1} = 0.$$

30. (a) Since f is continuous at p and $f(p) \neq 0$, there exists a $\delta > 0$ with

$$|f(x) - f(p)| < \frac{|f(p)|}{2},$$

for $|x - p| < \delta$ and $a < x < b$. We restrict δ so that $[p - \delta, p + \delta]$ is a subset of $[a, b]$. Thus, for $x \in [p - \delta, p + \delta]$, we have $x \in [a, b]$. So

$$-\frac{|f(p)|}{2} < f(x) - f(p) < \frac{|f(p)|}{2} \quad \text{and} \quad f(p) - \frac{|f(p)|}{2} < f(x) < f(p) + \frac{|f(p)|}{2}.$$

If $f(p) > 0$, then

$$f(p) - \frac{|f(p)|}{2} = \frac{f(p)}{2} > 0, \quad \text{so} \quad f(x) > f(p) - \frac{|f(p)|}{2} > 0.$$

If $f(p) < 0$, then $|f(p)| = -f(p)$, and

$$f(x) < f(p) + \frac{|f(p)|}{2} = f(p) - \frac{f(p)}{2} = \frac{f(p)}{2} < 0.$$

In either case, $f(x) \neq 0$, for $x \in [p - \delta, p + \delta]$.

(b) Since f is continuous at p and $f(p) = 0$, there exists a $\delta > 0$ with

$$|f(x) - f(p)| < k, \quad \text{for} \quad |x - p| < \delta \quad \text{and} \quad a < x < b.$$

We restrict δ so that $[p - \delta, p + \delta]$ is a subset of $[a, b]$. Thus, for $x \in [p - \delta, p + \delta]$, we have

$$|f(x)| = |f(x) - f(p)| < k.$$

Exercise Set 1.2, page 28

1. We have

	Absolute error	Relative error
(a)	0.001264	4.025×10^{-4}
(b)	7.346×10^{-6}	2.338×10^{-6}
(c)	2.818×10^{-4}	1.037×10^{-4}
(d)	2.136×10^{-4}	1.510×10^{-4}

2. We have

	Absolute error	Relative error
(a)	2.647×10^{1}	1.202×10^{-3}arule
(b)	1.454×10^{1}	1.050×10^{-2}
(c)	420	1.042×10^{-2}
(d)	3.343×10^{3}	9.213×10^{-3}

3. The largest intervals are
   (a) (149.85,150.15)
   (b) (899.1, 900.9)
   (c) (1498.5, 1501.5)
   (d) (89.91,90.09)

4. The largest intervals are:
   (a) $(3.1412784, 3.1419068)$
   (b) $(2.7180100, 2.7185536)$
   (c) $(1.4140721, 1.4143549)$
   (d) $(1.9127398, 1.9131224)$

5. The calculations and their errors are:
   (a) (i) 17/15 (ii) 1.13 (iii) 1.13 (iv) both 3×10^{-3}
   (b) (i) 4/15 (ii) 0.266 (iii) 0.266 (iv) both 2.5×10^{-3}
   (c) (i) 139/660 (ii) 0.211 (iii) 0.210 (iv) 2×10^{-3}, 3×10^{-3}
   (d) (i) 301/660 (ii) 0.455 (iii) 0.456 (iv) 2×10^{-3}, 1×10^{-4}

6. We have

	Approximation	Absolute error	Relative error
(a)	134	0.079	5.90×10^{-4}
(b)	133	0.499	3.77×10^{-3}
(c)	2.00	0.327	0.195
(d)	1.67	0.003	1.79×10^{-3}

7. We have

	Approximation	Absolute error	Relative error
(a)	1.80	0.154	0.0786
(b)	-15.1	0.0546	3.60×10^{-3}
(c)	0.286	2.86×10^{-4}	10^{-3}
(d)	23.9	0.058	2.42×10^{-3}

8. We have

	Approximation	Absolute error	Relative error
(a)	1.986	0.03246	0.01662
(b)	-15.16	0.005377	3.548×10^{-4}
(c)	0.2857	1.429×10^{-5}	5×10^{-5}
(d)	23.96	1.739×10^{-3}	7.260×10^{-5}

9. We have

	Approximation	Absolute error	Relative error
(a)	3.55	1.60	0.817
(b)	-15.2	0.0454	0.00299
(c)	0.284	0.00171	0.00600
(d)	0	0.02150	1

10. We have

	Approximation	Absolute error	Relative error
(a)	1.983	0.02945	0.01508
(b)	-15.15	0.004622	3.050×10^{-4}
(c)	0.2855	2.143×10^{-4}	7.5×10^{-4}
(d)	23.94	0.018261	7.62×10^{-4}

11. We have

	Approximation	Absolute error	Relative error
(a)	3.14557613	3.983×10^{-3}	1.268×10^{-3}
(b)	3.14162103	2.838×10^{-5}	9.032×10^{-6}

12. We have

	Approximation	Absolute error	Relative error
(a)	2.7166667	0.0016152	5.9418×10^{-4}
(b)	2.718281801	2.73×10^{-8}	1.00×10^{-8}

13. (a) We have

$$\lim_{x\to 0} \frac{x\cos x - \sin x}{x - \sin x} = \lim_{x\to 0} \frac{-x\sin x}{1-\cos x} = \lim_{x\to 0} \frac{-\sin x - x\cos x}{\sin x} = \lim_{x\to 0} \frac{-2\cos x + x\sin x}{\cos x} = -2$$

(b) $f(0.1) \approx -1.941$

(c) $\dfrac{x(1-\frac{1}{2}x^2) - (x - \frac{1}{6}x^3)}{x - (x - \frac{1}{6}x^3)} = -2$

(d) The relative error in part (b) is 0.029. The relative error in part (c) is 0.00050.

14. (a) $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x}}{x} = \lim_{x\to 0} \frac{e^x + e^{-x}}{1} = 2$

(b) $f(0.1) \approx 2.05$

(c) $\frac{1}{x}\left(\left(1+x+\frac{1}{2}x^2+\frac{1}{6}x^3\right)-\left(1-x+\frac{1}{2}x^2-\frac{1}{6}x^3\right)\right)=\frac{1}{x}\left(2x+\frac{1}{3}x^3\right)=2+\frac{1}{3}x^2$; using three-digit rounding arithmetic and $x = 0.1$, we obtain 2.00.

(d) The relative error in part (b) is $= 0.0233$. The relative error in part (c) is $= 0.00166$.

15.

	x_1	Absolute error	Relative error	x_2	Absolute error	Relative error
(a)	92.26	0.01542	1.672×10^{-4}	0.005419	6.273×10^{-7}	1.157×10^{-4}
(b)	0.005421	1.264×10^{-6}	2.333×10^{-4}	-92.26	4.580×10^{-3}	4.965×10^{-5}
(c)	10.98	6.875×10^{-3}	6.257×10^{-4}	0.001149	7.566×10^{-8}	6.584×10^{-5}
(d)	-0.001149	7.566×10^{-8}	6.584×10^{-5}	-10.98	6.875×10^{-3}	6.257×10^{-4}

16.

	Approximation for x_1	Absolute error	Relative error
(a)	1.903	6.53518×10^{-4}	3.43533×10^{-4}
(b)	-0.07840	8.79361×10^{-6}	1.12151×10^{-4}
(c)	1.223	1.29800×10^{-4}	1.06144×10^{-4}
(d)	6.235	1.7591×10^{-3}	2.8205×10^{-4}

	Approximation for x_2	Absolute error	Relative error
(a)	0.7430	4.04830×10^{-4}	5.44561
(b)	-4.060	3.80274×10^{-4}	9.36723×10^{-5}
(c)	-2.223	1.2977×10^{-4}	5.8393×10^{-5}
(d)	-0.3208	1.2063×10^{-4}	3.7617×10^{-4}

17.

	Approximation for x_1	Absolute error	Relative error
(a)	92.24	0.004580	4.965×10^{-5}
(b)	0.005417	2.736×10^{-6}	5.048×10^{-4}
(c)	10.98	6.875×10^{-3}	6.257×10^{-4}
(d)	-0.001149	7.566×10^{-8}	6.584×10^{-5}